Sliding Mode Observer-based Multi-fault Reconstruction of Nonlinear System

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Abstract: For a class of nonlinear system with multi-dimensional actuator faults and multi-dimensional unknown input disturbances, the twice-linear coordinate transformation is made and the sliding mode observer is designed to obtain two dynamic deviation equations. One equation only deals with the faults and the other deals with both the faults and the disturbances. By means of the principle of equivalence of sliding mode observer and sliding mode variable structure, both the actuator faults and the unknown input disturbances can be reconstructed. The effectiveness of such method can be verified by the simulation example.

Key Words: coordinate transformation, robust fault diagnosis, fault reconstruction, sliding mode observer

1 INTRODUCTION

The robust fault diagnosis aims at detecting the minor fault as possible under the condition of insensitive unknown input disturbance such as model uncertainty. Relative to linear system, the nonlinear system is more complicated and the research is more difficult. However, the study of the nonlinear system with identified variables is practicable as some new control methods have been introduced in recent decades. For example, the sliding mode control [1] and adaptive control have been successfully applied in the fault detection and isolation. As the given phase locus is irrelevant to the external factors such as the parameters and disturbance of the controlled object, the system is robust to the unknown input disturbance when it moves on the sliding mode surface. Since the sliding mode theory was advanced to design the nonlinear system of observer[2], it had been successfully introduced in the model-based fault diagnosis.

The sliding mode observer was employed to maintain the sliding mode motion and obtain the fault reconstruction signal by analyzing the equivalent intervenient output signal, namely the equivalent control system occurred in the sliding model feedback control system[3]-[4]. Based on this method, a nonlinear system with uncertain variables was discussed in [5]. Such uncertain variables could be regarded as the actuator faults and unknown input disturbances. A sliding mode observer was designed to estimate the status and its intrinsic nature was used to reconstruct the faults. A coordinate transformation method was proposed and applied to the estimation of the uncertainties[6], this transformation was employed to launch a disturbance decoupling method for the weak and incipient actuator fault and carry out the fault detection by means of the residue signal generated by the Luenberge observer[7]. The nonlinear system in[8] comprised an actuator fault and an unknown disturbance only. The equivalent principle of sliding mode variable structure and the coordinate transformation were used to reconstruct the actuator fault and estimate the unknown input disturbance. A nonlinear system with more faults was discussed in [9]. The system contained the faults only. The faults were reconstructed by the designed sliding mode observer and their status was estimated. The nonlinear systems in [10] and [11] contain either an actuator fault and an unknown disturbance or multi-dimensional faults only. The nonlinear systems in [12] and [13] comprised multi-dimensional faults and multi-dimensional disturbances. Two new systems were obtained by linear coordinate transformation. The faults are reconstructed by the designed adaptive observer and sliding mode observer but the disturbances are not estimated.Compare to the nonlinear system, the linear system is more simple. In [14] four observers were designed for a linear over-actuated system to perform the positioning and estimation of fault.In [15] and [16], a type of linear systems are discussed to carry out fault reconstruction by the designed observer. The above solution is limited in case of the occurrence of both multi-dimensional faults and multi-dimensional unknown input disturbances in nonlinear system. Based on Literatures [7] and [8], a fault diagnosis scheme of nonlinear system including a model with uncertain fault and unknown input disturbance is initiated in this paper. Providing the geometric conditions are satisfied, the nonlinear system model is converted to two subsystems via linear coordinate transformation twice, in which one is relevant to the faults only and the other is affected by both the actuator faults and unknown input disturbances. The sliding mode observers are constructed respectively for the two subsystems to obtain the dynamic deviation equations and reconstruct and estimate the multi-dimensional fault and multi-dimensional disturbance.

The rest of this paper is organized as follows. Section

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2 describes the system and introduces some assumptions. Section 3 presents the coordinate transformation method and the design of sliding mode observer respectively. Section 4 proposes the method for reconstruction of faults and disturbances. In Sec. 5, computer simulation experiment is presented to demonstrate the effectiveness of the proposed strategy. Section 6 makes some concluding remarks.

