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Sliding mode compensation to preserve dynamic decoupling of stable systems

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Abstract

Dynamic decoupling of linear multiple-input and multiple-output systems involves both static and transient decoupling between inputs and outputs of the system. Different control techniques exist in order to achieve this specification when ideal actuators are considered. However, input saturation changes the direction of the plant input with respect to the controller output and, as a consequence, decoupling is lost. This paper presents a method that allows, by means of a sliding mode (SM) auxiliary loop, maintaining dynamic decoupling even in the presence of actuator saturation. Since the SM compensation is confined to the low-power side of the control system, the discontinuous signal can be implemented with fast switching devices or, in the case of digital controllers, within a microprocessor algorithm. Furthermore, due to the robustness properties of SM, the compensation loop dynamics may be assigned independently of the main control loop.

Keywords: Process control; Dynamic decoupling; Input saturation; Sliding mode

1. Introduction

A frequent design specification in multivariable control systems is input–output decoupling. There are different degrees of decoupling, ranging from partial to full decoupling, and from static (only in steady state) to dynamic decoupling (at all frequencies). Clearly, full dynamic decoupling is the strongest demand. It implies that any change in the set-point value of a controlled variable of the system leads to a response only in that process variable, while all the other controlled variables remain unaffected.

The advantages of full dynamic decoupling are intuitive. Nevertheless, as might be expected, it is very sensitive to modeling errors (Skogestad and Postlethwaite, 2005) and it has a performance cost, which depends on the poles and zeros of the plant in the right-hand plane (RHP) (Seron et al., 1997). The evaluation of these difficulties is outside the scope of this article, where we will consider nominal dynamic decoupling to be the main control objective.[3]

Different control techniques exist to design a controller that achieves full dynamic decoupling of a system when ideal actuators are considered (Hautus and Heymann, 1983; Eldem, 1994; Goodwin et al., 1997; Wang, 2003). However, the problem becomes considerably more complicated if physical limits of the actuators are taken into account. In fact, multiple input saturation changes the amplitude and the direction of the control signal that is necessary to achieve dynamic decoupling. Hence, in addition to the known problem of windup (Kothare et al., 1994), the change of directionality problem appears, which brings about the loss of the decoupling obtained for the ideal case.

Among the earliest efforts to preserve control directionality in constrained multivariable systems, Hanus and Kinnært (1989) firstly proposed to modify the reference conditioning technique (originally devised as a single-input and single-output (SISO) anti-windup method) to deal with the problems of multiple-input and multiple-output (MIMO) systems. Therein, an artificial nonlinearity placed just before the real

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[3] A proposal to relax the cost of diagonal decoupling in non-minimum phase systems has been proposed in Garelli et al. (2006a).

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nonlinearity is designed in such a way that the conditioned reference remains as close as possible to the original reference, under some criterion. Afterwards, Walgama and Sternby (1993) integrated the ideas of Hanus and Kinnaert with a generalization of the conditioning technique by introducing a "ltered setpoint. A posterior contribution worthy of mention was made by Peng et al. (1998), where a parametrization of anti-windup compensators and an optimal design are addressed in a simple manner. Other techniques such as Linear Matrix Inequalities (LMI), Linear Parameter-Varying (LPV) and reference conditioning for Internal Model Control (IMC) were also applied to this topic (Mulder et al., 2001; Wu and Grigoriadis, 1999; Zheng et al., 1994). More recently, in Soroush et al. (2005), an optimization-based control method for discrete-time input-constrained processes was presented.

Most of the optimal design methods (e.g., Hanus and Kinnært, 1989; Walgama and Sternby, 1993; Peng et al., 1998) successfully avoid the change of control directionality by conditioning the whole reference vector. However, when dynamic decoupling is the main control objective, preservation of the control directionality is necessary but not suf"cient. Although they solve the problem that originally caused the loss of decoupling, the methodology used may, in less degree, also affect the decoupling of the system. In fact, when a reference changes, the simultaneous correction of the whole reference vector may lead to shape the unchanged references, thus producing transient effects in controlled variables that should not change. When the process to be controlled allows reaching the operation points by successive changes of individual reference components, which is very common in chemical processes, an improved degree of decoupling is achievable. We will therefore focus on this way of operation, which is also taken into account in Goodwin et al. (2001), where decoupling preservation is attained by scaling down the error vector (which is equivalent to introducing a nonlinear controller gain). Such method is shown to be effective and preserves dynamic decoupling. However, as it is claimed in Goodwin et al. (2001), for some processes it may happen that no scaling of the error brings the control back to the linear region.

In this paper we present a technique to preserve dynamic decoupling of constrained multivariable processes, assuring that the controlled variables whose set-points did not change remain unaffected. The algorithm combines a reference conditioning technique with variable structure systems (VSSs) and sliding mode (SM) related concepts. These ideas were earlier put together to limit crossed interactions in decentralized control systems (Garelli et al., 2006b) and in 2× 2 partially decoupled systems with RHP zeros (Garelli et al., 2006a). Differing from these two previous works, input constraints are considered here, where it is aimed to eliminate interactions rather than bounding them. Moreover, in the present approach the model of the plant is not needed for the SM compensation design. Finally, the development of the current MIMO strategy imposes restrictions neither with respect to the number of inputs/outputs of the (square) system, nor to the type of the centralized controller. For the latter, some conditions are derived in order to guarantee the stability of the SM compensation.

A very interesting property of the proposal is that, due to the robustness properties of the SM to reject disturbances, the SM conditioning dynamics is not affected by the main control loop. Thus, the dynamics of the compensation loop may be designed independently of the main loop design. Another distinctive feature is that, for strictly proper transfer functions, the method allows one to easily determine the rate of approach to the constraint by designing the sliding surface, thus avoiding hard-hitting the limit. In addition, differing from other variable structure control schemes, there are neither chattering problems nor reaching mode in the current application (usually considered drawbacks of variable structure control).

The next section presents the basic ideas of the conditioning algorithm. Section 3 develops the methodology proposed in this paper to preserve dynamic decoupling in presence of input saturation, which is thereafter evaluated through a pair of non-minimum phase examples in Section 4. Finally, conclusions are drawn.

2. Basic idea of the proposal

A VSS is composed of various continuous subsystems with a switching logic. The resulting discontinuous control action is a function of the system state. A particular operation is achieved when switching occurs at a very high frequency constraining the system state to a surface, named the sliding surface. This kind of operation is called SM and has many attractive properties. It is robust to parameter uncertainties and external disturbances, the closed-loop system is an order-reduced one, and its dynamics depends on the designer-chosen sliding surface (Sira-Ramírez, 1988; Utkin, 1977; Edwards and Spurgeon, 1998). Because of its interesting features, a large number of papers presenting practical applications of SM control have been reported. For instance, Herrmann et al. (2003), Chen and Peng (2005, 2006) and Picó et al. (2005) have discussed the application of SM to chemical process control.

In the present work, we take advantage of the interesting features and the con"ned dynamics of a system operating on SM to address an important issue concerning multivariable systems: the problem of input constraint and system decoupling. However, differing from conventional SM applications, sliding regimes are exploited here as a transitional mode of operation, in which the discontinuous signal is used for conditioning the reference signal instead of using it as the main control action.

In order to keep the presentation as simple as possible, we "rst describe conceptually how the SM reference conditioning operates when it is applied to an SISO system controlled by a biproper controller. The more general MIMO technique for proper controllers will be discussed in the next section.

