Proceedings of the IEEE
International Conference on Automation and Logistics
Shenyang, China August 2009

Real-Time Controlling of Inverted Pendulum by Fuzzy Logic*

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***Abstract* - Inverted pendulum is a typical multivariable, nonlinear, strong-coupling, instable system. The basic aim of our work was to balance a real pendulum in the position in center of course. For this purpose we used fuzzy logic controller. The fuzzy logic controller designed in the Matlab-Simulink environment. In this paper, the inverted pendulum mathematical model is built. MATLAB based Hardware in Loop simulation system is designed. A novel expert fuzzy control scheme was proposed. The proposed control scheme was implemented in Matlab and showed good performance in the real-time fuzzy control of the inverted pendulum. The results of simulation and experiment indicated that the control method has good control ability**.

***Index Terms* - inverted pendulum, mathematical model fuzzy control, real-time,**

I. INTRODUCTION

As a typical unstable nonlinear system, inverted pendulum system is often used as a benchmark for verifying the performance and effectiveness of a new control method because of the simplicities of the structure [1-2]. Stabilizing control of inverted pendulum is to balance the pendulum in the upright position. This is a very typical and academic nonlinear control problem, and many techniques already exist for its solution [3], for example, model-based control, fuzzy control, neural network(NN) control, genetic algorithms (GAs)-based control, and so on. However, but the controller was difficult to completely stabilize a pendulum system within a short time. In this paper, a data-driven Takagi-Sugeno fuzzy model called ANFIS is employed for the stabilizing control, and a semi-physical simulation test system for liner inverted pendulum is studied and constructed by use of the semi-physical theory. The rapid prototyping designing technology consists of software exploitation, hardware development and the algorithm designing and realizing, which offers a way to help develop swiftly [4-6].

II. MATHEMATICAL MODELING OF INVERTED PENDULUM

The inverted pendulum system is a classic control problem that is used in universities around the world. It is a suitable process to test prototype controllers due to its high non-linearities and lack of stability. The system consists of an inverted pole hinged on a cart which is free to move in the x direction as shown in Figure 1.

Fig.1 The inverted pendulum used in control laboratories

In this section, the model of the single inverted pendulum is established and the dynamical equations of the system will be derived. Figure 2 shows the free body diagram of the system.

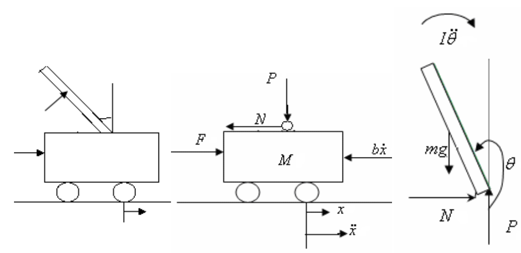

Fig.2 Free body diagrams of the system.

The single inverted pendulum system is made of a cart on top of which pendulum is pivoted. The cart is constrained to move only in the horizontal x direction, while the pendulum can only rotate in the x-y plane. The single inverted pendulum system has two degrees of freedom and can therefore be fully represented using two generalized coordinates: horizontal displacement of the cart, and rotational displacement of pendulum. The physical properties of the system are fixed in Table 1.

Table 1. Properties of the Cart and Pendulum

M	Mass of cart	1.096 Kg
m	Mass of pendulum	0.109Kg
b	Friction of cart	0.1 N/m/sec
l	Length to pendulum centre of mass	0.25m
I	inertia of the pendulum	0.0034 kg*m*m
F	Force applied to cart	
x	Cart position coordinate	
φ	Pendulum angle from the vertical	
θ	Pendulum angle from the vertical downwards	

From Figure 2, applying Newton's second law of motion to the cart system and by assuming the (nonlinear) coulomb friction applied to the linear cart is assumed to be neglected.

* This work is partially supported by project of science and technology of education department of Liaoning province #2008552 to Liu Yanmei.

978-1-4244-4795-4/09/$25.00 © 2009 IEEE

Moreover, the force on the linear cart due to the pendulum's action has also been neglected in the presently developed model. the following dynamic equations in horizontal direction (1) and vertical direction (2) are determined.

