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ORIGINAL PAPER

Adaptive sliding mode control of dynamic system using RBF neural network

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Abstract This paper presents a robust adaptive sliding mode control strategy using radial basis function (RBF) neural network (NN) for a class of time varying system in the presence of model uncertainties and external disturbance. Adaptive RBF neural network controller that can learn the unknown upper bound of model uncertainties and external disturbances is incorporated into the adaptive sliding mode control system in the same Lyapunov framework. The proposed adaptive sliding mode controller can on line update the estimates of system dynamics. The asymptotical stability of the closed-loop system, the convergence of the neural network weight-updating process, and the boundedness of the neural network weight estimation errors can be strictly guaranteed. Numerical simulation for a MEMS triaxial angular velocity sensor is investigated to verify the effectiveness of the proposed adaptive RBF sliding mode control scheme.

Keywords Radial basis function · Adaptive neural network · Sliding mode control

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1 Introduction

Dynamic systems such as robot manipulators have unknown and time varying. nonlinearities. Conventional feedback controllers do not have good performance and robustness when facing with the unknown nonlinearities and external disturbances. Adaptive control schemes have been applied to various dynamic systems since adaptive control schemes can automatically adjust parameters of the controller according to the changing system dynamics. Sliding mode control is a robust control technique which has many attractive features such as robustness to parameter variations and insensitivity to external disturbance. Adaptive sliding mode control has the advantages of combining the robustness of sliding mode method with the tracking capability of adaptive control strategies. However, adaptive control depends on explicit model structure, sliding mode control needs the information of upper bound of model uncertainties and external disturbances and has chattering in practical applications.

Model uncertainties require the controller to be either adaptive or robust to these model uncertainties. Intelligent control approaches such as neural network and fuzzy control have ability to approximate nonlinear systems. Therefore, intelligent control approaches have been applied to represent complex plants and construct advanced controllers. Adaptive fuzzy sliding mode control schemes have been developed for robot manipulators [1, 2]. Lewis et al. [3] proposed neural network methodologies for robot manipulator. Neural network has the capability to approximate any nonlinear function over the compact input space. Therefore,neural network's learning ability to approximate arbitrary nonlinear functions makes it a useful tool for adaptive application. Neural adaptive sliding mode control technologies have been applied to the nonlinear dynamic systems [4–7]. Adaptive neural sliding mode controllers for robotic manipulators have been proposed in [4–6]. Adaptive neural controllers for robot manipulators and the magnetic levitation system have been developed in [7] and [8], respectively. Chien et al. [9] developed a robust adaptive controller design for a class of uncertain nonlinear systems using online T–S fuzzy-neural modeling approach, Dierks and Jagannathan [10] designed neural network output feedback controller for robot formation, Man et al. [11, 12] derived adaptive tracking controllers using neural networks for robot manipulator and a class of nonlinear systems. Wen and Liu [13] developed adaptive fuzzy-neural tracking control for uncertain nonlinear discrete-time systems in the NARMAX form. Wang et al. [14] developed a direct adaptive neural control for strict-feedback stochastic nonlinear systems. Forouzantabar et al. [15] proposed adaptive neural network control of bilateral teleoperation with constant time delay. Zhang and Ge [16] investigated adaptive neural network tracking control of MIMO nonlinear systems with unknown dead zones. Fei [17] derived robust adaptive vibration tracking control for a MEMS vibratory gyroscope with bound estimation. John and Vinay [18] presented an adaptive controller for a MEMS triaxial gyroscope which drive both axes of vibration and controls the entire operation.

This paper focuses on the design of a robust adaptive sliding mode control strategy using RBF neural network. The information of the upper bound of the model uncertainties and external disturbances does not need be known in advance. The control scheme integrates the adaptive sliding mode control and the nonlinear mapping of neural network. A RBF neural network is used to adaptively learn the unknown upper bound of model uncertainties and external disturbances to eliminate the chattering of sliding mode effectively. The control system can guarantee the convergence of trajectory tracking error and robustness for model uncertainties and external disturbances. Meanwhile, the proposed adaptive sliding mode controller can online update the unknown system dynamics. A key property of this scheme is that the prior knowledge of the upper bound of the system uncertainties is not required but online estimated using RBF network. The main motivations are highlighted as follows:

1. An adaptive neural sliding mode control is adopted to approximate the unknown upper bound of the uncertainties and external disturbances. The advantage of using adaptive neural sliding mode control is that we need not know the upper bound of uncertainties and disturbances in advance. It will be convenient for us to control the dynamic systems since the upper bound of the uncertainties and external disturbances can be adaptively tuned. This is the most important feature of the proposed control as compared with the existing work.
2. A neural network control is incorporated into the adaptive sliding control system to strengthen the robustness of the control system. An adaptive sliding mode controller is derived to identify unknown system parameters and an adaptive RBF control method based on the sliding-mode control is developed to estimate the optimal upper bound of model uncertainties and external disturbances.

The paper is organized as follows. In Sect. 2, nominal sliding mode controller is designed. In Sect. 3, the derivation and stability analysis of an adaptive sliding mode controller using RBF neural network are given. Simulation results are presented in Sect. 4 to verify the effectiveness of the proposed adaptive neural sliding mode control. Conclusions are provided in Sect. 5.

2 Nominal sliding mode control

Consider the system with multiple inputs with parametric uncertainties and external disturbances:

$$\dot{X}(t) = AX(t) + Bu(t) + Bf_m(t) \tag{1}$$

where $X(t) \in R^n$, $u(t) \in R^m$ and $A \in R^{n\times n}$, $B \in R^{n\times m}$ are unknown constant matrices, $f_m(t)$ is an unknown lumped model uncertainties and external disturbances.