Fig. 1 illustrates the scheme of an SISO control system where the SM conditioning technique was added to avoid surpassing actuator limits. P is the plant under control with linearized model of the form

$$P: \quad \begin{aligned} \dot{x}_p &= A_p x_p + b_p u, \\ y &= c_p x_p, \end{aligned} \qquad (1)$$

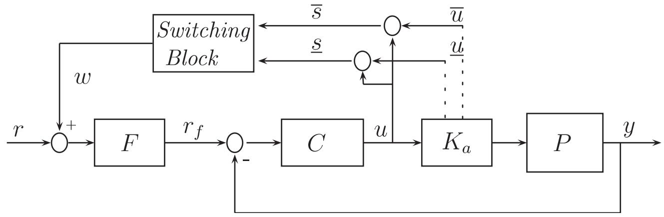

Fig. 1. SM reference conditioning for SISO system and biproper controller.

K_a the actuator with saturation, C a linear biproper minimum-phase controller

$$C: \begin{array}{l} x_c = A_c x_c + b_c e, \\ u = c_c x_c + d_c e, \end{array} \quad (2)$$

and F a first-order filter:

$$F: \begin{array}{l} x_f = - {}_f x_f + r + w, \\ r_f = {}_f x_f \,. \end{array} \quad (3)$$

The dynamics of this linear filter, whose purpose is to smooth out the conditioned reference r_f, is designed to be faster than the closed loop in such a way that the response of the system is not deteriorated during linear operation of the actuators.

For our particular purpose, the following commutation law is implemented in the switching block:

$$\begin{array}{ll} w = w^- & \text{if } \bar{s} < 0, \\ w = w^+ & \text{if } \underline{s} > 0, \\ w = 0 & \text{otherwise,} \end{array} \quad (4)$$

where

$$\begin{array}{l} \bar{s} = \bar{u} - u, \\ \underline{s} = \underline{u} - u, \end{array} \quad (5)$$

being $\bar{u}$ the upper limit of the actuator and $\underline{u}$ the lower one.

Note that because of the first-order filter and the biproper controller, the trivial sliding functions (5) are of relative degree 1 with respect to w, which is a necessary condition for the establishment of SM (Sira-Ramírez, 1989). Conversely, in the case of strictly proper controllers other controller states should be considered in the switching functions in order to guarantee this necessary condition.

According to (4), when the actuator operates in its linear region ($\underline{u} < u < \bar{u}$) the signal w is 0 and no correction is made, that is, the conditioning loop is inactive. However, when u tries to exceed its upper bound, which makes $\bar{s} < 0$, the signal w changes to w^- (similarly, if u intends to fall bellow its lower limit, making $\underline{s} > 0$, w switches to w^+). Whenever the trajectories of the system continue trying to cross the controller output bounds, the signal w will be switching between 0 and w^- (or w^+) at high frequency and a sliding regime will establish transiently on surface $\bar{s}=0$ (or $\underline{s}=0$). In this manner, the filtered reference will be continuously adjusted in such a way that the controller output never exceeds the actuator limits.

The values w^+ and w^- have to be sufficient to redirect the state trajectories towards the interior of the linear region, i.e., they must assure:

$$\begin{array}{ll} \bar{s} > 0 & \text{if } \bar{s} < 0, \\ \underline{s} < 0 & \text{if } \underline{s} > 0. \end{array} \quad (6)$$

Assuming $w^- < 0 < w^+$ (this choice corresponds to a positive controller gain), (6) will be satisfied provided $w^- \;\; \overline{w_{eq}}$ and $\underline{w_{eq}} \;\; w^+$ (Utkin, 1977), where the continuous equivalent control w_{eq} (stands either for $^-$ or _) are obtained differentiating once (5) with respect to time and equaling 0. Hence,

$$w_{eq} = r_f - r - [d_c \;{}_f\,]^{-1}\{c_c x_c - d_c y\}|_{s=0}. \quad (7)$$

It is important to remark that to satisfy the latter inequalities, the selection of w^+ and w^- can be made in a conservative manner because the SM is restricted to the low-power side of the system.

A sufficient condition for the reestablishment of linear operation can be derived from Eq. (7). With this aim, it is assumed that SM is established on surface $\bar{s} = \bar{u} - u = 0$, which means that the continuous equivalent control satisfies $w^- \;\; \overline{w_{eq}} < 0$. The system recovers linear operation and the nominal closed-loop dynamics provided $\overline{w_{eq}}$ crosses 0. Thus, we will study the asymptotic behaviour of $\overline{w_{eq}}$ to find under which conditions it will be greater than 0 in finite time. If the system continued undergoing SM on $\bar{s} = 0$, $\overline{w_{eq}}$ would tend towards

$$\lim_{t} \overline{w_{eq}} = \lim_{t} (r_f - r). \quad (8)$$

In addition, since $u = \bar{u}$ during SM

$$\lim_{t} r_f = \lim_{t} (e + y) = ((d_c - c_c A_c^{-1} b_c)^{-1} - c_p A_p^{-1} b_p)\bar{u} = S_u(0)^{-1}\bar{u}, \quad (9)$$

where $S_u(0)$ is the control sensitivity evaluated in s = 0, i.e., the dc-gain from r to u. Therefore, for step references $r < S_u(0)^{-1}\bar{u}$ the asymptotic value $\lim_t \;\; \overline{w_{eq}}$ will be greater than 0, which guarantees that the system reenters linear region in finite time.

Reasoning in the same fashion for $\underline{s} = 0$ yields the following sufficient condition for the reestablishment of linear operation:

$$|r| < |S_u(0)^{-1} u|. \quad (10)$$

3. Preservation of dynamic decoupling in MIMO systems

Based on the basic ideas presented earlier, this section develops an SM reference conditioning method to maintain dynamic decoupling of stable multivariable systems even in presence of input saturation. The general case of proper (biproper or strictly proper) controllers is considered.

3.1. Method formulation

Fig. 2 represents the MIMO control scheme suggested to avoid directionality changes when actuator limits are reached. As in Fig. 1, two loops can be distinguished: the main control loop with output feedback and the SM conditioning loop.

In the main control loop P represents the stable process under control with m inputs and m outputs. K_a are now m actuators

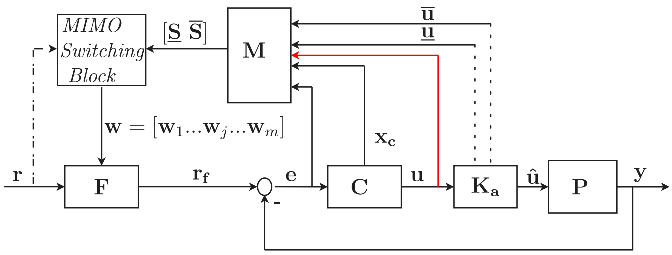

Fig. 2. MIMO control system with SM conditioning loop.

with amplitude saturation, whilst C is a centralized $m \times m$ proper controller designed to obtain full dynamic decoupling during the linear operation of the actuators K_a. F represents a "rst-order linear "lter in each channel. Signals r, r_f, e, u, u and y are vectors of m scalar functions of time. The lower and upper limits of the actuators are represented by the constant vectors $\underline{u} \in R^m$ and $\bar{u} \in R^m$, respectively. Individually, the nonlinearity introduced by the ith actuator can be modeled in a simple manner as:

$$K_{a_i}: \begin{array}{lll} u_i = \overline{u_i} & \text{if} & u_i > \overline{u_i}, \\ u_i = u_i & \text{if} & \underline{u_i} \le u_i \le \overline{u_i}, \\ u_i = \underline{u_i} & \text{if} & u_i < \underline{u_i} \end{array} \tag{11}$$

with $i = 1, \ldots, m$ and where $\underline{u_i}$ and $\overline{u_i}$ are the corresponding elements of $\underline{u}$ and $\bar{u}$.

The SM conditioning loop de"nes in this case two sliding surfaces like the ones described in the previous section for each of the m^2 transfer functions of the controller. This leads to some differences in the conditioning loop structure with respect to the SISO case.

