# Simulation of Synchronous Machine in Stability Study for Power System

Tin Win Mon, and Myo Myint Aung

Abstract—This paper presents a linear mathematical model of the synchronous generator with excitation system for power system stability. For large systems the state space model has been used more frequently in connection with system described by linear differential equations. The flux linkage of each circuit in the machine depends upon the exciter output voltage. The excitation system models described use a pu system. The complete system is mainly included synchronous machine, exciter and transmission line. The system is whether stable or unstable is determined by eigen-values of the system coefficient matrix A. These eigen-values are identified with the parameters of the machine and are not depend on the exciter parameters. Complete state equations are obtained first order differential equation from the system model by identifying appropriate state variables. The conversion of state space equation to transfer function is applied in the MATLAB/SIMULINK program. The first-order approximations are made for the system equations. The current and flux linkage are used to as state variables the overall system. Stability characteristics of the system are determined by examining the eigen-values. It can be prove that root locus plot. The current state-space model is specifically concerned with the 35 MVA, 11 kV synchronous generator modeling aspects of the boostbuck excitation system. MATLAB/SIMULINK software is used to compute the state-space model of the system.

*Keywords*—Park's transformation, Per-unit conversion, statespace model, transfer function.

#### I. INTRODUCTION

CONTROL system design and analysis technologies are Very useful to be applied in real time development. A stable power system is one in which the synchronous machines, when perturbed will either return to their original state if there is no net change of power or will acquire a new state without losing synchronism. If the system equations are linear (have been linearized ), the techniques of linear system analysis are used to study dynamic behavior. The most common method is to simulate each component by its transfer function. The system performance may then be analyzed by such methods as root-locus plots, frequency domain analysis (Nyquist criterion), and Routh-Hurwitz criterion. These methods have been frequently used in studies pertaining to small systems or a small number of machines. For large systems the state-space model has been used more frequently in connection with system described by linear differential equations. By the MATLAB modeling results of synchronous machine-infinite system stability is determined by root-locus plot.

This overall system consists of synchronous machine, exciter, amplifier portion and the components of regulator. The mathematical modeling of this system is described in this paper and then this system is transferred into transfer function Simulation result (time response) obtains from this transfer function.

## II. MODELING OF SYNCHRONOUS MACHINE-INFINITE BUS System

To be modeling a Synchronous machine-infinite bus system, simple circuit of its electrical diagram as shown in Fig. 1 is considered. To be Modeling and Simulation of Synchronous Machine-infinite bus system, the following steps to be made step by step;

Step 1: Represent Synchronous Machine Circuit diagram

- Step 2: Represent System Mathematical equations
- Step 3: Calculate the Transfer Function of overall system
- Step 4: Convert to model block
- Step 5: Create the M file to simulate the model



Fig. 1 A close loop system that representing the Synchronous Machine and its excitation System

## A. Description of Synchronous Machine

Large scale power is generated by three phase synchronous generator, known as alternator, driven either by steam turbines, hydro turbines, or gas turbines. The armature windings are placed on the stationary part called stator. The armature windings are designed for generation of balance three phase voltages and are arranged to develop the same

Tin Win Mon is with the Mandalay Technological University Myanmar, (corresponding author to provide phone: 095-02-88702; fax: 095-02-88702; e-mail: monmon.23@ gmail.com and nyinyi.29@ gmail.com).

Myo Myint Aung, was with Yangon Technological University, Myanmar. He is now with the Head of Department of Electrical Power Engineering, Mandalay Technological University, Myanmar.

number of magnetic poles as the field winding that is on the rotor. The rotor is also equipped with one or more shortcircuited windings known as damper winding. The rotor is driven by prime mover at constant speed and its field circuit is excited by direct current flow. The excitation may be provided through slip rings and brushes by means of AC generators mounted on the same shaft as the rotor of the synchronous machine. Modern excitation system usually use AC generator with rotating rectifiers, are known as brushless excitation.