The reference model is given by

$$\dot{X}_m(t) = A_m X_m(t) + B_m r(t) \tag{2}$$

where $X_m(t) \in R^n$, $r(t) \in R^m$, $A_m \in R^{n\times n}$, $B_m \in R^{n\times m}$ are known constant parameter matrices.

We make the following assumptions:

A1. All eigenvalues of A_m are in the open left-half complex plane, and $r(t)$ is bounded and piecewise continuous;

A2. There exists a constant matrix $K_1^* \in R^{n\times m}$ and a non-zero constant matrix $K_2^* \in R^{m\times m}$ such that the following equations are satisfied $A + BK_1^{*T} = A_m, BK_2^* = B_m$;

A3. There is a known matrix $Q \in R^{m\times m}$ such that $K_2^* Q$ is symmetric and positive definite:

$$M = K_2^* Q = \left(K_2^* Q\right)^T = Q^T K_2^{*T} > 0. \tag{3}$$

A4. The matched lumped uncertainty and disturbance f_m is bounded by unknown positive parameter $\bar{\rho}$, $\|f_m\| \le \bar{\rho}$.

The control task is to design a feedback control $u(t)$ for the plant (1), the plant state $X(t)$ asymptotically tracks a given state $X_m(t)$of the reference model.

The tracking error is defined as

$$e(t) = X(t) - X_m(t) \tag{4}$$

the derivative of tracking error is

$$\dot{e} = A_m e + (A - A_m)X + Bu + Bf_m - B_m r. \tag{5}$$

The integral sliding surface is defined as

$$s(t) = \lambda e - \int_0^t \lambda A_m e \, d\tau. \tag{6}$$

The derivative of the sliding surface is

$$\dot{s} = \lambda(A - A_m)X + \lambda Bu + \lambda Bf_m - \lambda B_m r. \tag{7}$$

Setting $\dot{s} = 0$ to solve equivalent control u_{eq} gives

$$\begin{aligned} u_{eq} &= -(\lambda B)^{-1}\lambda(A - A_m)X + (\lambda B)^{-1}\lambda B_m r - f_m \\ &= K_1^{*T} X(t) + K_2^* r(t) - f_m, \end{aligned} \tag{8}$$

where $K_1^* = (\lambda B)^{-1}\lambda(A_m - A)$, $K_2^* = (\lambda B)^{-1}\lambda B_m$. The nominal control signal u is proposed as

$$u(t) = K_1^{*T} X(t) + K_2^* r(t) - \rho \frac{s}{\|s\|}, \tag{9}$$

where ρ is constant, $\|\cdot\|$ is the Euclidean norm, $\rho \frac{s}{\|s\|}$ is the unit sliding mode signal.

3 Adaptive sliding mode controller

In this section, we will address the design procedure of RBF neural network based adaptive sliding mode control. Because of the great advantages of neural networks in dealing with the nonlinear system, an adaptive neural sliding mode controller is designed and its stability is analyzed. The block diagram of an adaptive sliding mode control using RBF network is shown in Fig. 1.

According to (9), the adaptive control signal u is proposed as

$$u(t) = K_1^T(t)X(t) + K_2(t)r(t) - \rho \frac{s}{\|s\|}, \tag{10}$$

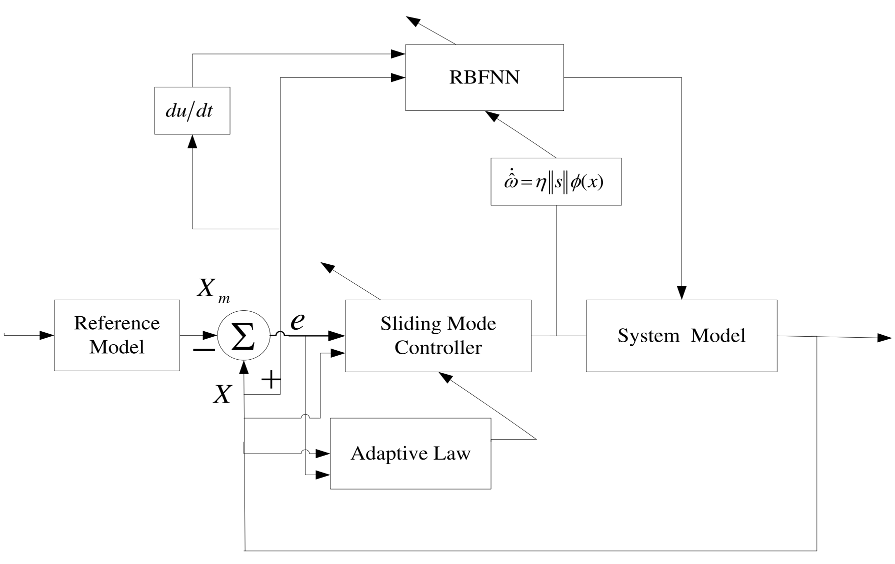

Fig. 1 Block diagram of adaptive sliding mode control using RBF network

where $K_1(t)$ and $K_2(t)$ are the estimates of K_1^* and K_2^*, respectively.