The block M represents a constant matrix operator which produces two switching function vectors, $\bar{S}$ and $\underline{S} \in R^{m^2}$, the former comprising the switching functions associated with the upper limits of the actuators and the latter the ones corresponding to the lower bounds. Within $S = [s_1^T \cdots s_j^T \cdots s_m^T]^T$, each $s_j = [s_{1j} \cdots s_{ij} \cdots s_{mj}]^T$ is a vector composed of the switching functions associated with the j th-reference conditioning (note that each s_{ij} comes from a different actuator K_{a_i}).

In order to preserve control directionality, the discontinuous vector $w = [w_1^T \cdots w_j^T \cdots w_m^T]^T$, wherein each $w_j = [w_{1j} \cdots w_{ij} \cdots w_{mj}]^T$ comprises the discontinuous signals that shape the j th reference, is governed by the following switching law (implemented in the MIMO switching block):

$$\begin{array}{lll} w_{ij} = w_{ij}^- & \text{if } \overline{s_{ij}} < 0, & \\ w_{ij} = w_{ij}^+ & \text{if } \underline{s_{ij}} > 0, & i = 1, \ldots, m; \; j = j_0, \\ w_{ij} = 0 & \text{otherwise}, & \end{array} \tag{12}$$

whilst $w_{ij} = 0$ if $j \ne j_0$, where r_{j0} is the changed reference.

For strictly proper transfer functions of the controller, the multivariable version of Eq. (5) has relative degree greater than 1 with respect to w. Thus, the sliding functions will have to include other controller states in order to enable the establishment of sliding regimes. Although this makes the sliding functions a bit more complex, it also provides degrees of freedom that may be used to control the rate of approach to the saturation limits, as we will see in the next subsection. Hence, the sliding functions are reformulated as follows:

$$s_{ij} = u_i - u_i \quad \text{if} \quad \rho = 0, \tag{13}$$

$$s_{ij} = u_i - u_i - \sum_{\ell=1}^{\rho} k_{ij\ell} \, u_i^{(\ell)} \quad \text{if} \quad \rho \ge 1, \quad \forall i, j = 1, \ldots, m, \tag{14}$$

where ρ is the relative degree of the transfer function between the controller output u_i and input e_j, its dependence on i and j is not made explicit in order to simplify notation, $u_i^{(\ell)}$ the derivative of order ℓ of u_i and $k_{ij\ell}$ constant gains.[4] It is important to remark that no differentiation is needed for generating $u_i^{(\ell)}$, which are obtained as a linear combination of controller states and inputs in the constant matrix operator represented by the block M of Fig. 2. The inclusion of $u_i^{(\rho)}$ guarantees that the sliding functions are of relative degree 1 with respect to w_j. This property can be easily checked by verifying that $u_i^{(\rho)}$ is proportional to $e_j = r_{f_j} - y_j$, and that e_j depends on w_j because F represents a "rst-order "lter for each channel.

Similar to the previous section, (12)–(14) state that SM will establish transiently over the surfaces $s_{ij} = 0$ to shape the reference signal and prevent controller outputs from crossing their limits.

3.2. SM dynamics

The "lter F may be represented in state-space as:

$$F: \begin{array}{l} \dot{x}_f = A_f x_f + B_f r + B_w w, \\ r_f = C_f x_f, \end{array} \tag{15}$$

where $A_f = -C_f = -\lambda_f \cdot I_m$, $B_f = I_m$ and B_w is block diagonal (blocks of $1 \times m$) with dimension $m \times m^2$.

Consider also the following column realization (Chen, 1999) of the controller C:

$$C: \begin{array}{l} \dot{x}_c = A_c x_c + B_c e, \\ u = C_c x_c + D_c e, \end{array} \tag{16}$$

wherein $A_c = \mathrm{diag}(A_j)$, $B_c = \mathrm{diag}(b_j)$, $C_c = [C_1 \cdots C_m]$ and $D_c = [d_1 \cdots d_m]$, with $j = 1, \ldots, m$. A_j and C_j are matrices of $r_{d_j} \times r_{d_j}$ and $m \times r_{d_j}$, respectively, being r_{d_j} the degree of the least common denominator of the transfer functions of the j th column of C(s). Also, b_j and d_j are column vectors of r_{d_j} and m elements, respectively. Hence, (A_j, b_j, C_j, d_j) is a realization of the transfer vector between the error e_j and the controller outputs u. Picking out the ith row of C_j and the ith element of d_j (called c_{ij} and d_{ij}, respectively), it results in a realization of the transfer function between e_j and the output u_i of the controller:

$$C_{ij}(s): \begin{array}{l} \dot{x}_{c_j} = A_j x_{c_j} + b_j e_j, \\ u_i = c_{ij} x_{c_j} + d_{ij} e_j. \end{array} \tag{17}$$

[4] The indexing of these constants corresponds to the controller input–output pair (ij) and the derivative order of $u(\ell)$ in the sliding function.

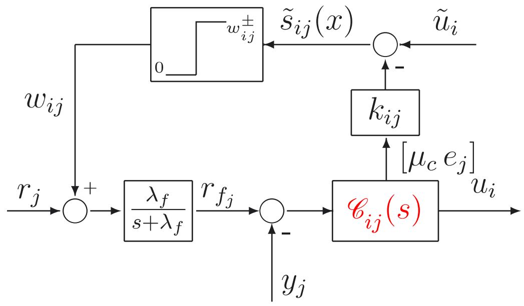

Fig. 3. Active SM conditioning loop when a limit value (u_i) is reached in u_i due to a change in r_j. $_c$ comprises the controller output u_i and its first ($-$ 1) derivatives.

As we argued in the introduction, individual changes of the reference components are considered, since this way of operation is very common in decoupled designs and it allows achieving an improved degree of decoupling in presence of input saturation. Then, and according to (12), the SM correction loop does always shape the reference which last changed, that we will call r_j from now on instead of $r_{j\,0}$ to simplify notation. In this way, each time the system is about to reach saturation in the ith component of the control vector u, the SM compensation operates over the controller transfer function between e_j and u_i, as shown in Fig. 3. The other components of the error e are not in the SM loop because they remain unchanged, and so they do not affect the SM dynamics.

Therefore, the open loop dynamics of the conditioning loop when a change in reference r_j leads the controller output u_i near its limit value is determined from (15) and (17) by:

$$\begin{bmatrix} x_{c_j} \\ e_j \end{bmatrix} = \begin{bmatrix} A_j & b_j \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_{c_j} \\ e_j \end{bmatrix} + \begin{bmatrix} 0 \\ c_{f_j} A_f x_f + c_{f_j} B_f r - y_j \end{bmatrix} + \begin{bmatrix} 0 \\ c_{f_j} B_w w \end{bmatrix}, \tag{18}$$

$$u_i = c_{ij} x_{c_j} + d_{ij} e_j, \tag{19}$$

where c_{f_j} is the j th row of C_f, and so $c_{f_j} B_f r = - {}_f r_j$ and $c_{f_j} B_w w = - {}_f [1 \ldots 1] w_j$.

If $C_{ij}(s)$ is biproper, during SM we have $e_j = d_{ij}^{-1}(u_i - c_{ij} x_{c_j})$. This results from solving (19) for e_j and equaling (13) to 0. Thus, the last row of (18) becomes redundant. Replacing the previous expression of e_j in the first r_{d_j} rows of (18), the following sliding regime dynamics is obtained:

$$x_{c_j} = Q_{c0} x_{c_j} + b_j d_{ij}^{-1} u_i, \qquad Q_{c0} = (A_j - b_j d_{ij}^{-1} c_{ij}). \tag{20}$$

It is easy to show that the eigenvalues of Q_{c0} are the zeros of $C_{ij}(s)$. Then, if the biproper transfer function involved in the compensation loop is of minimum phase the SM conditioning loop will be stable. Furthermore, SM dynamics presents interesting robustness properties. Notice that it is governed only by controller parameters, and that it is not seen from the controller output, because $u_i = u_i$ during the sliding regime.