2 Problem description

Consider an uncertain nonlinear system affected by actuator faults and unknown disturbances

$$\begin{cases} \dot{x}(t) = Ax + \Phi(x,u) + Ef_a(t) + Dd(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where $x \in R^n$, $u \in R^m$ and $y \in R^p$ denote respectively the state variables, inputs and outputs; $\Phi(x,u) \in R^n$ is a known nonlinear function. $d(t) \in R^q$ is an unknown nonlinear function lumped uncertainties and disturbances experienced by the system which is assumed to be bounded, i.e. there exists a positive constant γ_1 such that $\|d(t)\| \le \gamma_1$. The unknown nonlinear function $f_a(t) \in R^q$ represents actuator faults, which supposed to be norm bounded,i.e. there exists a constant γ_2 such that $\|f_a(t)\| \le \gamma_2$. $A \in R^{n\times n}$, $B \in R^{n\times m}$, $C \in R^{p\times n}$, $D \in R^{n\times q}$ and $E \in R^{n\times q}$ are known constant matrices,where D is the known distribution matrix of disturbances, E is the known constant distribution matrix of actuator faults and $n > p > q$.

Assumption 1.D is a column full rank matrix,and $rank\,(CD) = rank\,(D)$.

Assumption 2. The matrix pair (A, C) is observable.

There exist the two transform matrixes T and S [6] such that:

$x(t) = T^{-1}z(t) = T^{-1}\begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix}$ $y(t) = S^{-1}\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}$

and system (1) can be transformed as:

$$\begin{cases} \dot{z}(t) = \begin{bmatrix} \dot{z}_1(t) \\ \dot{z}_2(t) \end{bmatrix} = TAT^{-1}z(t) \\ \quad + T\Phi(z,u) + TEf_a(t) \\ \quad + TDd(t) + TBu(t) \\ v(t) = \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = SCT^{-1}z(t) \end{cases} \tag{2}$$

where $SCT^{-1} = \begin{bmatrix} C_{11} & 0 \\ 0 & C_{22} \end{bmatrix}$, and C_{22} is a invertible matrix.

Selected a nonsingular transformation matrix T [6]:

$$T = \begin{bmatrix} I_{n-q} & -D_1D_2^{-1} \\ 0 & I_q \end{bmatrix}$$

and each matrix in (2) is:

$TAT^{-1} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix}$

$TD = \begin{bmatrix} 0 \\ \bar{D}_2 \end{bmatrix}, TB = \begin{bmatrix} \bar{B}_1 \\ \bar{B}_2 \end{bmatrix}$

$TE = \begin{bmatrix} \bar{E}_1 \\ \bar{E}_2 \end{bmatrix}, T\Phi(x,u) = \begin{bmatrix} \bar{\Phi}_1(x,u) \\ \bar{\Phi}_2(x,u) \end{bmatrix}$

System (1) can be transformed into the following two subsystems:

$$\begin{cases} \dot{z}_1(t) = \bar{A}_{11}z_1(t) + \bar{A}_{12}z_2(t) + \bar{\Phi}_1(z,u) \\ \quad + \bar{E}_1f_a(t) + \bar{B}_1u(t) \\ v_1(t) = C_{11}z_1(t) \end{cases} \tag{3}$$

$$\begin{cases} \dot{z}_2(t) = \bar{A}_{21}z_1(t) + \bar{A}_{22}z_2(t) + \bar{\Phi}_2(z,u) \\ \quad + \bar{E}_2f_a(t) + \bar{D}_2d(t) + \bar{B}_2u(t) \\ v_2(t) = C_{22}z_2(t) \end{cases} \tag{4}$$

3 Sliding mode observer design

Assumption 3. $(\bar{A}_{11}, C_{11})$ and $(\bar{A}_{22}, C_{22})$ are observable[7].

Assumption 4. The nonlinear functions $\bar{\Phi}_1$ and $\bar{\Phi}_2$ are assumed to be known and satisfy the Lipschtiz conditions such that

$$\left\|\bar{\Phi}_1(z,u) - \bar{\Phi}_1(\hat{z},u)\right\| \le \gamma_3\|z - \hat{z}\| \le \gamma_3(\|e_1\| + \|e_2\|)$$

$$\left\|\bar{\Phi}_2(z,u) - \bar{\Phi}_2(\hat{z},u)\right\| \le \gamma_4\|z - \hat{z}\| \le \gamma_4(\|e_1\| + \|e_2\|)$$

where γ_3 and γ_4 are known positive constants called Lipschtiz constant.

Assumption 5. F_1 and F_2 satisfy the following equations

$$P_1\bar{E}_1 = C_{11}^TF_1^T$$

$$P_2\bar{E}_2 = C_{22}^TF_2^T$$

where P_1 and P_2 are symmetric positive definite matrices.