Horizontal direction:

$$M\ddot{x} = F - b\dot{x} - N$$

$$N = m\frac{d^2}{dt^2}(x + l\sin\theta)$$

$$N = m\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta$$

$$(M + m)\ddot{x} + b\dot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = F \quad (1)$$

Vertical direction:

$$P - mg = m\frac{d^2}{dt^2}(l\cos\theta)$$

also: $P - mg = -ml\ddot{\theta}\sin\theta - ml\dot{\theta}^2\cos\theta$

$$-Pl\sin\theta - Nl\cos\theta = I\ddot{\theta}$$

$$\theta = \pi + \phi, \cos\phi = -\cos\theta, \sin\phi = -\sin\theta$$

$$I = \tfrac{1}{3}ml^2$$

$$\tfrac{4}{3}ml^2\ddot{\theta} + mgl\sin\theta = -ml\ddot{x}\cos\theta \quad (2)$$

Suppose : $\theta = \pi + \phi$, 《1,

$\cos\theta = -1$, $\sin\theta = -\phi$, $(\frac{d\theta}{dt})^2 = 0$,u=F

The linearized model can be obtained by considering the small variations about the equilibrium point when the pendulum is at upright position and neglecting higher order term. After linearization, the dynamic equations (3) and equation (4) are obtained:

$$\begin{cases} \frac{4}{3}l\ddot{\phi} - g\phi = \ddot{x} \\ (M + m)\ddot{x} + b\dot{x} - ml\ddot{\phi} = u \end{cases} \quad (3)$$

Applying the Laplace transform, yields the transfer function, such that:

$$\begin{cases} (I + ml^2)\Phi(s)s^2 - mgl\Phi(s) = mlX(s)s^2 \\ (M + m)X(s)s^2 + bX(s)s - ml\Phi(s)s^2 = U(s) \end{cases} \quad (4)$$

By manipulating the dynamics equations (3) and equation (4), and substituting the parameter values of the cart and pendulum, the following transfer function of the pendulum's angle and transfer function of cart's position are obtained as shown in equation (5) and equation (6) respectively.

$$\frac{\Phi(s)}{X(s)} = [\frac{4}{3}l - \frac{g}{s^2}]$$

$$\frac{\Phi(s)}{U(s)} = \frac{\frac{ml}{q}s^2}{s^4 + \frac{\frac{4}{3}bml^2}{q}s^3 - \frac{(M + m)mgl}{q}s^2 - \frac{bmgl}{q}s} \quad (5)$$

$$q = [(M + m)(I + ml^2) - (ml)^2]$$

$$\frac{\Phi(s)}{X(s)} = \frac{0.02725s^2}{0.0102125s^2 - 0.26705}$$

$$\frac{\Phi(s)}{U(s)} = \frac{2.35655s}{s^3 + 0.0883167s^2 - 27.9169s - 2.30942} \quad (6)$$

The transfer functions can be represented in state–space form and output equation as stated by equation (7) and equation (8).

$$\begin{cases} \dot{x} = \dot{x} \\ \ddot{x} = \frac{-4b}{(4M + m)}\dot{x} + \frac{3mg}{(4M + m)}\phi + \frac{4}{(4M + m)}u \\ \dot{\phi} = \dot{\phi} \\ \ddot{\phi} = \frac{-3b}{(4M + m)l}\dot{x} + \frac{3g(M + m)}{(4M + m)l}\phi + \frac{3}{(4M + m)l}u \end{cases} \quad (7)$$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\phi} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-4b}{(4M + m)} & \frac{3mg}{(4M + m)} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-3b}{(4M + m)l} & \frac{3g(M + m)}{(4M + m)l} & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{4}{(4M + m)} \\ 0 \\ \frac{3}{(4M + m)l} \end{bmatrix} u$$

$$y = \begin{bmatrix} x \\ \phi \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u \quad (8)$$

Equation (1) and equation (2) can be used to design the fuzzy logic controller while equations (5), (6), (7) and (8) can be used to design other controllers such as PID , LQR etc.

III. Fuzzy Controller Design

From the practical point of view, real-time control requires some simplification of the experimental model, and human intervention is always necessary for this type of control. In general, a controller based on the experience of the human operator is desired for the practical purpose. Fuzzy controllers use heuristic information in developing design methodologies for control of non-linear dynamic systems. This approach eliminates the need for comprehensive knowledge and mathematical modeling of the system. A fuzzy control system is shown in Figure 3.

