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REDUCED ORDER KALMAN FILTERING WITHOUT MODEL REDUCTION

D. Simon*

Abstract

This paper presents an optimal discrete time reduced order Kalman filter. The reduced order filter is used to estimate a linear combination of a subset of the state vector. Most previous approaches to reduced order filtering rely on a reduction of the model order. However, this paper takes the full model order into account. The reduced order filter is obtained by minimizing the trace of the estimation error covariance.

Key Words

Kalman filter, state estimation, order reduction

1. Introduction

For linear dynamic systems with process and measurement noise that are white and uncorrelated, the Kalman filter is known to be an optimal estimator [1]. In case the process and measurement noise are colored or correlated, the Kalman filter can be generalized and remains optimal [2]. However, computational constraints can make the full order Kalman filter difficult to implement in real time, especially when the implementation platform is a microcontroller or digital signal processor [3]. In addition, some Kalman filter applications (e.g., meteorology and oceanography applications) can involve millions of states [4]. This has led to considerable effort on methods of reducing the order of the Kalman filter.

The earliest efforts at reduced order Kalman filters recognized that the Kalman filter for a system where some of the measurements are noise free is equivalent to a Kalman filter for a system with a reduced number of states [5, 6]. On a related note, the Riccati equation associated with the Kalman filter can be reduced if some of the states are unobservable [7] although the resulting filter still estimates all the states. Similarly, the Riccati equation can be reduced if a matrix decomposition is performed on some of the matrices in the Kalman filter, especially if those matrices are rank deficient [4, 8–10].

Most reduced order filters are designed on the basis of a reduction in the order of the system model. If we can find a reduced order system model that approximates the full order system model, then we can design a state estimator on the basis of the reduced order system model that approximates the full order Kalman filter. This approach has been applied to motor state estimation [3, 11], navigation system alignment [12], image restoration [13, 14], and audio analysis of respiratory data [15].

Some aerospace system equations have states that can be semi-decoupled such that the derivative of the first state partition is independent of the second state partition (although the derivative of the second state partition is still dependent on the first state partition). In that case a reduced order Kalman filter can be designed to estimate the first state partition by generalizing an approach to Kalman filtering with time correlated measurement noise and correlated process and measurement noise [16].

This paper does not make any approximations in the order of the model and does not assume any special structure in the system dynamics. This is similar to the approaches proposed by Sims [17], Bernstein [18], Nagpal [19], and Keller [20, 21]. Sims' approach involves the solution of a two point boundary value problem, which can be numerically difficult. Bernstein's approach guarantees stability, but it is limited to steady state and involves the solution of simultaneous Riccati equations, which again can be numerically difficult. In addition, his reduced order estimator may be biased. Nagpal does not assume any special form for the state transition matrix of the estimator. This leads to the minimization of the error covariance subject to constraints that guarantee an unbiased estimate. However, Nagpal's approach is limited to cases where the number of estimated states is greater than or equal to the number of independent observations. In addition, estimator stability is still an open question. Keller [20] presents a simplification of Nagpal's approach along with convergence and stability results, although the restriction on the number of estimated states remains. Keller presents another approach [21] that is restricted to time invariant systems where the number of measurements is greater than the number of unestimated states.

The present paper assumes that the state transition matrix is a certain matrix. This guarantees an unbiased estimate and so the error covariance is minimized without any constraints. Like Nagpal, we cannot prove anything about stability for the reduced order filter presented here. The optimal reduced order filter equations involve the si-

* Department of Electrical Engineering, Cleveland State University, 2121 Euclid Avenue, SH 332, Cleveland, OH 44115-2214, USA; e-mail: d.j.simon@csuohio.edu
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multaneous iteration of six-time varying Riccati equations. In general, these iterations are computationally more demanding than a full order filter. However, if a steady state filter is desired, then the Riccati equations can be iterated off line until they converge to steady state values, at which point a reduced order constant filter gain can be computed for on-line implementation.