From Assumption 3 we know that there exist matrices L_1 and L_2 which make A_{01} and A_{02} stable matrices:

$$\bar{A}_{11} - L_1\bar{C}_{11} = A_{10}$$

$$\bar{A}_{22} - L_2\bar{C}_{22} = A_{20}$$

Based on the transformed systems (3)(4),the present study proposes the following two sliding mode observers described as:

$$\begin{cases} \dot{\hat{z}}_1(t) = \bar{A}_{11}\hat{z}_1(t) + \bar{A}_{12}\hat{z}_2(t) + \bar{\Phi}_1(\hat{z},u) \\ +\bar{E}_1r_1(t) + \bar{B}_1u(t) + L_1(v_1(t) - \hat{v}_1(t)) \\ \hat{v}_1(t) = C_{11}\hat{z}_1(t) \end{cases} \tag{5}$$

$$\begin{cases} \dot{\hat{z}}_2(t) = \bar{A}_{21}\hat{z}_1(t) + \bar{A}_{22}\hat{z}_2(t) + \bar{\Phi}_2(\hat{z},u) \\ +\bar{E}_2r_2(t) + \bar{B}_2u(t) + L_2(v_2(t) - \hat{v}_2(t)) \\ \hat{v}_2(t) = C_{22}\hat{z}_2(t) \end{cases} \tag{6}$$

where $\hat{z}_1$, $\hat{z}_2$ and $\hat{v}_1(t)$, $\hat{v}_2(t)$ denote respectively the estimated states and outputs, and $r_1(t)$, $r_2(t)$ represents the input signals of sliding mode observers:

$$\begin{cases} r_1(t) = \rho_1 sign(F_1(C_{11}z_1 - C_{11}\hat{z}_1)) \\ r_2(t) = \rho_2 sign(F_2(C_{22}z_2 - C_{22}\hat{z}_2)) \end{cases} \tag{7}$$

where ρ_1 and ρ_2 are all to be designed. By defining the state estimation errors $e_1(t) = z_1(t) - \hat{z}_1(t)$, $e_2(t) = z_2(t) - \hat{z}_2(t)$ and output estimation errors $e_{v1}(t) = v_1(t) - \hat{v}_1(t) = C_{11}e_1(t)$, $e_{v2}(t) = v_2(t) - \hat{v}_2(t) = C_{22}e_2(t)$. we order $e = \begin{pmatrix} e_1 & e_2 \end{pmatrix}^{\mathrm{T}}$. The observation-error dynamic equation can be computed from (3) to (6) as:

$$\dot{e}_1(t) = \left(\bar{A}_{11} - L_1C_{11}\right)e_1(t) + \bar{A}_{12}e_2(t) + \bar{\Phi}_1(z,u) - \bar{\Phi}_1(\hat{z},u) + \bar{E}_1f_a(t) - \bar{E}_1r_1(t) \quad (8)$$

$$\dot{e}_2(t) = \bar{A}_{21}e_1(t) + \left(\bar{A}_{22} - L_2C_{22}\right)e_2(t) + \bar{\Phi}_2(z,u) - \bar{\Phi}_2(\hat{z},u) + \bar{E}_2f_a(t) - \bar{E}_2r_2(t) + \bar{D}_2d(t) \quad (9)$$

Lemma 1 Consider the error dynamics system (8), (9), assumption 4-5, and if the following LMI is satisfied

$$\begin{bmatrix} \bar{A}^TP + P\bar{A} + \xi\gamma^2I_n & P \\ P & -\xi I_n \end{bmatrix} < 0 \quad (10)$$

the parameters ρ_1 and ρ_2 satisfy

$$\rho_1 > \gamma_2$$

$$\rho_2 > \gamma_2 + \frac{\left\|\bar{D}_2\right\|}{\left\|\bar{E}_2\right\|}\gamma_1$$

then e_1 and e_2 are asymptotically convergent, that is $\lim\limits_{t\to\infty} e_1(t) = 0$, $\lim\limits_{t\to\infty} e_2(t) = 0$

where ξ is a positive constant and I_q is a q-dimensional identity matrix.