Define the parameter errors as

$$\tilde{K}_1(t) = K_1(t) - K_1^*, \tag{11}$$

$$\tilde{K}_2(t) = K_2(t) - K_2^*. \tag{12}$$

Substituting (10), (11), (12) into (1) obtains

$$\begin{aligned}\dot{X}(t) &= AX(t) + B\big[K_1^T(t)X(t) + K_2(t)r(t)\big] \\ &\quad + Bf_m - B\rho\frac{s}{\|s\|}, \\ &= A_m X(t) + B_m r(t) \\ &\quad + B_m\big[K_2^{*-1}\tilde{K}_1^T(t)X(t) + K_2^{*-1}\tilde{K}_2(t)r(t)\big] \\ &\quad + Bf_m - B\rho\frac{s}{\|s\|}.\end{aligned} \tag{13}$$

Substituting (13), (10) into (5), yields the tracking error equation

$$\begin{aligned}\dot{e}(t) &= A_m e(t) \\ &\quad + B_m\big[K_2^{*-1}\tilde{K}_1^T(t)X(t) + K_2^{*-1}\tilde{K}_2(t)r(t)\big] \\ &\quad + Bf_m - B\rho\frac{s}{\|s\|}\end{aligned} \tag{14}$$

and the derivative of $s(t)$ becomes

$$\begin{aligned}\dot{s}(t) &= \lambda B_m\big[K_2^{*-1}\tilde{K}_1^T(t)X(t) + K_2^{*-1}\tilde{K}_2(t)r(t)\big] \\ &\quad + \lambda Bf_m - \lambda B\rho\frac{s}{\|s\|}.\end{aligned} \tag{15}$$

Suppose $\bar{\rho}(t)$ is the upper bound of model uncertainties and external disturbances. If the upper bound value $\bar{\rho}(t)$ can not be measured properly and unknown, RBF neural network can be used to adaptively learn the upper bound $\bar{\rho}(t)$. The structure of RBF neural network is a three-layer feedforward network shown as in Fig. 2. The input layer is the set of source nodes. The second layer is a hidden layer of high dimension. The output layer gives the response of the network to the activation patterns applied to the input layer. In this paper, the advantage of RBF neural network is to adjust the value of the upper bound of model uncertainties and external disturbances.

The estimate of the upper bound $\bar{\rho}(t)$ is

$$\hat{\bar{\rho}}(x,\omega) = \hat{\omega}^T\phi(x), \tag{16}$$

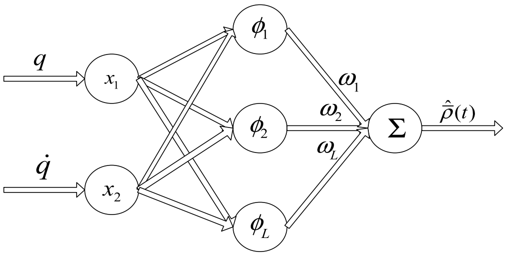

Fig. 2 The structure of RBF network

where $x = [q\ \dot{q}]$ is the input of RBF neural network, $\hat{\omega}^T$ are weights of RBF neural network and $\phi(x)$ is Gaussian function,

$$\phi_i(x) = \exp\left(-\frac{\|x - m_i\|^2}{\sigma_i^2}\right), \quad i = 1, 2, \ldots, L \tag{17}$$

where L is the number of output node, $\phi(x) = [\phi_1, \phi_2, \ldots, \phi_L]^T$, $\phi_i(x)$ is ith Gaussian function, m_i is the ith center vector, and σ_i is ith standard deviation.

We make the following assumptions:

Assumption 1 The optimal weights of RBF satisfy:

$$\omega^{*T}\phi(x) - \bar{\rho}(t) = \varepsilon(x) \quad \text{and} \quad |\varepsilon(x)| < \varepsilon_1. \tag{18}$$

Assumption 2 The upper bound $\bar{\rho}(t)$ satisfies

$$\bar{\rho}(t) - \|f_m\| > \varepsilon_0 > \varepsilon_1. \tag{19}$$

Define a Lyapunov function

$$\begin{aligned}V &= \frac{1}{2}s^T s + \frac{1}{2}\mathrm{tr}\big[\tilde{K}_1 M^{-1}\tilde{K}_1^T\big] \\ &\quad + \frac{1}{2}\mathrm{tr}\big[\tilde{K}_2 M^{-1}\tilde{K}_2^T\big] + \frac{1}{2}\eta^{-1}\mu\tilde{\omega}^T\tilde{\omega},\end{aligned} \tag{20}$$

where $\tilde{\omega} = \omega^* - \hat{\omega}$, $\eta = \varepsilon_0 - \varepsilon_1 > 0$, $M = M^T > 0$, $\mu = \|\lambda B\|$, M is positive definite matrix, tr$[M]$ denoting the trace of M.

The derivative of the Lyapunov function is

$$\begin{aligned}\dot{V} &= s^T\dot{s} + \mathrm{tr}\big[\tilde{K}_1 M^{-1}\dot{\tilde{K}}_1^T\big] + \mathrm{tr}\big[\tilde{K}_2 M^{-1}\dot{\tilde{K}}_2^T\big] \\ &\quad - \eta^{-1}\sigma\tilde{\omega}^T\dot{\hat{\omega}} \\ &= s^T\Big\{\lambda B_m\big[K_2^{*-1}\tilde{K}_1^T(t)X(t) + K_2^{*-1}\tilde{K}_2(t)r(t)\big] \\ &\quad + \lambda Bf_m - \lambda B\rho\frac{s}{\|s\|}s\Big\}\end{aligned}$$