In the equations in this paper, we use the notation $(\cdots)$ to indicate the quantity that appeared in the previous set of parentheses. For example, the expression $[(A+B)+(\cdots)^T]$ is shorthand for $[(A+B)+(A+B)^T]$. The notation $E(\cdot)$ indicates the expected value of the quantity in parentheses. The notation $\mathrm{Tr}(A)$ indicates the trace of the matrix A. Section 2 presents the reduced order filter, Section 3 presents some simulation results, and Section 4 presents some concluding remarks.

2. Reduced Order Kalman Filtering

Suppose we have a linear discrete time dynamic system. For ease of notation we will assume that the system is time invariant, although the derivation applies equally well to time-varying systems. The system is given as:

$$\begin{aligned} \bar{x}_{k+1} &= \bar{F}\bar{x}_k + \bar{G}w_k \\ z_k &= \bar{H}\bar{x}_k + v_k \end{aligned} \tag{1}$$

where $\{w_k\}$ and $\{v_k\}$ are uncorrelated zero mean white noise processes with covariance matrices Q and R, respectively. The state vector x has $m+n$ elements, and the measurement vector z has p elements. We desire to estimate m linear combinations of the state $T_1^T\bar{x}, \ldots, T_m^T\bar{x}$, where each T_i is an $m+n$ element column vector, and all of the T_i vectors are linearly independent. We form the $(m+n)\times(m+n)$ matrix:

$$T = \begin{bmatrix} T_1^T \\ \vdots \\ T_m^T \\ S \end{bmatrix} \tag{2}$$

where S is an $n\times(m+n)$ matrix that is arbitrary as long as T is invertible. We make the linear transformation $x = T\bar{x}$ which results in the algebraically equivalent system:

$$\begin{aligned} x_{k+1} &= Fx_k + Gw_k \\ z_k &= Hx_k + v_k \end{aligned} \tag{3}$$

where $F = T\bar{F}T^{-1}$, $G = T\bar{G}$, and $H = \bar{H}T^{-1}$. Now we want to estimate the first m elements of x, and we do not care about the last n elements of x. The equivalent system can be partitioned as:

$$\begin{aligned} \begin{bmatrix} \tilde{x}_{k+1} \\ \tilde{\tilde{x}}_{k+1} \end{bmatrix} &= \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \begin{bmatrix} \tilde{x}_k \\ \tilde{\tilde{x}}_k \end{bmatrix} + \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} w_k \\ z_k &= \begin{bmatrix} H_1 & H_2 \end{bmatrix} \begin{bmatrix} \tilde{x}_k \\ \tilde{\tilde{x}}_k \end{bmatrix} + v_k \end{aligned} \tag{4}$$

We want to estimate $\tilde{x}$ but we do not care about estimating $\tilde{\tilde{x}}$. The dynamic system for $\tilde{x}$ and $\tilde{\tilde{x}}$ can be written as:

$$\begin{aligned} \tilde{x}_{k+1} &= F_{11}\tilde{x}_k + (F_{12}\tilde{\tilde{x}}_k + G_1 w_k) \\ \tilde{\tilde{x}}_{k+1} &= F_{21}\tilde{x}_k + F_{22}\tilde{\tilde{x}}_k + G_2 w_k \\ z_k &= H_1\tilde{x}_k + (H_2\tilde{\tilde{x}}_k + v_k) \end{aligned} \tag{5}$$

where F_{11} is the transition matrix of $\tilde{x}_k$ and the quantities in parentheses are considered to be noise terms [22]. With this perspective we can design a filter for $\tilde{x}_k$ that uses the known part of its transition matrix F_{11}, and that uses the measurement matrix H_1 that relates $\tilde{x}_k$ to z_k. This motivates the proposed reduced order estimator:

$$\hat{\tilde{x}}_{k+1}^+ = F_{11}\hat{\tilde{x}}_k^+ + K_{k+1}(z_{k+1} - H_1F_{11}\hat{\tilde{x}}_k^+) \tag{6}$$