Proof. Consider the following Lyapunov function

$$V = e_1^TP_1e_1 + e_2^TP_2e_2 \quad (11)$$

Let

$$\bar{A} = \begin{bmatrix} \bar{A}_{11} - L_1C_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} - L_2C_{22} \end{bmatrix}$$

$$P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}$$

The derivative of V along with the error dynamic systems (8) and (9) is

$$\begin{aligned} \dot{V} &= e^T(\bar{A}^TP + P\bar{A})e + 2e^TP(\bar{\Phi}(z,u) - \bar{\Phi}(\hat{z},u)) \\ &+ 2e_1^TP_1(\bar{E}_1f_a(t) - \bar{E}_1r_1(t)) \\ &+ 2e_2^TP_2(\bar{E}_2f_a(t) + \bar{D}_2d(t) - \bar{E}_2r_2(t)) \end{aligned} \quad (12)$$

we obtain

$$\dot{V} \le e^T(\bar{A}^TP + P\bar{A} + \frac{1}{\xi}P^2 + \xi\gamma^2I_n)e \quad (13)$$

$\dot{V}$ turns out to be negative definite by imposing

$$\bar{A}^TP + P\bar{A} + \frac{1}{\xi}P^2 + \xi\gamma^2I_n < 0 \quad (14)$$

The linear matrix inequality is satisfied

$$\begin{bmatrix} \bar{A}^TP + P\bar{A} + \xi\gamma^2I_n & P \\ P & -\xi I_n \end{bmatrix} < 0 \quad (15)$$

This completes the proof.

Thus $\dot{V}_1 < 0$ as long as $e(t) \neq 0$, so that $e(t) = 0$ is a globally asymptotically stable equilibrium point. That is

$$\lim_{t\to\infty} e_1(t) = 0 \qquad \lim_{t\to\infty} e_2(t) = 0 \quad (16)$$

Lemma 2 Choose the sliding mode surfaces $s_1 = F_1e_{v1}$, $s_2 = F_2e_{v2}$, based on Assumptions 4 and 5, then the error system (8)(9) will be driven to the sliding mode surface in finite time.

Proof. (1) Selecting the Lyapunov function as follows:

$$V_1 = \frac{1}{2}s_1^Ts_1 \quad (17)$$

The time derivative of V_1 along the trajectories of system (8) is given by

$$\begin{aligned} &\dot{V}_1 = (F_1C_{11}e_1)^T(F_1C_{11}\dot{e}_1) = \\ &(F_1C_{11}e_1)^TF_1C_{11}((\bar{A}_{11} - L_1C_{11})e_1(t) + \bar{A}_{12}e_2(t) \\ &+\bar{\Phi}_1(z,u) - \bar{\Phi}_1(\hat{z},u) + \bar{E}_1f_a(t) - \bar{E}_1r_1(t)) \end{aligned} \quad (18)$$

We design ρ_1 which satisfies

$$\rho_1 \ge \frac{(\left\|\bar{A}_{11} - L_1C_{11}\right\| + \gamma_4)\delta_1 + (\left\|\bar{A}_{12}\right\| + \gamma_3)\delta_2 + K_1}{\left\|\bar{E}_1\right\|} + \gamma_2 \quad (19)$$

then

$$\dot{V}_1 \le -K_1\left\|F_1C_{11}\right\|\left\|s_1\right\| \quad (20)$$

where K_1 is a positive constant.

This shows that the reachability condition [17] is satisfied. As a consequence, an ideal sliding motion will take place on the surface $s_{\mathrm{i}} = 0\,(i = 1,2)$ and after some finite time we have

$$F_1e_{v1} = F_1\dot{e}_{v1} = F_1C_{11}\dot{e}_1 = 0 \quad (21)$$

$$F_2e_{v2} = F_2\dot{e}_{v2} = F_2C_{22}\dot{e}_2 = 0 \quad (22)$$

4 Fault reconstruction and disturbance Estimation

From lemma 1, e_1 and e_2 will approach zero in some finite time and $\Phi(z,u) = \Phi(\hat{z},u)$. lemma 2 show that the system reaches the sliding mode surface, $s_i = \dot{s}_i = 0\,(i = 1,2)$ according to the sliding mode equivalent principle [17].With these conclusions,we can reconstruction the actuator faults and estimate the unknown input disturbances