$$
\begin{aligned}
&+\operatorname{tr}\big[\tilde K_1 M^{-1}\dot{\tilde K}_1^T\big]+\operatorname{tr}\big[\tilde K_2 M^{-1}\dot{\tilde K}_2^T\big]\\
&-\eta^{-1}\mu\tilde\omega^T\dot{\hat\omega}\\
&=s^T\left(\lambda B f_m-\lambda B\rho\frac{s}{\|s\|}\right)\\
&+\lambda B_m\big[K_2^{*-1}\tilde K_1^T(t)X(t)+K_2^{*-1}\tilde K_2(t)r(t)\big]\\
&+\operatorname{tr}\big[\tilde K_1 M^{-1}\dot{\tilde K}_1^T\big]+\operatorname{tr}\big[\tilde K_2 M^{-1}\dot{\tilde K}_2^T\big]\\
&-\eta^{-1}\mu\tilde\omega^T\dot{\hat\omega}\\
&=s^T\left(\lambda B f_m-\lambda B\rho\frac{s}{\|s\|}\right)\\
&+\big\{\lambda B_m K_2^{*-1}\tilde K_1^T(t)X(t)+\operatorname{tr}\big[\tilde K_1 M^{-1}\dot{\tilde K}_1^T\big]\big\}\\
&+\big\{\lambda B_m K_2^{*-1}\tilde K_2(t)r(t)+\operatorname{tr}\big[\tilde K_2 M^{-1}\dot{\tilde K}_2^T\big]\big\}\\
&-\eta^{-1}\mu\tilde\omega^T\dot{\hat\omega},
\end{aligned}
\tag{21}
$$

where we use the definition $M=K_2^*Q=M^T>0$ and the properties $\operatorname{tr}[N_1N_2]=\operatorname{tr}[N_2N_1]$, $\operatorname{tr}[N_3]=\operatorname{tr}[N_3^T]$ for any matrices N_1, N_2 and N_3.

To make $\dot V\le 0$, we choose the adaptive laws as

$$\dot{\tilde K}_1^T(t)=\dot K_1^T(t)=-Q^TB_m^T\lambda^Ts^TX^T \tag{22}$$

$$\dot{\tilde K}_2(t)=\dot K_2(t)=-Q^TB_m^T\lambda^Ts^Tr^T \tag{23}$$

with Q satisfying assumption A3 and $K_1(0)$ and $K_2(0)$ being arbitrary.

Substituting $\dot{\tilde K}_1^T(t)$, $\dot{\tilde K}_2^T(t)$ into $\dot V$ yields

$$
\begin{aligned}
\dot U&=s^T\left(\lambda B f_m-\lambda B\rho\frac{s}{\|s\|}\right)-\eta^{-1}\mu\tilde\omega^T\dot{\hat\omega}\\
&=s^T\left(\lambda B f_m+\lambda B\bar\rho-\lambda B\bar\rho-\lambda B\rho\frac{s}{\|s\|}\right)\\
&\quad-\eta^{-1}\mu\tilde\omega^T\dot{\hat\omega}\\
&\le\|s\|\bigg(\|\lambda B\|\|f_m\|+\|\lambda B\|\bar\rho-\|\lambda B\|\bar\rho\\
&\quad-\|\lambda B\|\rho\left\|\frac{s}{\|s\|}\right\|\bigg)-\eta^{-1}\mu\tilde\omega^T\dot{\hat\omega}\\
&=\|s\|\|\lambda B\|\big(\|f_m\|+\bar\rho-\bar\rho\big)-\|\lambda B\|\rho\|s\|\\
&\quad-\eta^{-1}\mu\tilde\omega^T\dot{\hat\omega}\\
&=+\|s\|\|\lambda B\|\big(\bar\rho-\|f_m\|\big)+\|\lambda B\|\|s\|(\bar\rho-\rho)\\
&\quad-\eta^{-1}\mu\tilde\omega^T\dot{\hat\omega}\\
&=-\|s\|\|\lambda B\|\big(\bar\rho-\|f_m\|\big)\\
&\quad+\|\lambda B\|\|s\|\big(\omega^{*T}\phi-\varepsilon-\hat\omega^T\phi\big)-\eta^{-1}\mu\tilde\omega^T\dot{\hat\omega}\\
&=-\|s\|\|\lambda B\|\big(\bar\rho-\|f_m\|\big)-\|\lambda B\|\|s\|\varepsilon\\
&\quad+\big[\|\lambda B\|\|s\|\tilde\omega\phi-\eta^{-1}\mu\tilde\omega^T\dot{\hat\omega}\big].
\end{aligned}
\tag{24}
$$

Using adaptive algorithm to adjust the weights online

$$\dot{\hat\omega}=\eta\|s\|\phi(x). \tag{25}$$

Substituting $\dot{\hat\omega}$ into $\dot V$ yields

$$
\begin{aligned}
\dot V&=-\|s\|\|\lambda B\|\big(\bar\rho-\|f_m\|\big)-\|\lambda B\|\|s\|\varepsilon\\
&\le-\|s\|\|\lambda B\|\big(\bar\rho-\|f_m\|\big)+\|\lambda B\|\|s\|\varepsilon\\
&=-\|s\|\|\lambda B\|\varepsilon_0+\|\lambda B\|\|s\|\varepsilon\\
&\le-\|s\|\|\lambda B\|\varepsilon_0+\|\lambda B\|\|s\||\varepsilon|\\
&\le-\|s\|\|\lambda B\|\varepsilon_0+\|\lambda B\|\|s\|\varepsilon_1\\
&=-\|s\|\|\lambda B\|(\varepsilon_0-\varepsilon_1)=-\eta\|s\|\|\lambda B\|\le 0.
\end{aligned}
\tag{26}
$$

$\dot V$ is negative definite implies that s, $\tilde K_1$ and $\tilde K_2$ converge to zero. $\dot V$ is negative semi-definite ensures that V, s, $\tilde K_1$ and $\tilde K_2$ are all bounded. From (15) it can be concluded that $\dot s$ is also bounded, the inequality (26) implies that s is integrable as $\int_0^t\|s\|dt\le\frac{1}{\eta}[V(0)-V(t)]$. Since $V(0)$ is bounded and $V(t)$ is nonincreasing and bounded, it can be concluded that $\lim_{t\to\infty}\int_0^t\|s\|\,dt$ is bounded. Since $\lim_{t\to\infty}\int_0^t\|s\|\,dt$ is bounded and $\dot s$ is also bounded, according to Barbalat lemma, $s(t)$ will asymptotically converge to zero, $\lim_{t\to\infty}s(t)=0$. From (6) $e(t)$ also converges to zero asymptotically. From the adaptive laws (22), (23), according to the persistence excitation theory, if X and $\dot X$ are persistent excitation signals, i.e., $\omega_1\ne\omega_2\ne\omega_3$, then it can be guaranteed that $\tilde K_1\to 0$, and $\tilde K_2\to 0$, K_1 and K_2 will converge to their true values asymptotically.