## III. MATHEMATICAL MODELING OF A MACHINE WITH INFINITE-BUS SYSTEM

The mathematical description of the synchronous machine is obtained if a certain transformation of variables is performed. Park's transformation is simply to transform all stator quantities from phase a, b and c into equivalent dq axis new variables. The Park's transformation P is defined as (1);

$$P = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \cos \dot{e} & \cos \left( \dot{e} - \frac{2 \ddot{g}}{3} \right) & \cos \left( \dot{e} + \frac{2 \ddot{g}}{3} \right) \\ \sin \dot{e} & \sin \left( \dot{e} - \frac{2 \ddot{g}}{3} \right) & \sin \left( \dot{e} + \frac{2 \ddot{g}}{3} \right) \end{bmatrix}$$
(1)

Angle  $\theta$  is given by,

$$\theta = \omega_R t + \delta + \pi/2 \tag{2}$$

where,

 ω<sub>R</sub> = rated angular frequency of synchronous machine
 δ =Synchronous torque angle

Equation (3) is the generator voltage equation in the rotor frame of reference is described in Per-unit. The machine equation in the rotor frame of reference becomes (4).

$$V = -ri - \lambda' + V_n \tag{3}$$

$$\begin{bmatrix} \mathbf{v}_{d} \\ -\mathbf{v}_{F} \\ \mathbf{0} \\ \mathbf{v}_{q} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{r} & \mathbf{0} & \mathbf{0} & \mathbf{i} \mathbf{L}_{q} & \mathbf{i} \mathbf{k} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{r}_{F} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\mathbf{i} \mathbf{L}_{d} & -\mathbf{i} \mathbf{k} \mathbf{M} & \mathbf{F} & -\mathbf{i} \mathbf{k} \mathbf{M} & \mathbf{D} & \mathbf{r} & \mathbf{0} \\ -\mathbf{i} \mathbf{L}_{d} & -\mathbf{i} \mathbf{k} \mathbf{M} & \mathbf{F} & -\mathbf{i} \mathbf{k} \mathbf{M} & \mathbf{D} & \mathbf{r} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{r}_{\mathbf{Q}} \end{bmatrix} \begin{bmatrix} \mathbf{i}_{d} \\ \mathbf{i}_{F} \\ \mathbf{i}_{D} \\ \mathbf{i}_{q} \\ \mathbf{i}_{Q} \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{L}_{d} & \mathbf{k} \mathbf{M} & \mathbf{F} & \mathbf{k} \mathbf{M} & \mathbf{D} & \mathbf{0} & \mathbf{0} \\ \mathbf{k} \mathbf{M} & \mathbf{F} & \mathbf{L} \mathbf{F} & \mathbf{M} & \mathbf{R} & \mathbf{0} & \mathbf{0} \\ \mathbf{k} \mathbf{M} & \mathbf{R} & \mathbf{L} \mathbf{D} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{L} & \mathbf{k} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{k} \mathbf{M} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{i}_{d} \\ \mathbf{i}_{F} \\ \mathbf{i}_{D} \\ \mathbf{i}_{q} \\ \mathbf{i}_{Q} \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{k} \mathbf{k} \mathbf{M} & \mathbf{k} \mathbf{K} & \mathbf{k} \mathbf{M} & \mathbf{0} & \mathbf{0} \\ \mathbf{k} \mathbf{K} & \mathbf{k} \mathbf{K} & \mathbf{k} \mathbf{K} & \mathbf{0} & \mathbf{0} \\ \mathbf{k} \mathbf{M} & \mathbf{k} & \mathbf{k} \mathbf{K} & \mathbf{k} \mathbf{M} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{k} \mathbf{M} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{i}_{d} \\ \mathbf{i}_{R} \\ \mathbf{i}_{Q} \\ \mathbf{i}_{Q} \end{bmatrix}$$

Where,

- Ld = Equivalent direct-axis Reactance
- LF = Filed winding Self –inductance
- $L_D$  = Self-inductance damper winding
- Lq = Equivalent quadrature axis reactance
- $L_Q$ = Self inductance of quadrature
- reactance  $kM_D$  = Stator to damper winding
  - resistance
- $kM_Q$  = Stator to quadrature winding resistance
- r = Stator winding current
- $r_F$  = Field winding resistance
- $r_D$  = resistance of d axis damper winding
- $r_Q$  = resistance of q axis damper winding
- iq = armature current in the q direction
- i<sub>F</sub> = Field current
- i<sub>D</sub> = d axis damper winding current
- $i_d = q$  axis damper winding current

This equation is in the state space form and can be written in compact form as (5) and (6).