The problem is to determine the optimal filter gain K_k. We define e_k as the error in the estimate of $\tilde{x}_k$. Combining the two previous equations and rearranging gives the estimation error as:

$$\begin{aligned} e_{k+1} &\equiv \tilde{x}_{k+1} - \hat{\tilde{x}}_{k+1}^+ \\ &= (I - K_{k+1}H_1)F_{11}(\tilde{x}_k - \hat{\tilde{x}}_k) + (F_{12} - K_{k+1}H_1F_{12} \\ &\quad - K_{k+1}H_2F_{22})\tilde{\tilde{x}}_k - K_{k+1}H_2F_{21}\tilde{x}_k - K_{k+1}v_{k+1} \\ &\quad + (G_1 - K_{k+1}HG)w_k \end{aligned} \tag{7}$$

Note that if $E(e_0)=0$, $E(\tilde{x}_0)=0$, and $E(\tilde{\tilde{x}}_0)=0$, then the estimator proposed in (6) is unbiased for all k regardless of the choice of K_k. Now we define the following covariance matrices:

$$\begin{aligned} P_k &= E(e_ke_k^T) \\ \tilde{P}_k &= E(\tilde{x}_k\tilde{x}_k^T) \\ \tilde{\tilde{P}}_k &= E(\tilde{\tilde{x}}_k\tilde{\tilde{x}}_k^T) \\ \Sigma_k &= E(\tilde{x}_k\tilde{\tilde{x}}_k^T) \\ \tilde{\Pi}_k &= E(\hat{\tilde{x}}_k\tilde{x}_k^T) \\ \tilde{\tilde{\Pi}}_k &= E(\hat{\tilde{x}}_k\tilde{\tilde{x}}_k^T) \end{aligned} \tag{8}$$

Our goal is to find the filter gain that minimizes some measure of P_k, the covariance of the estimation error. To simplify notation we define the auxiliary variables:

$$\begin{aligned} C &= H_1F_{12} + H_2F_{22} \\ D_k &= F_{12} - K_kC \\ E_k &= I - K_kH_1 \end{aligned} \tag{9}$$

where I is the appropriately dimensioned identity matrix. We can use (5)–(7) to obtain the following equations for the covariance matrices:

$$
\begin{aligned}
\tilde{\tilde{P}}_{k+1} &= F_{21}\tilde{P}_kF_{21}^T + F_{21}\Sigma_kF_{22}^T + F_{22}\Sigma_k^TF_{21}^T + F_{22}\tilde{\tilde{P}}_kF_{22}^T \\
&\quad + G_2QG_2^T \\
\tilde{P}_{k+1} &= F_{11}\tilde{P}_kF_{11}^T + F_{11}\Sigma_kF_{12}^T + F_{12}\Sigma_k^TF_{11}^T + F_{12}\tilde{\tilde{P}}_kF_{12}^T \\
&\quad + G_1QG_1^T \\
\Sigma_{k+1} &= F_{11}\tilde{P}_kF_{21}^T + F_{11}\Sigma_kF_{22}^T + F_{12}\Sigma_k^TF_{21}^T + F_{12}\tilde{\tilde{P}}_kF_{22}^T \\
&\quad + G_1QG_2^T \\
\tilde{\Pi}_{k+1} &= (I - K_{k+1}H_1)F_{11}(\tilde{\Pi}_kF_{11}^T + \tilde{\tilde{\Pi}}_kF_{12}^T) \\
&\quad + K_{k+1}(H_1F_{11} + H_2F_{21})(\tilde{P}_kF_{11}^T + \Sigma_kF_{12}^T) \\
&\quad + K_{k+1}(H_1F_{12} + H_2F_{22})(F_{11}\Sigma_k + F_{12}\tilde{\tilde{P}}_k)^T \\
&\quad + K_{k+1}HGQG_1^T \\
\tilde{\tilde{\Pi}}_{k+1} &= (I - K_{k+1}H_1)F_{11}(\tilde{\Pi}_kF_{21}^T + \tilde{\tilde{\Pi}}_kF_{22}^T) \\
&\quad + K_{k+1}(H_1F_{11} + H_2F_{21})(\tilde{P}_kF_{21}^T + \Sigma_kF_{22}^T) \\
&\quad + K_{k+1}(H_1F_{12} + H_2F_{22})(F_{21}\Sigma_k + F_{22}\tilde{\tilde{P}}_k)^T \\
&\quad + K_{k+1}HGQG_2^T \\
P_{k+1} &= E_{k+1}F_{11}P_kF_{11}^TE_{k+1}^T + [E_{k+1}F_{11}(\Sigma_k - \tilde{\tilde{\Pi}}_k)D_{k+1}] \\
&\quad + [\cdots]^T + [E_{k+1}F_{11}(\tilde{\Pi}_k - \tilde{P}_k)F_{21}^TH_2^TK_{k+1}^T] \\
&\quad + [\cdots]^T + D_{k+1}\tilde{\tilde{P}}_kD_{k+1}^T - (D_{k+1}\Sigma_k^TF_{21}^TH_2^TK_{k+1}^T) \\
&\quad - (\cdots)^T + K_{k+1}H_2F_{21}\tilde{P}_kF_{21}^TH_2^TK_{k+1}^T \\
&\quad + K_{k+1}RK_{k+1}^T + (G_1 - K_{k+1}HG)Q(G_1 \\
&\quad - K_{k+1}HG)^T \qquad (10)
\end{aligned}
$$