4 A Case Study for MEMS Triaxial Gyroscope

As an illustrative example, we use the triaxial MEMS gyroscope dynamic model for the study of RBF network adaptive sliding mode control. The gyroscope dynamic model [17, 18] is

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1578.5241 & -0.0251 & -97.4659 & -9.9975 & -116.9591 & -4.0025 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -97.4659 & -10.004 & -1396.1121 & -0.0402 & -136.4522 & -5.9948 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -116.9591 & -3.996 & -136.4522 & -6.0052 & 1184.9889 & -0.0523 \end{bmatrix} X + \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix}.$$

The control target for MEMS gyroscope is to maintain the proof mass to oscillate in the x, y and z direction at given frequency and amplitude $x_m = A_1 \sin(w_1 t)$, $y_m = A_2 \sin(w_2 t)$, $z_m = A_3 \sin(w_3 t)$.The desired motion trajectories are: $x_m = \sin(\omega_1 t)$, $y_m = 1.2 \sin(\omega_2 t)$, $z_m = 1.5 \sin(\omega_3 t)$. The reference model for the triaxial gyroscope is

$$\dot{X}_m = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -45.0241 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -26.1121 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -17.3889 & 0 \end{bmatrix} \times X_m,$$

where $X_m = [x_m\ \dot{x}_m\ y_m\ \dot{y}_m\ z_m\ \dot{z}_m]^T$.

In (6), the sliding parameters are chosen as: $\lambda = \mathrm{diag}[40,\ 40,\ 40,\ 40,\ 40,\ 40]$. The initial value of ω is $[0.1\ 0.1\ 0.1]^T$, the initial value of m is always chosen between -1 and $+1$, we choose

$$m = \begin{bmatrix} -0.1639 & 0.7487 & 0.5359 \\ -0.3900 & -0.9700 & 0.9417 \end{bmatrix} \quad \text{and}$$

$$\sigma = [0.2 \quad 0.2 \quad 0.2]^T.$$

Initial value of system states is $[0.5 \quad 0]$, external disturbance $f_m(t) = 5 \sin(2\pi t)$. The initial value of $K_1(t)$ is

$$K_1 = \begin{bmatrix} 1380.15 & 87.7193 & 105.2632 \\ 87.7193 & 1233 & 122.8070 \\ 105.2632 & 122.8070 & 1050.84 \\ 0.0226 & 8.9978 & 3.6023 \\ 9.0036 & 0.0362 & 5.3953 \\ 3.5964 & 5.4047 & 0.0471 \end{bmatrix}.$$

Within the simulation, the system dynamic function are assumed completely unknown and the adaptive fuzzy controller does not need the information of the dynamic model as in convention model-based adaptive controller, Actually, the dynamic model of the MEMS gyroscope system is only required for the purpose of simulation.

