

The Practice of MATLAB Simulation in Modern Control Theory Course Teaching

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Abstract—Modern control theory is one of the basic professional course for the senior undergraduates, such as automation, electrical engineering and automation specialties. It has the characteristics of higher theoretical definition and abstraction, bigger calculated amount, degree of depth and complexity, and the students should have well-knit mathematical foundation. The study can be made more interesting and challenging by introducing the simulation technology into course teaching. By applying MATLAB language and Simulink simulation tools, combining with the teaching examples, the system synthesis design problem in modern control theory course can be dealt with very easy. A closed-loop control system with full-order state observer is designed. The design process shows that fundamental changes in modern control theory course teaching have been made by MATLAB language and Simulink tools. The teaching process is converged to the combination of practical engineering case and vivid teaching from original dry pure theory teaching.

I. INTRODUCTION

MODERN control theory is one of the basic professional courses for the senior undergraduates, such as automation, electrical engineering and automation specialties. It is also suitable for some specialties as a public technology course, such as the electrical information class, electro-mechanical class and the economics and management specialties and so on. The basic task of this course is studying the methods of linear system analysis and synthesis. It has the characteristics of higher theoretical definition and abstraction, bigger calculated amount, degree of depth and complexity, and the students should have well-knit mathematical foundation [1], [2]. Due to the complexity of matrix operation and the abstraction of course, it is difficult for the students to understand and mastery the contents, even produce disgusted mood. By introducing the simulation software MATLAB into the modern control theory course teaching, it maybe helpful to solve these problems.

MATLAB is the abbreviation of Matrix Laboratory, and is a business mathematics software of America MathWorks company. It is commonly used in algorithm development, data visualization, data analysis, numerical calculation and so on, mainly contains two parts: MATLAB and Simulink. Now it has become one of the basic mathematical tools of automatic control, dynamic simulation, modern control theory, intelligent control and other advanced courses.

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Simulink is an important one of the components, which provides an integrated environment to the user, such as dynamic systems modeling, simulation and comprehensive analysis. In this environment, the user does not need to write a large number of procedures, but only to operate the mouse simply, select appropriate library modules. Then complex simulation model can be constructed. Simulink has many advantages, such as wide adaptability, structure and procedure clearer, more precise simulation etc [3], [4].

In the teaching process, graphical and interactive of the main teaching contents maybe realized easily through the organic combination of MATLAB language, Simulink and the multimedia courseware of modern control theory. To do this, the teaching process will be more vivid and visual, and the learning interest is excited. Meanwhile, it is helpful to strengthen the comprehension of abstract theory knowledge, obtain some engineering practice knowledge, and enhance analysis ability of the students. In this paper, MATLAB language and Simulink simulation tools are briefly introduced into the control system design applications. The practice proved that fundamental changes have been made by MATLAB language and Simulink tools in modern control theory course teaching. The teaching process is converged to the combination of practical engineering case and vivid teaching from original dry pure theory teaching.

II. SYNTHESIS DESIGN OF MODERN CONTROL THEORY

The study object of modern control theory is linear system, the study purpose is to construct a state space model of the linear system firstly, and then analysis and synthesis the linear system. The mainly contents are time analysis、Lyapunov stability analysis、controllability and observability analysis、state feedback and state observer design and so on. In modern control theory, two ways are mainly used to improve the system performance. One is state feedback, the other is output feedback. The attributes and feedback function of state feedback are higher than these properties of output feedback. Therefore, the state feedback method is usually used to improve the system performances during system synthesis design. For a completely controllable linear time-invariant system, the poles of closed loop system can be arbitrary assigned via state feedback, so that the desired dynamic characteristics can be obtained. This process is called a poles placement problem. However, in commonly cases, the system state is not necessarily measurable. This makes the realization of state feedback is difficult in physics. Therefore, the system states are needed to be reconstructed through some known variables. This process is called a state observer design problem. In the traditional teaching process,

the two methods above need to be analyzed in details, the workload is big. But, the poles placement problem and state observer design problem can be realized conveniently by applying the *place* function or the *Acker* function after introduce MATLAB software. Meanwhile the designed closed-loop system can be simulated visually by Simulink simulation tools.

A. Poles Placement Problem

The distribution situation of closed-loop poles determines the system stability and dynamic quality. Therefore, in the system design, the expected closed-loop poles can be determined firstly according to the requirement of system performance. Then the poles of closed-loop system would be assigned at desired positions via state feedback. The poles placement problem can be described as follows.