Now we use some properties of matrix calculus. If A and B are general matrices, then the following matrix derivatives hold:

$$
\begin{aligned}
\frac{\partial \mathrm{Tr}(BA^T)}{\partial A} &= B \\
\frac{\partial \mathrm{Tr}(AB)}{\partial A} &= B^T \qquad (11) \\
\frac{\partial \mathrm{Tr}(ABA^T)}{\partial A} &= AB + AB^T
\end{aligned}
$$

With these definitions we can choose the filter gain K_{k+1} to minimize the trace of P_{k+1} in (10). Taking the partial derivative of the trace of P_{k+1} in (10) with respect to K_{k+1} and setting the result equal to 0 results in:

$$K_{k+1}A_k = B_k \qquad (12)$$

where A_k and B_k are given as:

$$
\begin{aligned}
A_k &= H_1F_{11}P_kF_{11}^TH_1^T + [H_1F_{11}(\Sigma_k - \tilde{\tilde{\Pi}}_k)C^T] + [\cdots]^T \\
&\quad + [H_1F_{11}(\tilde{P}_k - \tilde{\Pi}_k)F_{21}^TH_2^T] + [\cdots]^T + C\tilde{\tilde{P}}_kC^T \\
&\quad + (C\Sigma_k^TF_{21}^TH_2^T) + (\cdots)^T + H_2F_{21}\tilde{P}_kF_{21}^TH_2^T + R \\
&\quad + HGQG^TH^T \\
B_k &= (F_{11}P_k + F_{12}\Sigma_k^T - F_{12}\tilde{\tilde{\Pi}}_k^T)F_{11}^TH_1^T + (F_{11}\Sigma_k^T \\
&\quad - F_{11}\tilde{\tilde{\Pi}}_k + F_{12}\tilde{\tilde{P}}_k)C^T + (F_{11}\tilde{P}_k - F_{11}\tilde{\Pi}_k \\
&\quad + F_{12}\Sigma_k^T)F_{21}^TH_2^T + G_1QG^TH^T \qquad (13)
\end{aligned}
$$

Equations (12) and (13) and the iterations in (10) can be solved for the optimal filter gain K_{k+1} at each time step.

In many cases the solution of these equations will be more computationally demanding than the full order filter, in which case the entire objective of reduced order filtering will be defeated. However, in some state estimation applications the total number of states is on the order of millions, while the number of estimated states is a tiny fraction of the total number of states [4]. In these cases the approach outlined here could result in a huge savings in computational effort.