Now, for deriving the SM dynamics corresponding to strictly proper $C_{ij}(s)$ (1, $d_{ij} = 0$), it is convenient to express (18)…(19) in its normal canonical form (Isidori, 1995):

$$\begin{aligned} u_{i_1} &= u_{i_2}, \\ u_{i_2} &= u_{i_3}, \\ \cdots &= \cdots, \\ u_{i\,-1} &= u_i, \\ u_i &= a\ _c + a\ _c + b e_j, \\ _c &= P_c\ _c + Q_c\ _c, \\ e_j &= c_{f_j} A_f x_f + c_{f_j} B_f r - y_j + c_{f_j} B_w w, \\ u_i &= u_{i_1}, \end{aligned} \tag{21}$$

where $_c = [u_{i_1}\ u_{i_2} \cdots u_i\ ]$ comprises the controller output u_i and its first ($-$ 1) derivatives, $_c$ are (r_{d_j} $-$) linearly independent states and b = 0.

When the system operates in its linear region, w = 0 and the conditioning loop is inactive. However, when a controller output reaches an actuator limit SM is established and the last equation of (21) becomes redundant. Effectively, by making (14) equal 0 the SM dynamics results:

$$\begin{aligned} u_{i_1} &= u_{i_2}, \\ u_{i_2} &= u_{i_3}, \\ \cdots &= \cdots, \\ u_{i\,-1} &= u_i, \\ u_i &= (u_i - u_i - \sum_{=2} k_{ij}\,_{(-1)}\, u_i\ )/k_{ij}\ , \\ _c &= P_c\ _c + Q_c\ _c, \\ u_i &= u_{i_1}, \end{aligned} \tag{22}$$

In this form, the zeros of $C_{ij}(s)$ are the eigenvalues of Q_c, and so they determine the hidden dynamics of the controller. Hence, either biproper or strictly proper $C_{ij}(s)$ related with a potential saturating actuator must be of minimum phase so that the SM dynamics is stable. Note, however, that this restriction on individual entries $C_{ij}(s)$ of the controller transfer matrix does not mean that the MIMO controller C or the process P must be of minimum phase. In fact, transmission zeros may be in the right half plane while all the zeros of the individual transfer functions are of minimum phase, as in the examples of the next section.

The first lines of (22) determine the evolution of the controller output u_i during SM. In Laplace domain, the constants k_{ij} are the coefficients of the characteristic polynomial of the transfer function from u_i to u_i. Thus, the dynamics of the controller output u_i only depends on the values chosen for k_{ij}. If k_{ij} are chosen properly, then once the SM is established the controller output u_i will tend towards its saturation limit u_i at a rate determined by k_{ij} and without overshooting it. Consequently, no differences will exist between u and u (they will coincide for all time), and the dynamic decoupling of the system will be preserved. This explains the choice made in (14) for the sliding surfaces. Just when the control action falls into the region delimited by the saturation limits without risk of abandoning linear operation, the SM conditioning loop will become inactive.

3.3. SM robustness and reestablishment of linear operation

A distinctive property of sliding regimes is that they are not affected by disturbances which are co-linear with the discontinuous action (i.e., that satisfy the so-called matching condition) (Sira-Ramírez, 1988). Effectively, it is said that the SM presents strong invariance to that kind of disturbances. Observe that this property is associated with the high-gain involved in sliding regimes. For our conditioning scheme r_j and y_j act as matched disturbances for the SM conditioning loop (see Fig. 3). This can also be seen from the right-hand side of (18), where the second and third terms, which can be interpreted respectively as the disturbance and the control vector fields of the conditioning loop, satisfy the matching condition. Thus, for suitable bounds on r, x_f, y and y, there always exist w_{ij}^{+} and w_{ij}^{-} that assure condition (6). If these values are properly chosen, w will be able to maintain the SM when constraints are reached. In this way, r_j and y_j affect neither the SM dynamics nor the conditioning loop stability. This is a particular feature of the current approach, and it is shown by Eqs. (20) and (22). Observe, however, that the SM generated by the switching law (12) does only reject those changes of r_j and y_j that would lead u_i to exceed its bounds. In fact, the system continues undergoing SM only if its own trajectories continue trying to cross the actuator limits. Conversely, whenever a change in r_j or y_j conducts them to reenter the linear region, the switching law (12) will make $w_{ij} = 0$ and the SM conditioning will become inactive.

During SM, the controller output u_i will coincide with or will tend towards the saturation limit u_i with the SM dynamics chosen, which is not affected by the main loop due to the robustness properties of the SM mentioned above. Actually, the limit value u_i acts as the input of the conditioning loop, whose output is u_i. Thus, the controlled variable y_j will evolve transiently (during SM) according to a serial connection of the saturation limit u_i (input), the conditioning dynamics (from u_i to u_i) and the stable dynamics of the plant (from u_i to y_j). Since control directionality does not change and only r_{f_j} is being conditioned, full decoupling is preserved and the other controlled variables remain unaffected. Thus, the whole dynamics will be stable during the transient SM operation. If the actuators are properly chosen for the control objective, i.e., either for positive or negative references

$$|[P(0)]_j\, u| > |r_j| \tag{23}$$

with $[P(0)]_j$ the j th row of the dc-gain of the plant, then the available control u will be sufficient for leading y_j close to its set-point r_j. Therefore, as was mentioned, the state trajectory will evolve naturally towards the linear region, and the SM conditioning loop will become inactive in finite time.[5] From then on, the system recovers the original closed-loop dynamics. Hence, the whole system is stable during both the transient SM operation and the normal unconstrained operation.

3.4. Additional comments

- Differing from most VSS approaches, there exists no reaching mode prior to the sliding regimes in the present strategy. In effect, since evolution within the linear region is the desired mode of operation, no control effort is taken to force the system to reach the sliding surfaces. This is an interesting property of the proposed SM conditioning algorithm because reaching phases commonly degrade the global response of VSS controllers (Mantz et al., 2015).
- Chattering, usually caused by limited frequency commutation or by unmodeled dynamics, is not present in the main control loop. On one hand, switching may be carried out at very high frequency because the discontinuous action is applied in the low power side of the system. On the other hand, the relative degree 1 of the sliding functions with respect to w is always guaranteed. Indeed, the sliding functions are obtained from feedback of the controller states, which are completely accessible, and the relative degree of the controller is perfectly known. Therefore, provided the reference filtering is fittingly designed, all the signals in the main control loop will be smooth.
- The same analysis of Section 3.2 is valid for triangularly decoupled systems with two inputs and two outputs, where the conditioning of a variable reference is performed in order to preserve decoupling of the other controlled variable. For partially decoupled systems of greater dimensions, interactions between the conditioned reference and the remaining coupled variables should be considered.