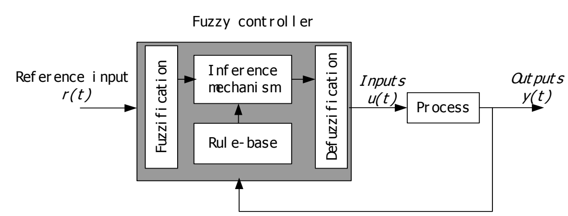

Fig.3 Fuzzy control system architecture

The aim of fuzzy control is normally to substitute a fuzzy rule-based system for a skilled human operator. When the pendulum leans in one direction, the control algorithm will try to move the cart under it with appropriate speed and direction. In this case, the algorithm will take the inputs i.e. the pendulum angle and cart position measured by encoders, then tell the cart which way and how fast to move. The paper presents the stages of development of the fuzzy controller for an inverted pendulum by developing a two-input, one output, Mamdani type system. The fuzzy sets of two-input($\theta, \dot{\theta}$) and one output(u) are designed in Figure 4.

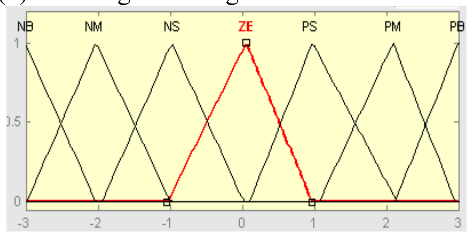

Fig.4 (a) Input($\theta, \dot{\theta}$) fuzzy sets

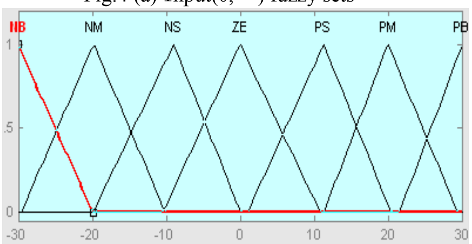

Fig.4 (b) Output(u) fuzzy sets

Note that we are using "NS" as an abbreviation for "negative small in size" and so on for the other variables. Such abbreviations help keep the linguistic descriptions short yet precise:

"NB" to represent "negative big"
"NM" to represent "negative medium"
"ZE" to represent "zero"
"PS" to represent "positive small".
"PM" to represent "positive medium"
"PB" to represent " positive big"

The expert knowledge is formulated as a collection of if-then rules and the associated membership functions. In addition, the choice of membership functions (MFs) and rule base of the Takagi-Sugeno fuzzy controller will affect the performance of the system. The fuzzy controller with ANFIS architecture can formalize a systematic approach to generate the fuzzy rule and MFs.

Figure 5 shows ANFIS model structure using ANFIS GUI editor from Math Work Inc. The input -output parameters and rule base cannot be shown here due to limited space.

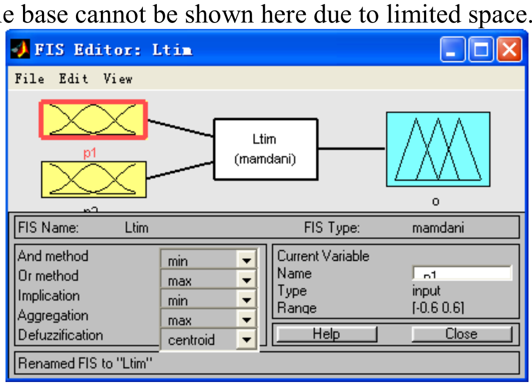

Fig.5 ANFIS model structure

The number of fuzzy control rules was significantly reduced by combining related variables, and the control performance was greatly improved by adopting suitable rule-base according to the angle of the pole with the vertical. In this paper, the initial linear relationship between input and output are described in 49 fuzzy-if-then rules which are represented in the format as table 2.

Table 2. The rule base

ec / e	NB	NM	NS	ZE	PS	PM	PB
NB	NB	NB	NB	NB	NB	NM	ZE
NM	NB	NB	NB	NB	NM	ZE	PM
NS	NB	NB	NB	NM	ZE	PM	PB
ZE	NB	NB	NM	ZE	PM	PB	PB
PS	NB	NM	ZE	PM	PB	PB	PB
PM	NM	ZE	PM	PB	PB	PB	PB
PB	ZE	PM	PB	PB	PB	PB	PB

Use either SI (MKS) or CGS as primary units. (SI units are encouraged.) English units may be used as secondary units (in parentheses). An exception would be the use of English units as identifiers in trade, such as “3.5-inch disk drive.”