Consider the LTI system which is described by

$$\begin{cases} \dot{X}(t) = AX(t) + bu(t) \\ y(t) = cX(t) \end{cases} \quad (1)$$

if we introduce the state feedback control

$$u(t) = -KX(t) + r(t) \quad (2)$$

into the open-loop system (1), the eigenvalues, or poles, of the closed-loop system

$$\begin{cases} \dot{X}(t) = (A - bK)X(t) + bv(t) \\ y(t) = cX(t) \end{cases} \quad (3)$$

can also be placed arbitrarily on the expected locations, $s_1^*, s_2^*, \dots, s_n^*$, and the state feedback gain vector can be constructed as

$$K = [k_1 \quad k_2 \quad \dots \quad k_n] \quad (4)$$

the transfer function of it is shown in (5).

$$G(s) = c[sI - (A - bK)]^{-1}b \quad (5)$$

B. Full-order State Observer Design Problem

For a completely controllable system, the closed-loop poles can be placed arbitrary. But due to the states can not be measured directly, or the restriction of measuring equipment in economy and fitness of use, the state feedback is difficult to realize in physics. One way of overcoming this difficulty is to reconstruct the system state. And then a desired state feedback can be realized by replacing the real states with the reconstruction states. The essence of state reconstruction is to construct a new system, let the measurable input variable $u(t)$ and output variable $y(t)$ as its input signals, make its output signal $\hat{X}(t)$ equal to the state $X(t)$ of original system in some conditions. The output signal $\hat{X}(t)$ is called a reconstructed state or state estimated value of the original state $X(t)$, and the new system is called a state observer.

Assume the states, which are used to feedback above, are impossible to be measured. A full-order observer can be designed to estimate them. The reconstructed system can be described by (6).

$$\begin{cases} \dot{\hat{X}}(t) = (A - Lc)\hat{X}(t) + bu(t) + Ly(t) \\ \hat{y}(t) = c\hat{X}(t) \end{cases} \quad (6)$$

Where, $\hat{X}(t)$ is the estimate of the actual state $X(t)$, and the feedback gain vector is shown in (7).

$$L = [l_1 \quad l_2 \quad \dots \quad l_n]^T \quad (7)$$

C. State Feedback Control System With State Observer

It is clear that if a system is controllable and the states are available for feedback, then the poles of the system can be placed arbitrarily by introducing a constant feedback control. However, in most practical applications, the states are not completely accessible and all the designer knows are the output $y(t)$ and input $u(t)$. A closed-loop state feedback control system is shown in Fig. 1, in which the state is estimated by a full-order observer.

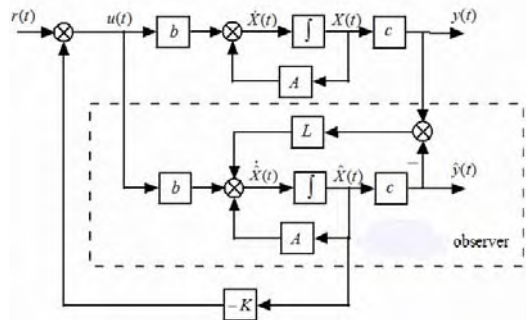


Fig. 1. Closed-Loop System via Estimated State Feedback

Where, the feedback control law is designed by the estimated state, it is

$$u(t) = -K\hat{X}(t) + v(t) \quad (8)$$

Obviously, the dimension of the closed-loop system in Fig.1 is $2n$, i.e. the system has n actual state variables and n estimated state variables. The system can be described by (9).

$$\begin{cases} \begin{bmatrix} \dot{X}(t) \\ \dot{\hat{X}}(t) \end{bmatrix} = \begin{bmatrix} A & -bK \\ Lc & A - Lc - bK \end{bmatrix} \begin{bmatrix} X(t) \\ \hat{X}(t) \end{bmatrix} + \begin{bmatrix} b \\ b \end{bmatrix} r(t) \\ y(t) = \begin{bmatrix} c & 0 \end{bmatrix} \begin{bmatrix} X(t) \\ \hat{X}(t) \end{bmatrix} \end{cases} \quad (9)$$

According to the separation principle, the transfer function of system (9) is equal to the transfer matrix of the state feedback system without using the observer, such as (6). and it can be calculated as (10).

$$G(s) = c[sI - (A - bK)]^{-1}b \quad (10)$$

In order to make the dynamic of the state error converge to zero as well as much faster than the closed-loop dynamic, the expected poles of observer are often chosen as (11).

$$\text{Re}[\lambda_i^*(A - Lc)] \leq (2 \sim 3) \text{Re}[\lambda_i^*(A - bK)] < 0 \quad (11)$$

Where, $\lambda_i^*(A - bK)$ are the expected poles of the close-loop system by introducing the state feedback control.