4. Examples

4.1. Example 1

Firstly, we consider a plant with the following nominal model:

$$P(s) = \frac{1}{(s+1)^2}\begin{bmatrix} s+2 & -3 \\ -2 & 1 \end{bmatrix}. \tag{24}$$

As can be easily checked, P(s) is a non-minimum phase plant with a transmission zero at $s = 4$. In order to synthesize a controller that decouples the system, we followed the ideas presented in Goodwin et al. (1997). We assume that the following complementary sensitivity is aimed:

$$T_0(s) = \frac{-9(s-4)}{4(s^2+4s+9)}\begin{bmatrix} 1 & 0 \\ 0 & \frac{10}{s+10} \end{bmatrix}. \tag{25}$$

This is accomplished with the following centralized controller:

$$C(s) = \frac{-9(s+1)^2}{4s} \times \begin{bmatrix} 1/(s+6.25) & 30/(s^2+14s+71.5) \\ 2/(s+6.25) & 10(s+2)/(s^2+14s+71.5) \end{bmatrix}. \tag{26}$$

With the objective of evaluating the proposed method we add to the system a filter like the one described in (15), even for the case where no SM correction is made. This will allow us to compare the system performance with and without SM compensation, ruling out the possibility that their differences are

[5] Observe that condition (23) is equivalent to the sufficient condition (10) derived in Section 2, because for static decoupling $S_u(0)^{-1}$ must equal P(0).

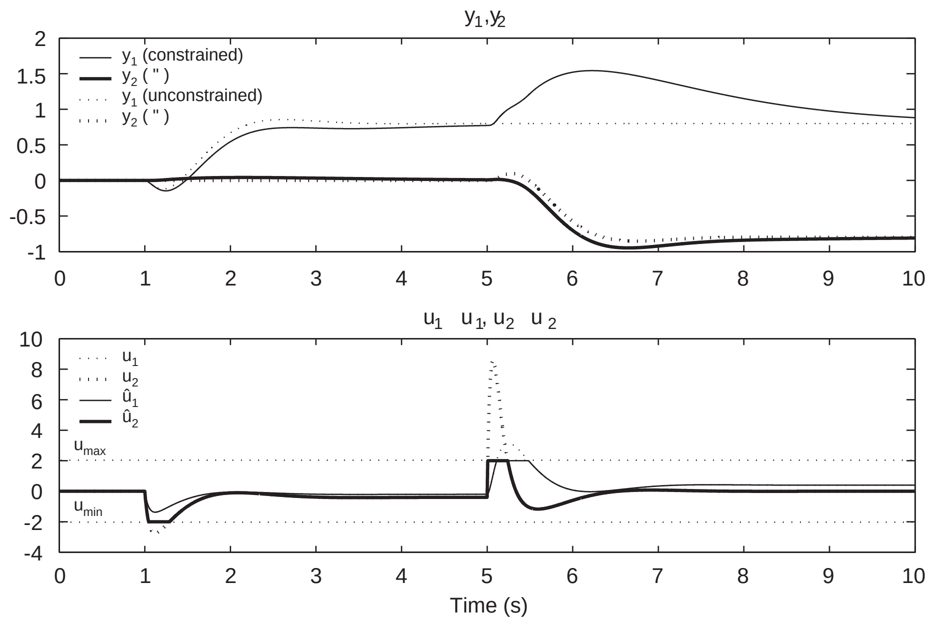

Fig. 4. Unconstrained and constrained system responses.

because of the added filter.[6] The filter dynamics was chosen to be faster than the closed loop one so that it does not affect the system response during the linear operation of the actuators. The matrices of the state-space representation of the filter are:

$$A_f = -20 \cdot I_2, \quad C_f = 20 \cdot I_2,$$
$$B_f = I_2, \quad B_w = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \tag{27}$$

where B_w only makes sense for the case with SM compensation, in which the discontinuous control vector w is generated.

The ideal actuator system was excited first with a positive step change in r_1 and then with a negative step change in r_2. Its response to such an input is shown with dotted lines in Fig. 4, where it can be verified the linear decoupled performance of the closed loop. The non-minimum phase behaviour of each channel output is the price to be paid for dynamic decoupling, and it depends on the poles and zeros of the plant in the right half plane (Morari and Zafiriou, 1989). The bottom half of the figure shows the time evolution of the control actions in both channels.

Considering now that the system has two identical actuators whose linear operations are confined to the range $[-2, 2]$, the performance is seriously deteriorated (solid lines of Fig. 4). In addition to longer settling times, the dynamic decoupling for which the controller had been designed is completely lost. The causes of this interaction can be found in the bottom half of the figure, where it is observed how the controller outputs $(\hat{u}_1, \hat{u}_2)$, which are the same as they were in the unconstrained case, differ from the plant inputs (u_1, u_2) as a consequence of saturation. The amplitude limits of the actuators are called $u_{max} = \overline{u}_1 = \overline{u}_2 = 2$ and $u_{min} = \underline{u}_1 = \underline{u}_2 = -2$.

In order to solve the change of directionality problem we applied the SM compensation proposed in Section 3 to the controller $C(s)$. From (24), the condition (23) guarantees recovery of linear operation provided $|\rho_j| < 2$, $j = 1, 2$. The matrix implemented in the block M is given by

$$M = \begin{bmatrix} -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & -.02 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ -1 & 0 & -.02 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix}, \tag{28}$$

where its input is $[\hat{u}_1\ \hat{u}_2\ \dot{\hat{u}}_1\ \dot{\hat{u}}_2\ u_{min}\ u_{max}]^T$ (canonical states were previously obtained). Note that the first four rows of M generate the sliding functions $\underline{s}_{ij}$, whilst the other four rows produce $\overline{s}_{ij}$. The order chosen for the outputs was $[s_{11}\ s_{21}\ s_{12}\ s_{22}]^T$ for both $\underline{s}_{ij}$ or $\overline{s}_{ij}$. Each component of the discontinuous control action $w = [w_{11}\ w_{21}\ w_{12}\ w_{22}]^T$ is generated in the MIMO switching block from the sliding functions. They switch according to the switching law (12), being for this example $w_{ij}^- = 1$ and $w_{ij}^+ = -1$ (these "inverted" values have to do with the negative gains of the transfer functions of $C(s)$). Only the functions $\underline{s}_{12}$ and $\overline{s}_{12}$ include $\dot{\hat{u}}_1$, in concordance with the relative degree of the transfer functions of $C(s)$.

The effectiveness of the SM proposal is verified in Fig. 5. Indeed, the outputs of the system remain dynamically decoupled and its performance is gracefully degraded (dotted curves y_1 and y_2 of Fig. 4 are repeated for comparative purposes). Besides, the controller output never exceeds the actuators

[6] A filter that avoids, by its own, the input saturation would result in an extremely conservative controller for small set-point changes.

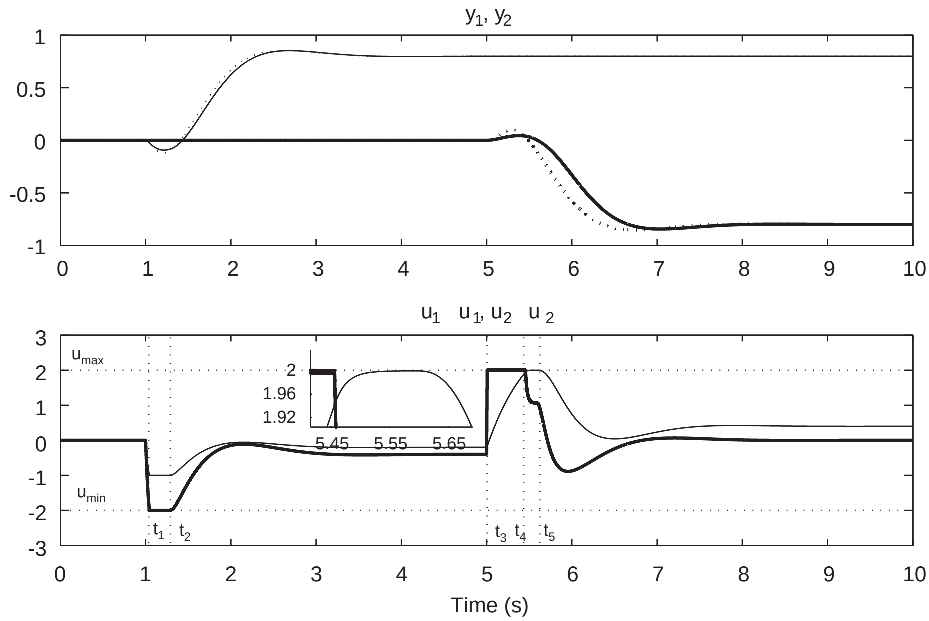

Fig. 5. System with actuators saturation and SM conditioning.