Avoid combining SI and CGS units, such as current in amperes and magnetic field in oersteds. This often leads to confusion because equations do not balance dimensionally. If you must use mixed units, clearly state the units for each quantity that you use in an equation.

IV. Simulation and Experimental Tests

The structure of the inverted pendulum system by Simulink is shown in Figure 6. Applying the dynamic model of inverted pendulum obtained in section 2, Figures 7 show control results of the inverted pendulum system by Simulink simulation.

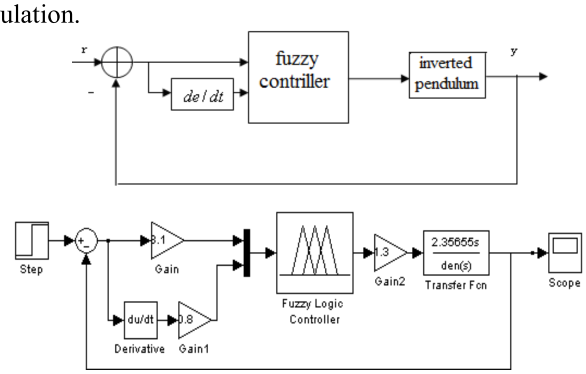

Fig.6. Simulation structure of the inverted pendulum

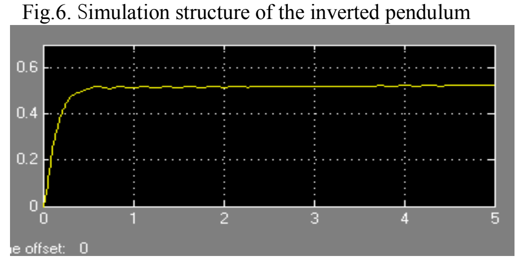

Fig.7 The simulation result of the inverted pendulum

The semi-physical simulation test system provides a sound environment to develop and test controller aimed to inverted pendulum. It integrates the controller model with the physical environment to form a simulation system with hardware in the loop. In this system, PC accomplishes three tasks, which are data acquisition、controlling output and implement of control policy, and provides an environment to develop and test the fuzzy controller. The virtual fuzzy controller is designed in the Simulink. Simulink not only have a capacity of numerical simulation, but also, when in the external mode，it can perform real-time simulation. Using MATLAB RTW real time toolbox to build a real-time control experimental platform, the semi-physical simulation structure of the inverted pendulum is shown in figure 8.

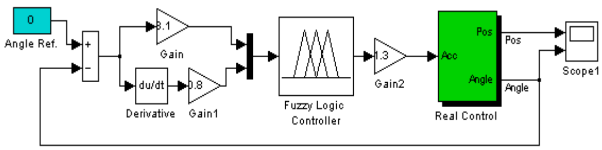

Fig.8 The semi-physical simulation structure of the inverted pendulum

The stabilization control of the pendulum is successfully realized in real time by the proposed fuzzy controller. Figure 9 shows the control system experimental results.

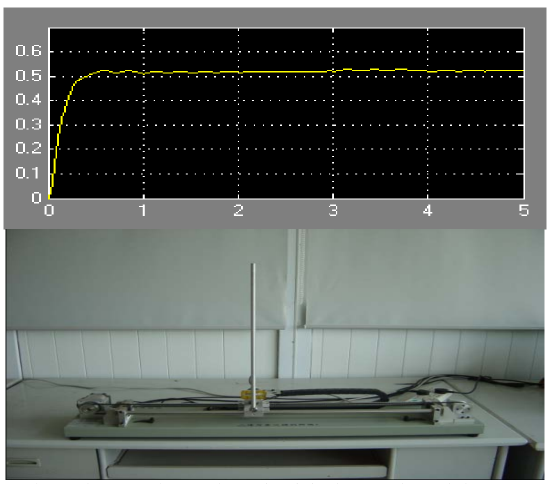

Fig.9 The experimental result of the inverted pendulum

The simulation and experimental results show that the fuzzy controller take about 1 sec. to bring the pendulum close to upright position.

V. CONCLUSION

The objective of this project was to design a stabilizing controller for inverted pendulum and this has been successfully achieved. The fuzzy controller is proved to be effective and feasible in both of the angular control of pendulum at upright position and position control of cart to its origin of rail. Because of the complexity of this system and the real-time requirement, MATLAB based Hardware in Loop Simulation is used to solve this problem. The effect of the control can be easily seen from the simulation and experimental test results.

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