$$v = -(R + \omega N)i - Li$$
(5)

$$i^{-} = -L^{-1} (R + \omega N)I - L^{-1} v$$
 (6)

Where,

 $L_{d} = L_{s} + M_{s} + 3/2 L_{m}$   $Lq = L_{s} + M_{s} - 3/2 L_{m}$  R = resistance matrixN = matrix of speed voltage inductance coefficients

L= matrix of constant inductances

The excitation system controls the terminal voltage of synchronous machine. This system is also plays an important role in the machine's mechanical oscillations, since its effects the electrical power. Two transfer functions are of vital importance. The first of these is the generator transfer function. The generator equations are nonlinear and the transfer function is a linearized approximation of the behaviour of the generator terminal voltage near a quiescent operating point or equilibrium state. The load equations are also nonlinear and reflect changes in the electrical output quantities due to changes in terminal voltage. Fig. 2 (a) and (b) show synchronous generator stator, field, and dq equivalent circuit.



(a) Stator and Field Circuit



#### (b) dq axis equivalent circuit

#### Fig. 2 Schematic diagram of Synchronous Machine

#### IV. SYNCHRONOUS MACHINE CONNECTED TO AN INFINITE BUS TROUGH A TRANSMISSION LINE

The differential equations for one machine connected to an infinite bus through a transmission line with impedance  $Z_e = R_e + j\omega_R L_e$ . In equation (7),  $i_d$  and  $i_q$  are dc currents obtained from the modified Park's transformation.

$$v_{d} = -\sqrt{3}V_{\alpha}Sin(\delta - \alpha) + Rei_{d} + \omega L_{e}i_{q}$$
$$v_{q} = \sqrt{3}V_{\alpha}Cos(\delta - \alpha) + Rei_{q} - \omega L_{e}i_{d}$$
(7)

If machine connected to an infinite bus with local load at machine terminal, the currents  $i_d$  and  $i_q$  should be replaced by the currents  $i_{td}$  and  $i_{tq}$ . (8) is used to add the transmission line in the synchronous machine.

$$vd = -\sqrt{3} V_{\alpha} Sin(\delta - \alpha) + Rei_{td} + \omega L_{e}i_{tq}$$
$$vq = \sqrt{3} V_{\alpha} Cos(\delta - \alpha) + Rei_{tq} - \omega L_{e}i_{td}$$
(8)

If the conditions at the machine terminals are given, the voltage of the infinite bus is then determined by subtracting the appropriate voltage drops to the machine terminal voltage. Otherwise, if the terminal conditions at the infinite bus are given as the boundary conditions, the position of the d and q axis currents, voltages and the machine terminal voltage can then be determined.

## A. Linearization of Load Equation for a Synchronous Machine with Infinite Bus System

When the Machine-Infinite bus system is subjected to a small load change, it tends to acquire a new operating state. The first order approximations for these have been illustrated in previous (7) and (8). Let the state variables  $x_i$  and  $x_j$  have the initial values  $x_{io}$  and  $x_{jo}$ . The changes in these variables are  $x_{i\Delta}$  and  $x_{j\Delta}$ . Therefore, (7) is used to linearize, the linearized system equations are as (9).

$$\begin{aligned} v_{d\Delta} &= -KCos(\delta_0 - \alpha)\delta_{\Delta} + R_e i_{d\Delta} + \omega_0 L_e \dot{i}_{q\Delta} + i_{q0} L_e \omega_{\Delta} \\ &+ L_e i_{d\Delta} \end{aligned}$$

$$v_{q\Delta} = -KSin(\delta_0 - \alpha)\delta_{\Delta} + R_e i_{q\Delta} - \omega_0 L_e i_{d\Delta} - i_{d0} L_e \omega_{\Delta}$$

$$+ L_e i_{a\Delta}$$
(9)

Since all variables are now small displacement. These equations are written in the following matrix form (10).

$$v = -Kx - Mx \tag{10}$$

These matrix M and K are included the transmission line constants, synchronous machine parameters and the infinite bus voltage. Therefore, state space model of the system is as (11) and (12);

$$x' = -M^{-1} K x - M^{-1} v \tag{11}$$

which is the same form as

$$x' = Ax + Bu \tag{12}$$

Stability characteristics may be determined by examining (11). x is an n vector denoting the states of the system and A is a coefficient matrix. The system inputs are represented by the r vector u, and these input are related mathematically to differential equations by an  $n \times r$  matrix B. This description has the advantage that A may be time varying and u may be used to represent several inputs if necessary.