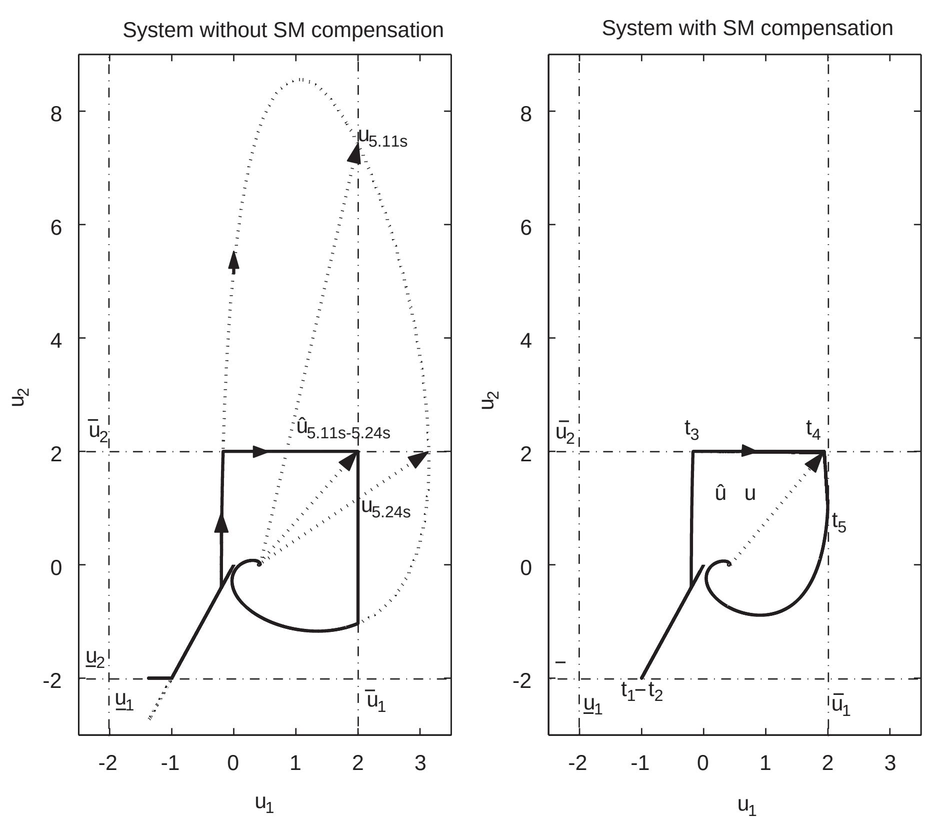

Fig. 6. Controller output $\hat{u}$ (dotted line) and plant input u (solid line) evolution.

limits, and so it coincides all the time with the plant input. When reference r_1 changes and u_2 reaches the lower limit of the second actuator, a sliding regime is established along the surface $\underline{s}_{21} = 0$ between t_1 and t_2. This SM shapes reference r_1 so that saturation of u_2 is prevented. Afterwards, SM establishes again with the step in r_2, now over two surfaces: $\overline{s}_{22} = 0$ (between t_3 and t_4) and $\overline{s}_{12} = 0$ (from t_4 to t_5). The latter surface constants were chosen for a time constant of 20 ms. This controller

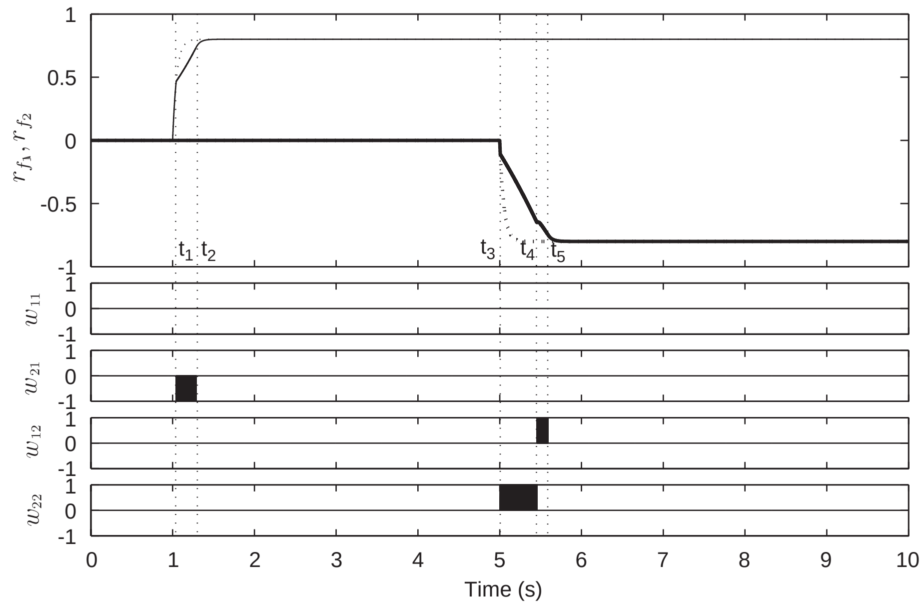

Fig. 7. Conditioned reference r_f and discontinuous action w.

output dynamics during SM avoids hard-hitting the actuator limits, as it can be appreciated in the zoomed area of the figure.

The left box of Fig. 6 demonstrates the change of directionality of the plant input $\tilde{u}$ with respect to the controller output u when no compensation is performed. Particularly, it can be seen how the controller output direction, the signal needed for decoupling, changes from $t = 5.11$ to 5.24 s (between the two longest arrows), whilst the plant input direction stays unchanged during this time period (shortest arrow). The right plot confirms that $\tilde{u} \parallel u$ when the SM compensator is added, which preserves dynamic decoupling (Fig. 5). It can also be observed how at t_4 SM establishes before reaching the limit $\bar{u}_1$ because of the dynamics imposed by $\bar{\sigma}_2 = 0$, thus controlling the rate of approach to that boundary (in concordance with the zoomed area of Fig. 5). Finally, Fig. 7 shows the reference r_f with the SM conditioning loop (solid lines) and without it (dotted lines). In addition, it presents the time evolution of the discontinuous control signals w. There it can be observed how the switching law (12) prevents from conditioning of the invariant references in each case.

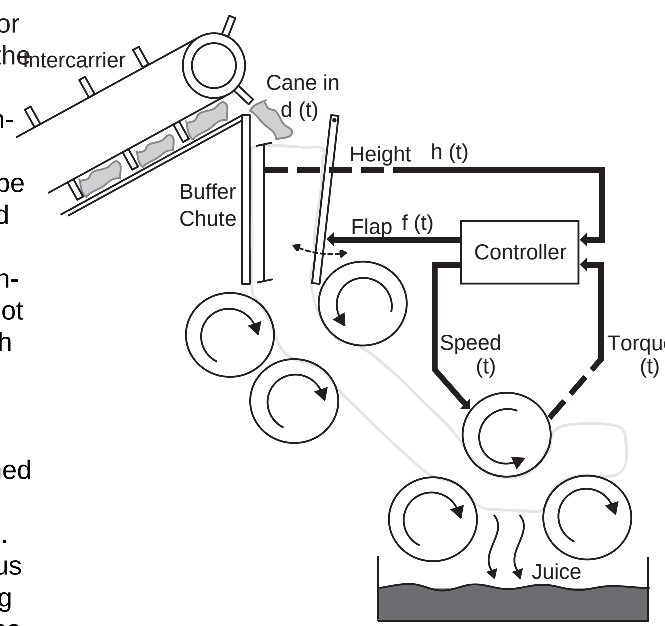

Fig. 8. A crushing station of sugar cane.