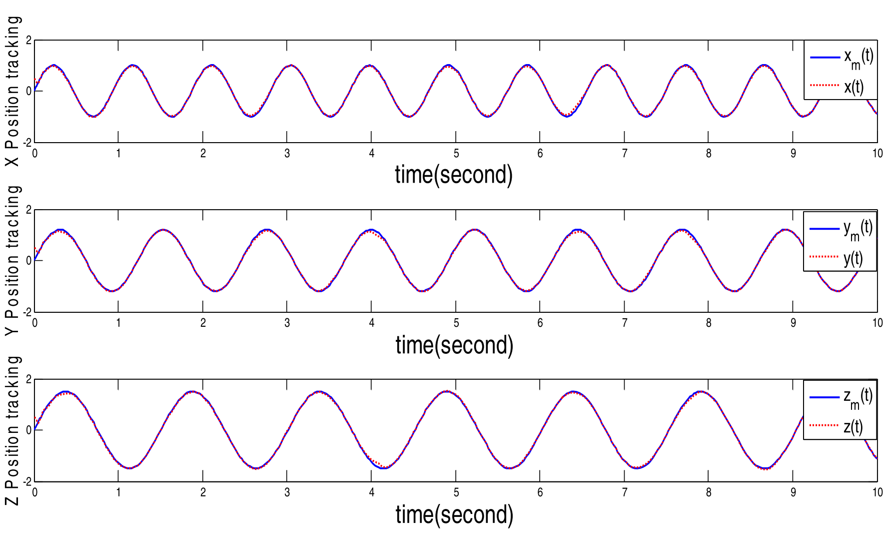

Fig. 3 Property of the trajectory tracking

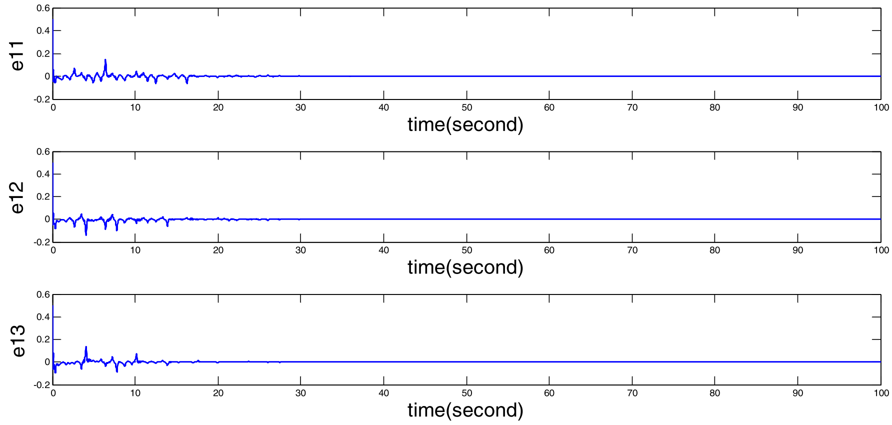

Fig. 4 Property of the tracking error $e(t)$

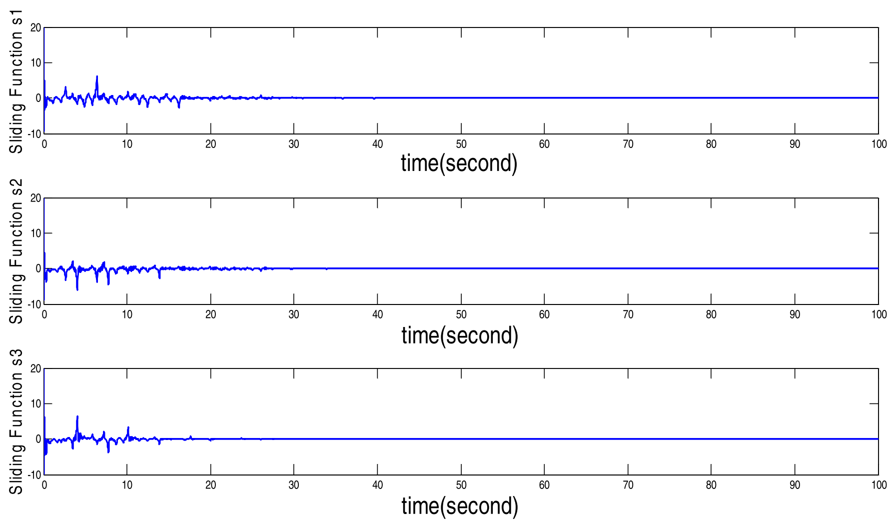

Fig. 5 Property of the sliding surface $s(t)$

Figure 3 is position tracking and Fig. 4 depicts the convergence of the tracking errors. It can be seen that tracking errors converge to zero asymptotically. It can be observed that the desired and actual trajectories almost overlap with each other; the position of x, y, and z can track the position of reference model in short time, and the tracking errors occurring at the beginning drop quickly in a few seconds. Therefore, it can be concluded that the MEMS gyroscope can maintain the proof mass to oscillate in the x, y, and z direction at given frequency and amplitude with the adaptive neural control and the control objective is well accomplished because neural network system has the strong ability to approximate the nonlinear system and compensate the system nonlinearities. It can be observed from Fig. 5 that $s(t)$ asymptotically converges to zero showing that the sliding control system reaches the sliding surface in short time. Adaptation of the controller parameters is described in Fig. 6 showing that $K_1(t)$ converge to their true values in short time with persistent excitation signals. The true value of the controller $K_1(t)$ is

$$K_1^* = \begin{bmatrix} 1533.5 & 97.4659 & 116.9591 \\ 97.4659 & 1370 & 136.4522 \\ 116.9561 & 136.4522 & 1167.6 \\ 0.0251 & 9.9975 & 4.0025 \\ 10.004 & 0.0402 & 5.9948 \\ 3.996 & 6.0052 & 0.0523 \end{bmatrix}.$$

The estimated upper bound of disturbances using RBF neural network is depicted in Fig. 7, where RBF neural network is used to adjust the gain of the switch part of adaptive sliding mode control input. Figure 8 is the angular velocity using RBF based adaptive slid-