## V. STATE-SPACE DESCRIPTION OF OVERALL SYSTEM

The excitation system controls the generated EMF of the generator and therefore controls not only the output voltage but also the power factor and current magnitude as well. The following equations (13) through (17) of the excitation control system transformed into S plane are given below. The corresponding model block diagram is shown in Fig. 3.

$$\dot{v_I} = \left(\frac{k_R}{\tau_R}\right) V_t - \left(\frac{l}{\tau_R}\right) V_I \tag{13}$$

$$\mathbf{v}_{3}^{\cdot} = \left(\frac{k_{F}}{\tau_{F}}\right) E_{FD}^{\cdot} - \left(\frac{1}{\tau_{F}}\right) V_{3}$$
(14)

$$v_{R}^{\cdot} = \left(\frac{K_{A}}{\tau_{A}}\right) V_{e} - \left(\frac{1}{\tau_{A}}\right) V_{R}$$
(15)

$$E_{FD}^{\cdot} = \left(\frac{1}{\tau_E}\right) V_R - \left(\frac{S_E + K_E}{\tau_E}\right) E_{FD}$$
(16)

$$V_E = V_{REF} + V_S - V_I - V_3 \tag{17}$$



Fig. 3 Closed Loop System that representing Synchronous Machine and Excitation System

The amplifier portion of excitation control system has an amplification factor  $K_A$  and time constant  $\tau_A$ , supplying an output voltage  $V_R$  which is limited by  $V_{Rmax}$  and  $V_{Rmin}$ . In the linearized model, the generator terminal voltage to its field voltage can be represented by a gain  $k_G$  and a time constant  $\tau_G$ . Typical values of  $K_A$  are in the range of 10 to 400. The amplifier time constant  $\tau_A$  is in the range of 0.02 to 0.1 second, and often is neglected. Gain  $K_G$  may vary between 0.7 to 1, and  $\tau_G$  is between 1.0 and 2.0 second from full load to no-load.  $\tau_R$  is very small, its range is between 0.01 to 0.06 second.

## A. Flow Diagram for Simulation for overall System



Fig. 4 Flow Diagram of Simulation Step

The procedure for modeling of Synchronous machine with feedback excitation system is as follows:

1. Mathematical models of synchronous generator with excitation systems are essential for the assessment of desired performance requirements.

2. The per-unit value of the system transfer function matrix is calculated from Equation (1 to 17).

3. The complete linear state space equation (4) is transferred to output stator and rotor current by using MATLAB programming.

4. Root-locus method is used to prove the system performance is either satisfaction or not.

6. The process is continued until all the system roots are located in the left hand s-plane.

Flow diagram of MATLAB programming step is shown in Fig. 4.

#### VI. MODELING AND SIMULATION

MATLAB/SIMULINK program was used in testing of the stability condition of the synchronous machine infinite bus system. The M-file and SIMULINK model can be combined by the commands [num, den]=ss2tf(A, B,C, D, 1);c=step(num, den, t); plot(t, c); title; x label and y label. The parameters for 35 MVA, 11kV generator, exciter and transmission line are listed in Table I and II. This input parameters are used to simulation program. The simulation results are shown in Fig.5 to Fig. 9. The out put field current of synchronous machine is shown in Fig. 5. The d axis stator current and damper winding current of the machine is shown in Fig. 6 and 7. The response is highly oscillatory, with a very large overshoot and a long settling time. It cannot have a small steady state error and a satisfactory transient response at the same time. The system is whether stable or unstable is determined not only eigen-values of overall system matrix A but also root locus plot. In Fig. 8, all the poles of the system transfer function have negative real part. Therefore, the system is stable. Fig. 9 is terminal voltage step response (with (or) without feed back stabilizer).

Transmission line impedance is 0.02 + j0.4. The real parts of all the eigen-values are negative, which means that the system is stable under the conditions assumed in the development of this model.