4.2. Example 2: sugar mill

As a second example, we consider the control of a sugar crushing station. A schematic diagram of this milling stage is shown in Fig. 8.

For maximal juice extraction, the buffer chute height $h(t)$ and the mill torque $\tau(t)$ are controlled by means of the position of a flap mechanism $f(t)$ and the turbine speed $\omega(t)$. While the control of the torque has significant influence on juice extraction, the main purpose of the chute height regulation is to filter out the main disturbance $d(t)$, generated by the fluctuating feed of sugar cane to the buffer chute.

The following linearized model was obtained for this process from experimental results (West, 1997):

$$\begin{bmatrix} \tau(t) \\ h(t) \end{bmatrix} = \begin{bmatrix} -\frac{5}{25s+1} & \frac{s^2-0.005s-0.005}{s(s+1)} \\ \frac{1}{25s+1} & -\frac{0.0023}{s} \end{bmatrix} \begin{bmatrix} f(t) \\ \omega(t) \end{bmatrix} + \begin{bmatrix} -\frac{0.005}{s} \\ -\frac{0.0023}{s} \end{bmatrix} d(t). \tag{29}$$

This plant has a right half plane zero at $s = 0.137$ with associated direction $h_{z0} = [1\ 5]^T$ (Morari and Zafiriou, 1989). The strong alignment of the zero with the "less important"

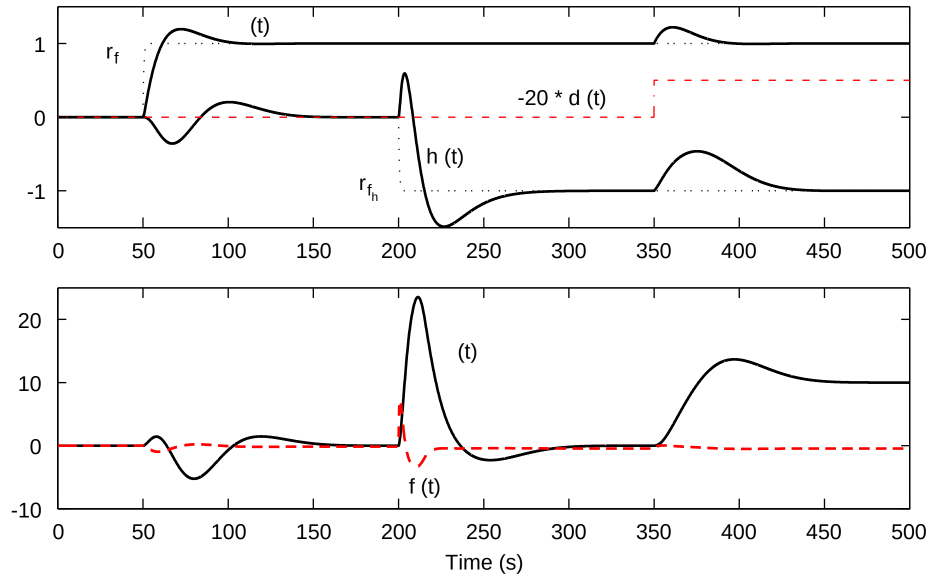

Fig. 9. Partially decoupled response with ideal actuators.

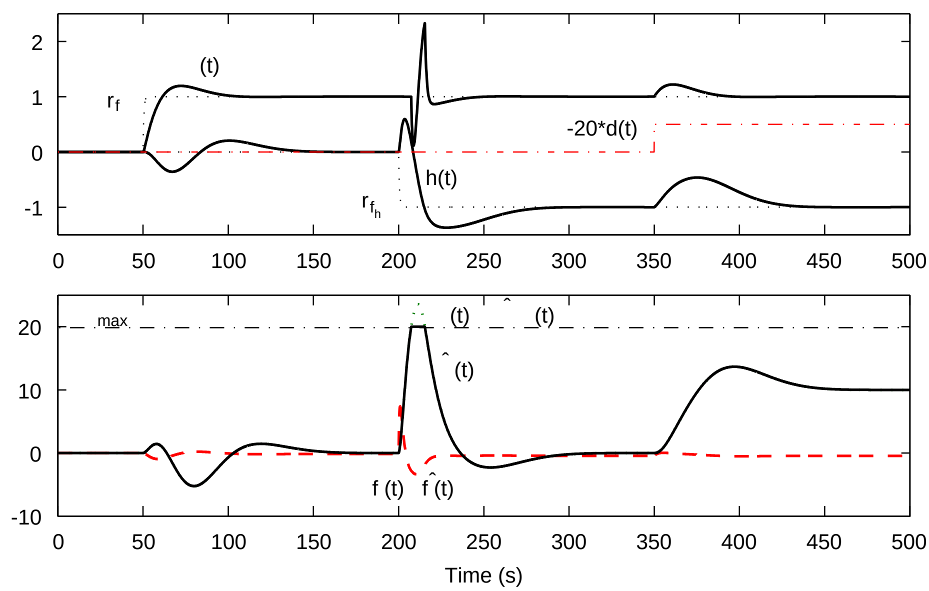

Fig. 10. Response degradation due to turbine speed saturation.

(regarding only juice extraction) variable h(t) indicates that triangular decoupling is desirable for this process, since small interactions will occur in h(t) if only (t) is decoupled. Furthermore, such a design will also push the non-minimum phase effect to the secondary variable h(t), avoiding the spreading over (t) that results from full dynamic decoupling (Garelli et al., 2006a). This was verified in Goodwin et al. (2001), where among the different designs tested on this plant, the triangular decoupling achieved the best performance.

We then took from Goodwin et al. (2001) the controller that attained triangular decoupling and ran simulations with it. The response of the closed-loop system with ideal actuators is shown in Fig. 9. It can be observed how the torque (t) is unaffected by changes in the chute height reference r_{f_h}. Moreover, the non-minimum phase zero spreading is avoided: only the chute height evolution shows step inverse response. The closed loop also compensates step disturbance d(t) rapidly, particularly in the torque channel. However, the bottom half of the figure shows that the turbine speed (t) presents quite large and sudden changes for the step on height channel, making input saturation possible. This risk of saturation is higher for greater bandwidth demands.