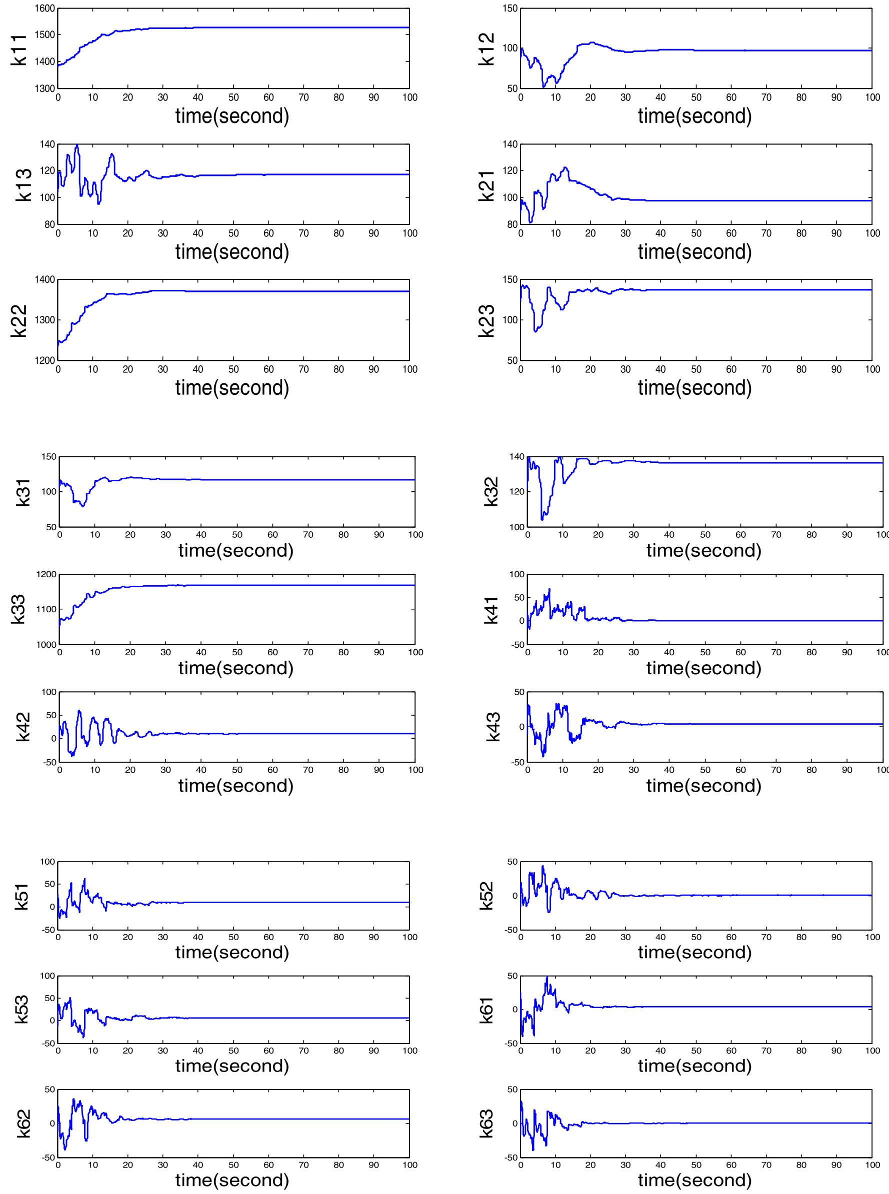

Fig. 6 Adaptation of the controller parameters

ing mode controller, it can be shown that the estimates of angular velocity converge to their true values $\Omega_x = 3.0$ rad/s, $\Omega_y = 2.0$ rad/s, $\Omega_z = 5.0$ rad/s with persistent excitation signals.

Figure 9 is the adaptive sliding control input with estimated upper bound of disturbances using RBF neural network and Fig. 10 is the adaptive sliding control input with fixed value upper bound of disturbances. Comparing these two figures, it can be found that the control input in Fig. 9 is better than that of Fig. 10 and the magnitudes of u1, u2 and u3 in Fig. 10 are larger than that in Fig. 9. All of the control inputs such as u1, u2, and u3 in Fig. 10 have obvious chattering because of fixed value of upper bound of disturbances. Therefore, it can be concluded that the adaptive learned upper bound of disturbances using RBF neural network can reduce chattering significantly. Using RBF neural network based adaptive sliding mode control can generate the smooth sliding mode control force which can create a small boundary layer about the switching surface to eliminate the chattering.