TARLEI

INPUT PARAMETERS OF MACHINE BASE VALUE		
Rated MVA (MVA)	35	
Rated voltage (kV)	11	
Excitation voltage (V)	400	
stator current (kA)	1.837	
field current (A)	385	
power factor	0.85	

TABLE II

INPUT PARAMETERS OF EXCITER AND REGULATOR		
proportionality constant K <sub>R</sub>	1.0	
Regulator input fitter time $\tau_R$	0.01	
(sec)		
regulator gain K <sub>A</sub>	40	
regulator amplifier time constant $\tau_A$	0.05	
(sec)		
exciter constant related to self excited	-0.05	
field K <sub>E</sub>		
exciter time constant $ au_{\rm E}$	0.5	
(sec)		
generator stabilizing circuit gain K <sub>F</sub>	0.04	
regulator stabilizing time constant $\tau_F$	0.715	



Fig. 5 Output Field Current of Machine



Fig. 6 d axis stator current of Synchronous Machine



Fig. 7 Output d axis Damper Current of Machine



Fig. 8 Root Locus Plot for overall System



(a) without stabilizer feedback



(b) with stabilizer feedback

Fig. 9 Terminal Voltage Step Response of Synchronous Machine-Infinite bus System

## VII. CONCLUSION

The aim of this paper is to introduce Technicians to the Modeling of Synchronous machine with excitation system on stability studies and to use computer simulation as a tool for conducting transient and control studies. Next to having an actual system to experiment on, simulation is often chosen by engineers to study transient and control performance or to test conceptual designs. MATLAB/SIMULINK is used because of the short learning curve that most students require to start using it, its wide distribution, and its general-purpose nature.

This will demonstrate the advantages of using MATLAB for analyzing steady state power system stability and its capabilities for simulating transients in power systems, including control behavior. This paper mentioned only a part of the author's research and approaching of her studies in MATLAB software. The real application of this research paper is stability of synchronous machine-infinite bus system as mentioned in above.

#### ACKNOWLEDGMENT

Firstly the author would like to thank his parents: U Pann Nyunt and Daw Nyein Nu for their best wishes to join the PhD research. Special thanks are due to her supervisor/head of Electrical Power Engineering Department, MTU, Union of Myanmar: Dr Myo Myint Aung. The author would like to express her thank to her teachers: Mr. Than Zaw Htwe and Mr. Hla Myo Aung. The author greatly expresses her thanks to all persons whom will concern to support in preparing this paper.

#### References

- [1] Paul M. Aerson, A. A. Fouad, "Power System Control and Stability," *IEEE Power Systems Engineering series*, 1997.
- M. K. EI. Sherbiny and D. M. Mehta, "Dynamic system stability," *IEEE Trans; PAS- 92:1538-46*, 1973.
- [3] Hadi Saadat, "Power System analysis" *McGraw-Hill companies, Inc*, 1999.
- [4] Hadi Saadat, "Computational Aids in Control Systems using MATLAB," McGraw-Hill Companies, Inc, 1993.
- [5] G. T. Heydt, "Identification and Tracking of Parameters for a Large Synchronous Generator," *Final project Report, Department of Electrical Power Engineering, Arizona State University*, 2002.
- [6] P. Kundur, "Power System Stability and Control," McGraw-Hill companies, Inc, 1994.
- [7] Courtesy by Kinda Hydro Power Plant.
- [8] Math Works, 2001, Introduction to MATLAB, the Math Works, Inc.
- [9] Math Works, 2001, SIMULINK, the Math Works, Inc.
- [10] Math Works, 2001, What is SIMULINK, the Math Works, Inc.

**Tin Win Mon** was born in 1980, November 29. A.GTI Certificate in 2000, November. Graduated in 3<sup>rd</sup> November, 2003 with B.E (Electrical Power) and finished Master degree on March, 2006 with M.E (Electrical Power). At present, she is a Ph.D candidate of Electrical Power Engineering Department, MTU, Mandalay, Myanmar. I'm trying to accomplish my Ph.D. thesis.

She served as a Demonstrator at Mandalay GTC from May, 2002 to January, 2004 when she was attending the Special Engineering Course in MTU. From February, 2004 to now, she promoted as an Assistant Lecturer of Hpa-an Technological University, Department of Technical and Vocational Education, Myanmar. Now, she is making her PhD research at Mandalay Technological University (MTU), Myanmar with the title of "Simulation of Synchronous Machine in Stability Study for Power System".

Tin Win Mon made her first publication of International Paper at this paper "Simulation of Synchronous Machine in Stability Study for Power System".