III. DESIGN AND SIMULATION EXAMPLE OF CLOSED-LOOP CONTROL SYSTEM WITH STATE OBSERVER

A open-loop control system is shown in Fig.2.

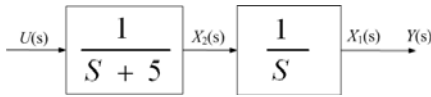


Fig. 2. block diagram of a control system

Please design a closed-loop control system with a full-order state observer, the performance indexes which should be satisfied are listed as follows:

1) Dynamic performance indexes: overshoot: $\sigma\% \leq 5\%$;
peak time: $t_p \leq 0.5s$; band width: $\omega_b \leq 10rad/s$.

2) Steady state performance index: static position error :
 $e_p = 0$ (for the step input).

The design and simulation process of the closed-loop control system with a full-order state observer is illustrated as follows.

A. Determine The State Space Description Of Original System

According to the system block diagram, we can obtain

$$X_1(s) = \frac{1}{s} X_2(s)$$

$$X_2(s) = \frac{1}{s+5} U(s)$$

$$Y(s) = X_1(s)$$

Rewriting them and by introducing inverse Laplace transform we can obtain that

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -5x_2 + u \\ y = x_1 \end{cases}$$

The differential equations above are expressed in matrix form notation as

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -5 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

This is the state space description of the original system.

B. Determine The Expected Closed-loop Poles

In order to satisfy the demand of overshoot: $\sigma\% \leq 5\%$, optimal damping ratio $\xi = 0.707$ is chosen, and $\sigma = 4.3\%$ satisfies the demand about overshoot.

Due to the demand of system band width: $\omega_b \leq 10rad/s$, and $\omega_b = \omega_n \sqrt{1 - 2\xi^2 + \sqrt{2 - 4\xi^2 + 4\xi^4}}$, substitutes $\xi = 0.707$ into it, $\omega_n = \omega_b \leq 10rad/s$ can be deduced.

$\omega_n = 10rad/s$ is selected.

Here, peak time $t_p = \frac{\pi}{\omega_n \sqrt{1 - \xi^2}} \approx 0.45s$ satisfies the demand of peak time $t_p \leq 0.5s$.

The expected poles of the closed-loop system are

$$s_{1,2} = -\xi\omega_n \pm j\omega_n \sqrt{1 - \xi^2} = -7.07 \pm j7.07$$

According to the constraint relationship (11) between the observer poles and the expected poles of closed-loop system, the observer poles can be set to be $s_{1,2} = -21$.

C. MATLAB Design Of The State Feedback Control Law And Observer

Input the instructions at the MATLAB command window:
a) % whether the system poles can be placed arbitrarily?

```
>>A=[0 1;-5];b=[0 1]';
>>Qc=ctrb(A,b); nc=rank(Qc)
nc =
    2
```

% poles placement is realized by applying *place* function

```
>>P=[-sqrt(50)+sqrt(-50);-sqrt(50)-sqrt(-50)];
>>K=place(A,b,P)
K =
```

```
100.0000  9.1421
```

% inspect whether the poles of the new system are the expected closed-loop poles

```
>>Ac=A-b*K;
>>D=eig(Ac)
D =
-7.0711 + 7.0711i
-7.0711 - 7.0711i
```

From the results, we can see that the poles of the new system are the expected poles. Thus, the state feedback gain matrix is

$$K = [100 \quad 9.14]$$

By introducing the state feedback control law (2), the poles of the closed-loop system can be placed at the expected locations, $-7.07 + j7.07, -7.07 - j7.07$.

b) % whether a full-order observer can be designed?

```
>>A=[0 1;-5];b=[0 1]';c=[1 0];
>>Qo=obsv(A,c); nc=rank(Qo)
nc =
```

2% calculate the feedback gain matrix by applying *acker* function

```
>>P1=[-21 -21]; A1=A';b1=c'; c1=b';
>>K=acker(A1,b1,P1);
>>L=K'
>>ALc=A-L*c
```

```
L =
    35
    225
ALc =
   -35    1
   -225   -5
```

Thus, the feedback gain matrix of the full order observer is

$$L = \begin{bmatrix} 35 \\ 225 \end{bmatrix}$$

and the full order observer is

$$\begin{aligned} \dot{\hat{X}}(t) &= (A - Lc)\hat{X}(t) + bu(t) + Ly(t) \\ &= \begin{bmatrix} -35 & 1 \\ -225 & -5 \end{bmatrix} \hat{X}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) + \begin{bmatrix} 35 \\ 225 \end{bmatrix} y(t) \end{aligned}$$

Block diagram of the closed-loop system with a full-order state observer is shown in Fig.3.