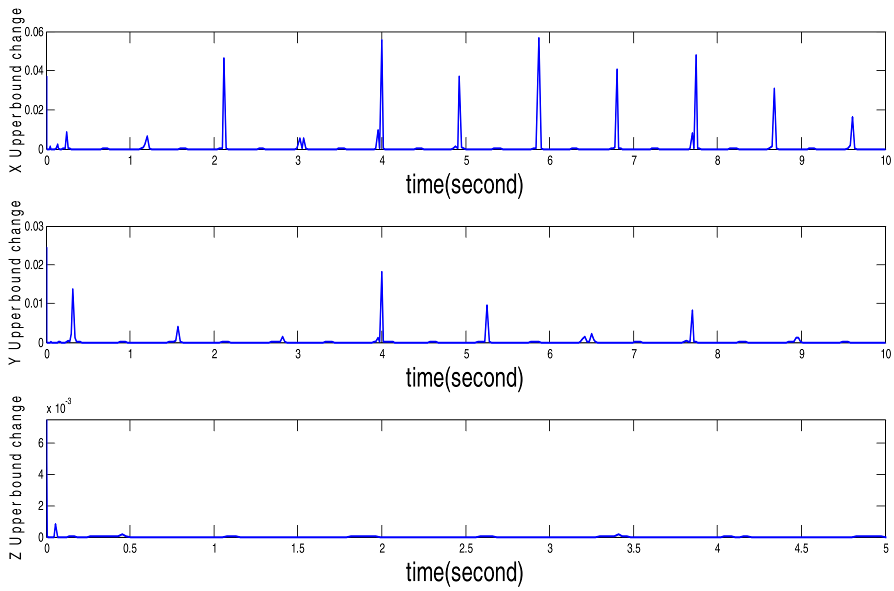

Fig. 7 Adaptation of upper bound of the disturbances

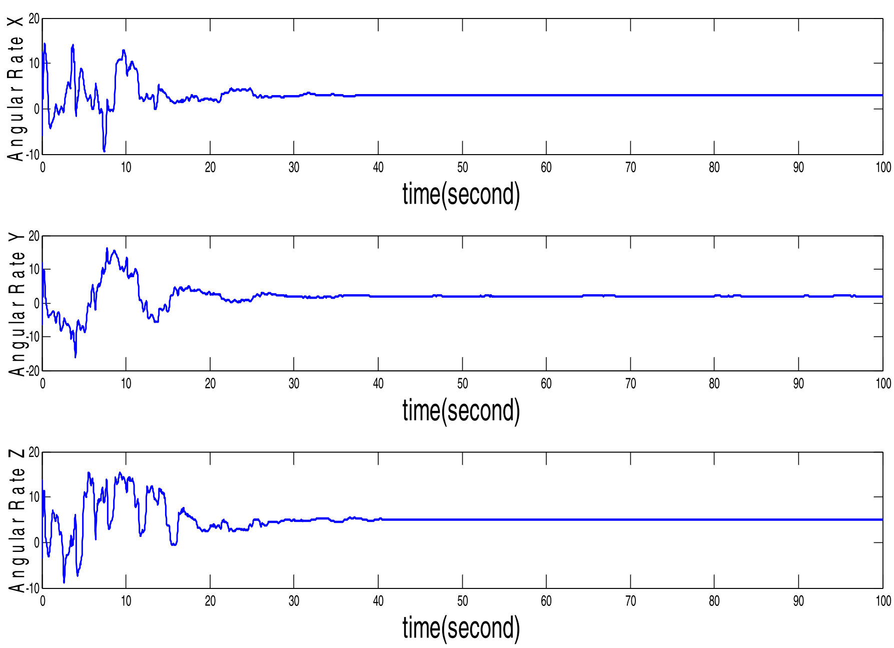

Fig. 8 Adaptation of angular velocity using RBF based adaptive sliding mode controller

It can be concluded that the dynamic system responses are as expected, and the simulation results demonstrated that adaptive RBF neural network control has good tracking performance and tracking errors converge to zero asymptotically. The satisfactory performance and improved robustness with regard to system nonlinearities such as parametric variations and external disturbances can be obtained with the proposed adaptive neural control.

5 Conclusion

This paper presents an adaptive design of RBF neural network based adaptive sliding mode control for

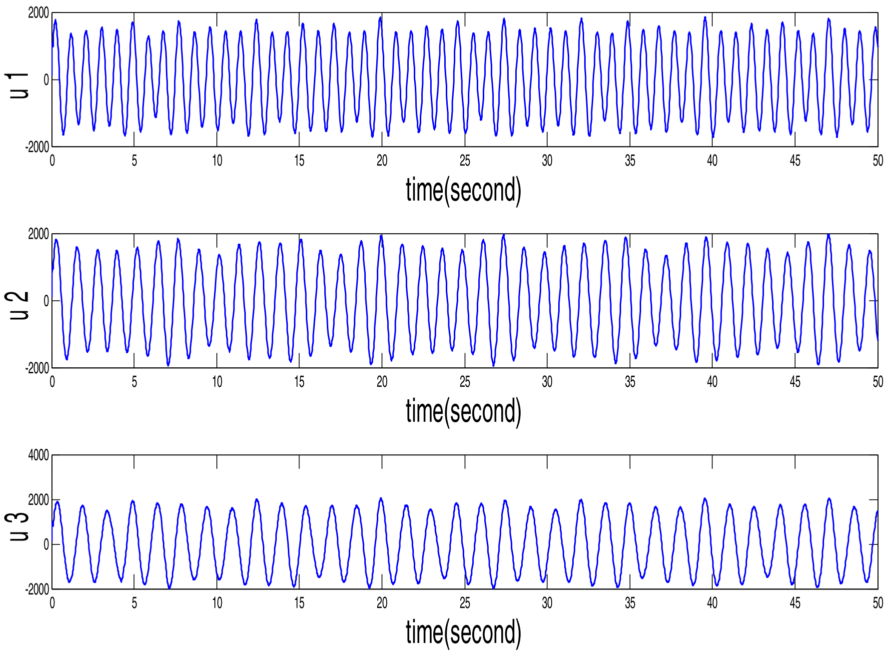

Fig. 9 Control input with estimated upper bound of the disturbances using RBF network

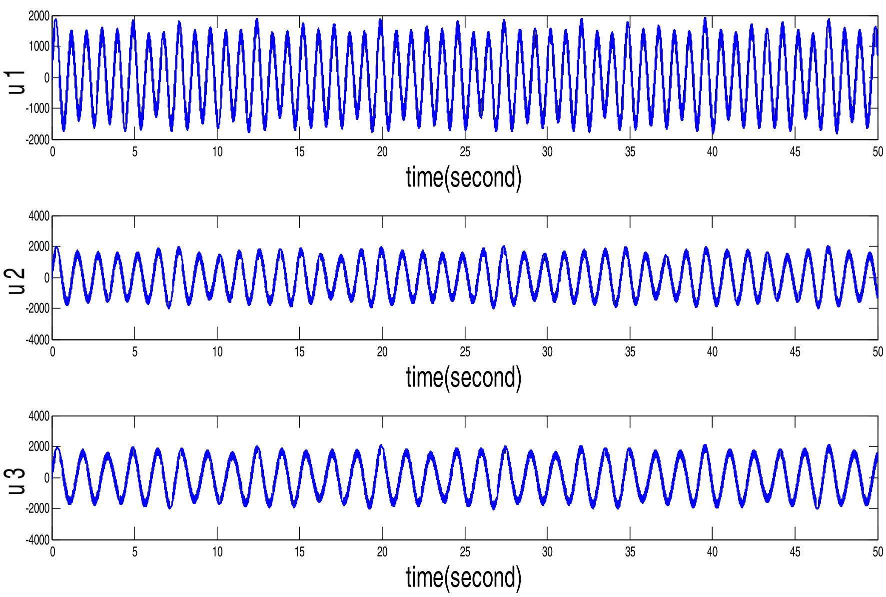

Fig. 10 Control input using fixed value of upper bound of the disturbances

dynamic systems. An adaptive RBF neural network is used to learn the upper bound of model uncertainties and external disturbances. The output of the neural network is used as compensator parameter and the effects of the model uncertainties and external disturbances can be eliminated. The stability of the closed-loop system can be guaranteed with the proposed adaptive RBF sliding mode control strategy. The simulation is implemented to verify the effectiveness of the proposed adaptive RBF sliding mode control for the triaxial angular velocity sensor. Simulation results demonstrates that tracking error and sliding surface all converge to zero asymptotically, the system parameters converges to their true values asymptotically

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