(1)The reconstruction of actuator faults

From (21), we have $F_1C_{11}\dot{e}_1 = 0$, then

$$\begin{aligned} &F_1C_{11}((\bar{A}_{11} - L_1C_{11})e_1(t) + \bar{A}_{12}e_2(t) + \bar{\Phi}_1(z,u) \\ &-\bar{\Phi}_1(\hat{z},u) + \bar{E}_1f_a(t) - \bar{E}_1r_1(t)) = 0 \end{aligned} \quad (23)$$

With lemma 1 and (7),we have

$$f_a(t) \approx r_1(t) = \rho_1 \mathrm{sgn}(F_1 e_{v1}) \tag{24}$$

(2)The estimation of disturbances
From (22),we have $F_2 C_{22} \dot{e}_2 = 0$, then

$$\begin{aligned} &\bar{A}_{21} e_1(t) + (\bar{A}_{22} - L_2 C_{22}) e_2(t) + \bar{\Phi}_2(z,u) \\ &- \bar{\Phi}_2(\hat{z},u) + \bar{E}_2 f_a(t) - \bar{E}_2 r_2(t) + \bar{D}_2 d(t) = 0 \end{aligned} \tag{25}$$

With lemma 1 and (7),we have

$$\begin{aligned} d(t) &\approx \bar{D}_2^{-1}(\bar{E}_2 r_2(t) - \bar{E}_2 f_a(t)) \\ &= \bar{D}_2^{-1} \bar{E}_2 (\rho_2 \mathrm{sgn}(F_2 e_{v2}) - \rho_1 \mathrm{sgn}(F_1 e_{v1})) \end{aligned} \tag{26}$$

To weaken the chatteringwe define

$$sign(e_y) = \frac{e_y}{|e_y| + \sigma_1} \qquad sign(e_{v2}) = \frac{e_{v2}}{|e_{v2}| + \sigma_2}$$

where σ_1 and σ_2 are two small positive constant.

5 Simulation result

Consider a single-link robotic arm with a revolute elastic joint rotating in a vertical plane whose motion equations are [18]

$$\begin{cases} J_1 \ddot{q}_1 + F_l \dot{q}_1 + k(q_1 - q_2) + mgl \sin q_1 = 0 \\ J_m \ddot{q}_2 + F_m \dot{q}_2 - k(q_1 - q_2) = u \end{cases} \tag{27}$$

where q_1 and q_2 are the link displacement and the rotor displacement, respectively. The link inertia J_1, the motor rotor inertia J_m, the elastic constant k, the link mass m, the gravity g, the connecting rod length l, and the viscous friction coefficients F, F_m are all positive constant parameters. The control u is the torque delivered by the motor.

We define the state variables $x_1 = q_1$, $x_2 = \dot{q}_1$, $x_3 = q_2$ and $x_4 = \dot{q}_2$, It is known in practice the parameters of robot vary as time changes with handling different objects. Such changing parameters are defined as unknown input disturbances and actuator fault. Assuming there are two actuator faults and two unknown input disturbances on the single-articulation robot, the system equation as shown Equation (1) can be obtained.

Correspond (1), the parameter matrixes are

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k}{J_1} & \frac{-F_1}{J_1} & \frac{k}{J_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{J_m} & 0 & \frac{-k}{J_m} & \frac{-F_m}{J_m} \end{bmatrix}$$

$$f(x,u,t) = \begin{bmatrix} 0 \\ -0.2205 \sin x_1 \\ 0 \\ 0 \end{bmatrix}$$

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 2 & 0 \\ 0 & 1 \end{bmatrix} D = \left[\begin{array}{cc} 1 & 0 \\ 0 & 0 \\ \hline 1 & 0 \\ 0 & 2 \end{array}\right] = \left[\begin{array}{c} D_1 \\ \hline D_2 \end{array}\right]$$

$$B = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$f_a(t) = \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix} d(t) = \begin{bmatrix} d_1(t) \\ d_2(t) \end{bmatrix}$$

The transformational matrix T

$$T = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Then

$$TE = \left[\begin{array}{cc} -1 & 0 \\ 0 & 2 \\ \hline 2 & 0 \\ 0 & 1 \end{array}\right] = \left[\begin{array}{c} E_1 \\ \hline E_2 \end{array}\right]$$

$$TD = \left[\begin{array}{cc} 0 & 0 \\ 0 & 0 \\ \hline 1 & 0 \\ 0 & 2 \end{array}\right] = \left[\begin{array}{c} 0 \\ \hline D_2 \end{array}\right]$$

So we obtain

$$D_2^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}$$

The transformational matrix S is

$$S = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Design the matrices as $P_1 = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$, $P_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $F_1 = \begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix}$, $F_2 = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$, $L_1 = \begin{bmatrix} 3 & -1 \\ 2 & 3.25 \end{bmatrix}$ and $L_2 = \begin{bmatrix} 4 & -1 \\ 2 & -1 \end{bmatrix}$.