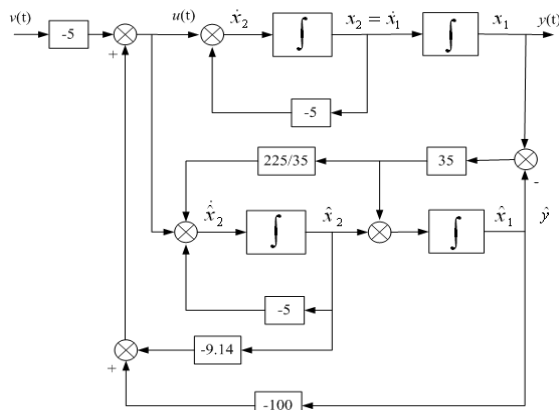


Fig. 3. Block Diagram of the Closed-loop System with a Full-order State Observer

D. Simulation Calculation By Applying Simulink Tools

Open the model editing window, pull all the needed modules from the related modules to this window, connect them and modify the module parameters, the simulation diagram of the closed-loop system with a full-order state observer is obtained, as shown in Fig.4. Double-click the scope module, and the simulation results can be got. At the same time, input the following instructions at the command window:

```
>>set(0,'ShowHiddenHandles','On')
```

```
>>set(gcf,'menubar','figure')
```

the hidden graphics editing tools is opened, edit the graphic and the unit step response curve can be got, as shown in Fig.5.

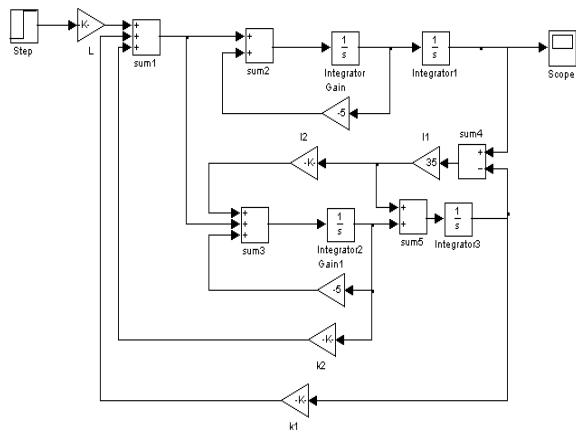


Fig. 4. The Simulation Diagram of the Closed-loop System with a Full-order State Observer

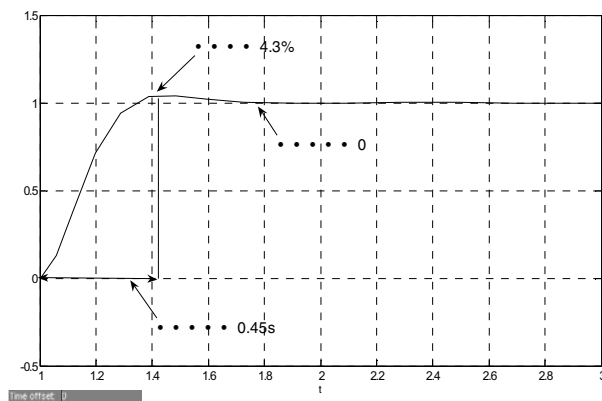


Fig. 5. The Unit Step Response Curve of the Closed-loop System

From Fig.5, we can see that the overshoot, peak time and static position error of the closed-loop system with the full-order state observer all satisfy the design demands, the expected design target is achieved. The example above is a typical system synthesis design problem by using MATLAB and Simulink simulation tools. It can be seen that the poles placement and observer design process can be vividly and visually seen through the simulation tools. Meanwhile, the unit step response curve can be mapped directly. And then, from the given unit step response curve, various performance indexes can be directly read out, tedious computation is avoided, enthusiasm of students and teaching quality are greatly improved.

IV. CONCLUSION

The applications of MATLAB in modern control theory teaching are not limited to the synthesis design problem, more additional applications will be infiltrated in the teaching of every chapter. Through creation of teaching situations, using MATLAB for the curriculum design and typical problems teaching, the quality of teaching is improved, the students' learning initiative is strengthened, comprehensive practical ability and creative thinking ability are trained, comprehensive quality is improved, thus the teaching goal is achieved. To make this course become fully reflect the modern control law and characteristic, closely combining with engineering practice, to cultivate "application type, compound talents". The course is one of the Shandong University of Technology class excellent courses and provincial excellent courses.

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