Hence, we introduced an isolated saturation element to the turbine speed in order to evaluate its effects on system performance. As Fig. 10 reveals, just slight saturation of the turbine speed (t) leads to great interactions with the torque when a

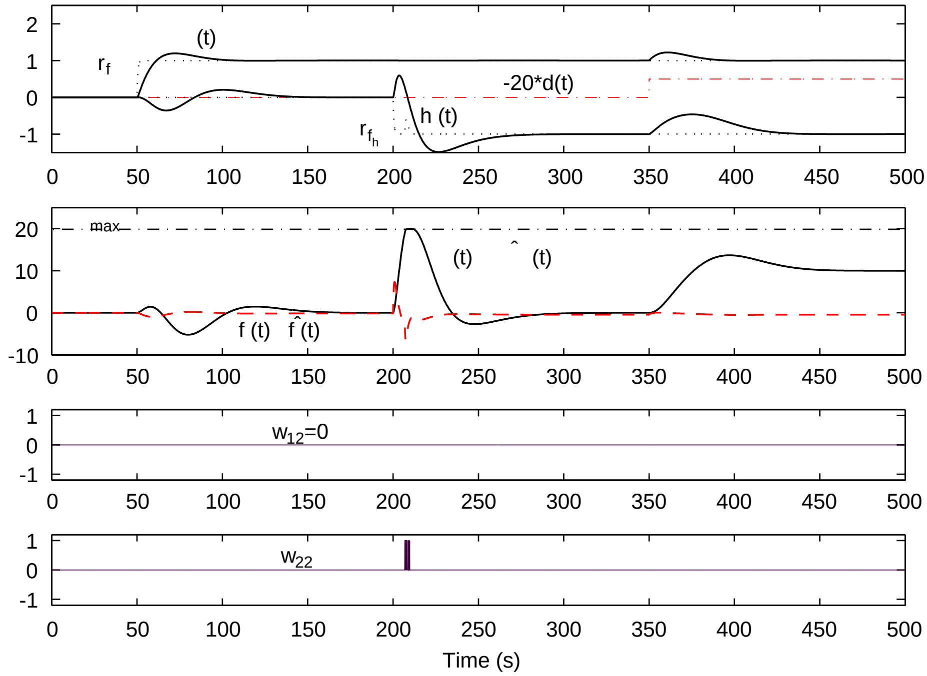

Fig. 11. Improvement of the constrained system response by means of the proposed SM conditioning.

step is applied to the height set-point. Like in the previous example, the desired decoupling is lost as a consequence of input saturation and the associated change of control directionality.

Once more, the proposed SM conditioning was added, this time in a simpli“ed con“guration. Because only torque decoupling preservation is desired, it was suf“cient to generate discontinuous signals w_{12} and w_{22} to shape $r_{f_h} = r_2$ when $f(t) = u_1$ or $(t) = u_2$ reach their bounds, respectively. The results presented in Fig. 11 show that the SM approach effectively preserves triangular decoupling in the presence of input saturation by shaping r_{f_h} with the discontinuous signal w_{22}. Notice that disturbance rejection is not affected at all since the original closed-loop performance is recovered once the system reenters the linear region.

5. Conclusions

A method to preserve dynamic decoupling of stable MIMO systems in the presence of input saturation was presented. The approach exploits the attributes of sliding regime as a transitional mode of operation, in which a discontinuous signal is used for conditioning the reference signal. The method is applicable to a great variety of centralized controllers, including strictly proper and non-minimum phase controllers, provided some derived conditions hold. As a consequence of the SM conditioning, input constraints and control directionality changes are effectively avoided without affecting the decoupling of the system.

Some remarkable features of the proposals are that: (1) the reference is conditioned only if controller outputs reach their bounds, otherwise, the SM loop is inactive and the original control system is not altered at all; (2) the dynamics of the SM conditioning loop can be designed independently of the main control loop dynamics; (3) there are neither chattering problems nor reaching mode, which usually degrade VSS responses; (4) the SM implementation is extremely simple because the conditioning loop is con“ned to the low-power side of the system.

Problems for further research are a detailed analysis of the performance costs and the derivation of an invariant set of the state-space in which control signals limits are assured, in order to be able to apply the algorithm (locally) to constrained unstable systems.

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References

Chen, C., 1999. Linear System Theory and Design, third ed. Oxford University Press, New York.

Chen, C., Peng, S., 2015. Design of a sliding mode control system for chemical processes. Journal of Process Control 15, 515…530.

Chen, C., Peng, S., 2006. A sliding mode control scheme for non-minimum phase non-linear uncertain input-delay chemical processes. Journal of Process Control 16 (1), 37…51.

Edwards, C., Spurgeon, S.K., 1998. Sliding Mode Control: Theory and Applications, “rst ed. Taylor & Francis, UK.

Eldem, V., 1994. The solution of diagonal decoupling problem by dynamic output feedback and constant precompensator the general case. IEEE Transactions on Automatic Control 39 (3), 503…511.

Garelli, F., Mantz, R., De Battista, H., 2016a. Partial decoupling of non-minimum phase processes with bounds on the remaining coupling. Chemical Engineering Science 61, 7706…7716.

Garelli, F., Mantz, R., De Battista, H., 2006b. Limiting interactions in decentralized control of MIMO systems. Journal of Process Control 16 (5), 473…483.

Goodwin, G., Feuer, A., Gómez, G., 1997. A state-space technique for evaluation of diagonalizing compensators. Systems and Control Letters 32 (3), 173…177.

Goodwin, G., Graebe, S., Salgado, M., 2001. Control System Design, “rst ed. Prentice-Hall, New Jersey.

Hanus, R., Kinnært, M., 1989. Control of constrained multivariable systems using the conditioning technique. In: Proceedings of the American Control Conference, Pittsburgh, pp. 1712…1718.

Hautus, M., Heymann, M., 1983. Linear feedback decoupling-transfer function analysis. IEEE Transactions on Automatic Control 28 (8), 823…832.

Herrmann, G., Spurgeon, S., Edwards, C., 2003. A model-based sliding mode control methodology applied to the HDA-plant. Journal of Process Control 13 (2), 129…138.

Isidori, A., 1995. Nonlinear Control Systems, third ed. Springer, London.

Kothare, M., Campo, P., Morari, M., Nett, K., 1994. A uni“ed frammework for the study of anti-windup designs. Automatica 30 (12), 1869…1883.

Mantz, R., De Battista, H., Bianchi, F., 2005. VSS global performance improvement based on AW concepts. Automatica 41 (6), 1099…1103.

Morari, M., Za“riou, E., 1989. Robust Process Control. Prentice-Hall, New Jersey.

Mulder, E., Kothare, M., Morari, M., 2001. Multivariable anti-windup controller synthesis using LMI. Automatica 37, 1407…1416.

Peng, Y., Vrancic, D., Hanus, R., Weller, S., 1998. Anti-windup design for multivariable controllers. Automatica 34, 1559…1565.

Picó, J., Picó-Marco, E., Navarro, J., De Battista, H., 2015. Control of fedbatch bioreactors. In: Nonlinear and Adaptive Control: Theory and Algorithms for the User. Imperial College Press, London.

Seron, M., Braslavsky, J., Goodwin, G., 1997. Fundamental Limitations in Filtering and Control, “rst ed. Springer, London (Chapter 4).

Sira-Ramírez, H., 1988. Differential geometric methods in variable structure systems. International Journal of Control 48 (4), 1359…1390.

Sira-Ramírez, H., 1989. Sliding regimes in general non-linear systems: a relative degree approach. International Journal of Control 50, 1487…1506.

Skogestad, S., Postlethwaite, I., 2005. Multivariable Feedback Control: Analysis and Design, second ed. Wiley, New York.

Soroush, M., Valluri, S., Mehranbod, N., 2005. Nonlinear control of input-constrained systems. Computers and Chemical Engineering 30, 158…181.

Utkin, V., 1977. Variable structure systems with sliding modes. IEEE Transactions on Automatic Control 22 (2), 212…222.

Walgama, K., Sternby, J., 1993. Conditioning technique for MIMO processes with input saturation. IEE Proceedings on Control Theory and Applications 140, 231…241.

Wang, Q.G., 2003. Decoupling Control, “rst ed. Springer, Berlin, Heidelberg.

West, M., 1997. Modelling and control of a sugar crushing station. Master•s Thesis, Department of Electrical and Computer Engineering, The University of Newcastle, Australia.

Wu, F., Grigoriadis, K., 1999. LPV based control of systems with amplitude and rate actuator saturation constraints. In: Proceedings of the American Control Conference, San Diego, pp. 3191…3195.

Zheng, A., Kothare, M., Morari, M., 1994. Anti-windup design for internal model control. International Journal of Control 60 (5), 1015…1024.