The actuator fault reconstruction algorithm

$$\hat{f}_a(t) \approx \rho_1 sign\left(F_1 e_{v1}\right) = \begin{bmatrix} \rho_{11} \frac{-2e_1}{|-2e_1| + \sigma_1} \\ \rho_{12} \frac{2e_2}{|2e_2| + \sigma_1} \end{bmatrix}$$

and the unknown disturbances

$$\begin{aligned} &d(t) \approx \bar{D}_2^{-1} \bar{E}_2 (\rho_2 \mathrm{sgn}(F_2 e_{v2}) - \rho_1 \mathrm{sgn}(F_1 e_{v1})) \\ &= \begin{bmatrix} 2 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} \rho_{21} \frac{2e_3}{|2e_3| + \sigma_2} - \rho_{11} \frac{-2e_1}{|-2e_1| + \sigma_1} \\ \rho_{22} \frac{e_4}{|e_4| + \sigma_2} - \rho_{12} \frac{2e_2}{|2e_2| + \sigma_1} \end{bmatrix} \end{aligned}$$

The robot parameters are $k = 2Nm/rad$, $F_m = 1$, $F_1 = 0.5Nm/(rad/s)$, $J_\mathrm{m} = 1Nm^2$, $J_1 = 2Nm^2$, $m = 0.15kg$, $g = 9.8m/\mathrm{s}^2$ and $l = 0.3m$. The input $u = \sin(5t) + 4\sin(20t)$. The designed parameters $\rho_1 = [\ \rho_{11} \quad \rho_{12}\ ]^T$, $\rho_2 = [\ \rho_{21} \quad \rho_{22}\ ]^T$, where $\rho_{11} = \rho_{12} = 40$, $\rho_{21} = 80$ and $\rho_{22} = 40$, and $\sigma = [\ \sigma_1 \quad \sigma_2\ ]^T$, where $\sigma_1 = \sigma_2 = 0.01$. $e_{v1} = \left[\ e_{11} \quad e_{12}\ \right]^T$, $e_{v2} = \left[\ e_{21} \quad e_{22}\ \right]^T$. The initial value of state variable x in the simulation example are chosen as -2, -3, -3 and -2. Two overlapped sine signals are used to simulate the incipient fault f_1. where $f_1 = 2sin40t + 2sin5t$. A 0.5s-cycle square wave is used as the given data of fault f_2, which is the combination of abrupt fault and intermittent fault. A white noise with amplitude of 3 is used as the unknown input disturbance d_1. The unknown input disturbance d_2 is a sine wave, where $d_2 = sin5t$. The simulation results are as follows:

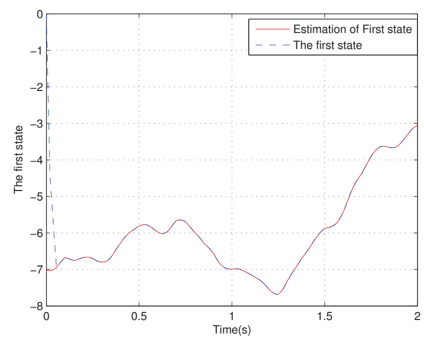

Figure 1: The first state x_1 and its estimation $\hat{x}_1$

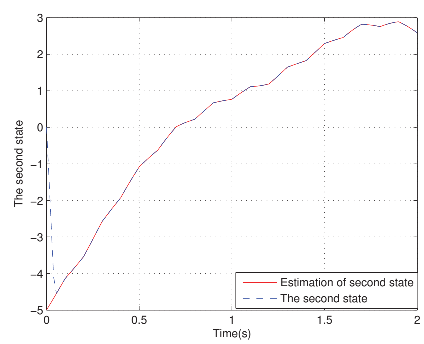

Figure 2: The second state x_2 and its estimation $\hat{x}_2$

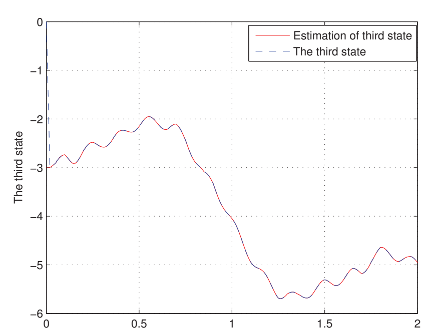

Figure 3: The third state x_3 and its estimation $\hat{x}_3$

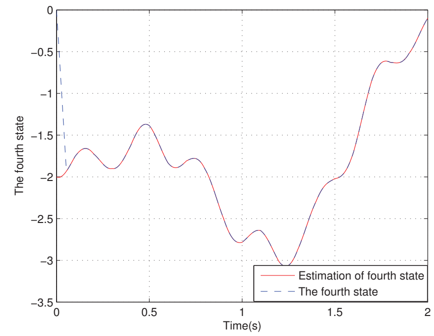

Figure 4: The forth state x_4 and its estimation $\hat{x}_4$

Figures 1, 2, 3 and 4 respectively indicate for the estimations of statuses x_1, x_2, x_3 and x_4 reach the given data rapidly at the different initial data and remain stable. The tracking performance is excellent. The above simulation results imply that convergence rate of observer is high, which sets a foundation for the reconstruction of the multi-dimensional fault and unknown input disturbance in the context below.

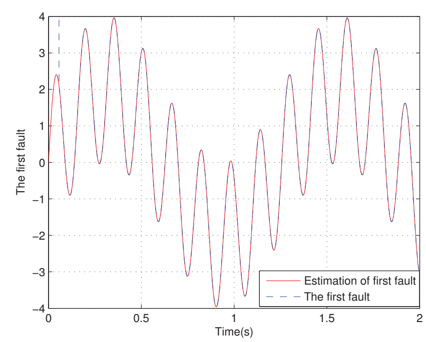

Figure 5: The fault f_1 and its estimation $\hat{f}_1$

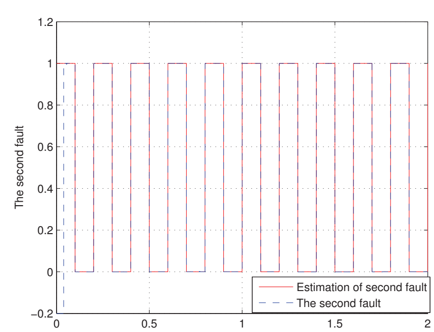

Figure 6: The fault f_2 and its estimation $\hat{f}_2$

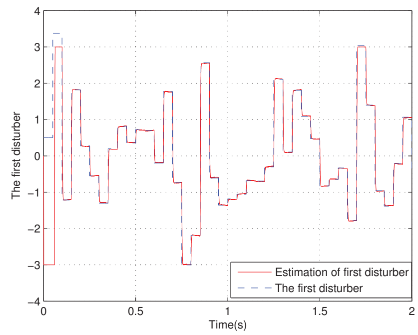

Figure 7: The disturbance d_1 and its estimation$\hat{d}_1$

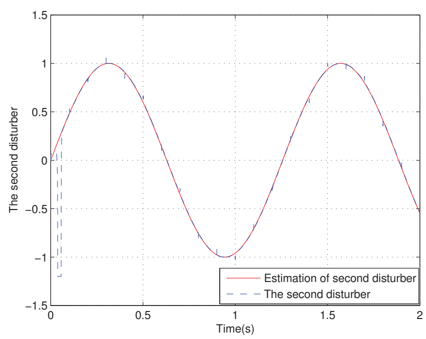

Figure 8: The disturbance d_2 and its estimation$\hat{d}_2$

Figures 5 respectively the reconstruction results of fault f_1 and Figures 6 indicate the reconstruction of Fault f_2.The above simulation results imply both incipient fault f_1 and abrupt fault f_2 can be precisely reconstructed. Figures 7 and 8 indicate the reconstruction of unknown input disturbances. The tracking performance is well.

6 Conclusions

In this paper, a nonlinear system with multi-fault and multi-disturbance is studied. The coordinate transformation is made for the original system twice, in which one subsystem deals with only fault and the other deals with both fault and disturbance. The precise reconstruction of fault and disturbance is obtained by means of the sliding mode variable structure equivalent principle. The nonlinear system in the simulation example comprises 2-dimensional fault and 2-dimensional disturbance, each being precisely reconstructed. The simulation results verify the effectiveness of such method.

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