In some applications of reduced order filtering, the number of estimated states is of the same order of magnitude as the total number of states. In these cases, the approach outlined here would not be computationally beneficial for real-time implementation. However, if the reduced order gain given in (12) converges during off-line calculations to a steady state value, then this constant reduced order gain can be used in real time to achieve computational savings. At this point, the only way to determine the convergence and stability of the reduced order filter is through numerical computation. Analytical results on convergence and stability remain as open research issues.

3. Simulation Results

3.1 Results for a Two State System

In this section, we present a simple example to demonstrate the effectiveness of the reduced order filter. Consider the second order system given by:

$$
\begin{aligned}
x_{k+1} &= Fx_k + \begin{bmatrix} 1 \\ 0 \end{bmatrix} w_k \\
z_k &= [0 \quad 1]x_k + v_k \\
w_k &\sim N(0, 0.1) \\
v_k &\sim N(0, 1) \\
F &= \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.7 \end{bmatrix} \qquad (14)
\end{aligned}
$$

We desire to find a reduced order estimator of the first element of x. The steady state full order Kalman filter is given as:

$$
\begin{aligned}
\hat{x}_{k+1}^- &= F\hat{x}_k^+ \\
\hat{x}_k^+ &= \hat{x}_k^- + K(z_k - [0 \quad 1]\hat{x}_k^-) \qquad (15) \\
K &\approx \begin{bmatrix} 0.1983 \\ 0.1168 \end{bmatrix}
\end{aligned}
$$

The steady state reduced order estimator proposed in this paper is given as:

$$
\begin{aligned}
\hat{\hat{x}}_{k+1}^+ &= 0.9\hat{\hat{x}}_k^+ + K_r(z_{k+1} - 0 \times 0.9 \times \hat{\hat{x}}_k^+) \\
&= 0.9\hat{\hat{x}}_k^+ + K_rz_{k+1} \qquad (16) \\
K_r &\approx 0.1420
\end{aligned}
$$

In this case the reduced order estimator is stable. Perhaps not coincidentally, the state transition matrix of the system given in (14) is stable. The reduced order filter is clearly computationally cheaper than the full order Kalman filter. The analytic one-sigma estimation error of the first element of x is 0.697 for the full order time-varying Kalman filter and 0.726 for the reduced order steady state filter. As expected, the use of the reduced order filter increases the estimation error, but the increase is only about 4%, while the decrease in computational effort is about 75%. Fig. 1 shows the convergence of the reduced order gain and Fig. 2 shows a comparison of the estimation error during a simulation of the time-varying full order Kalman filter and the steady state reduced order filter. If the state transition matrix is changed to:

$$F = \begin{bmatrix} 1.1 & -0.1 \\ 0.2 & 0.7 \end{bmatrix} \tag{17}$$

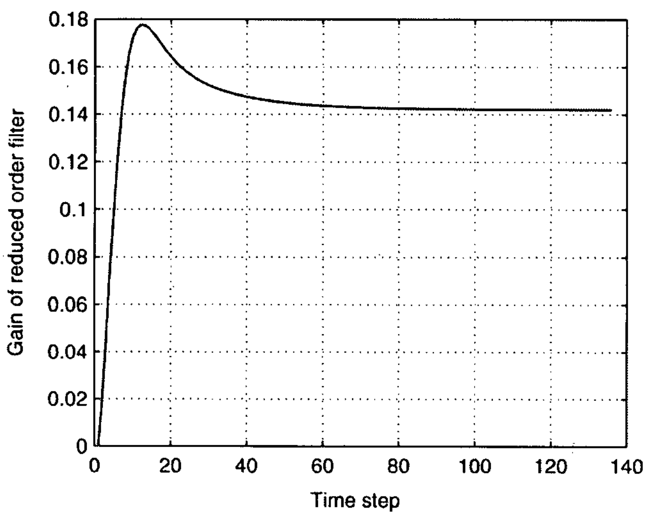

Figure 1. Convergence of reduced order gain.

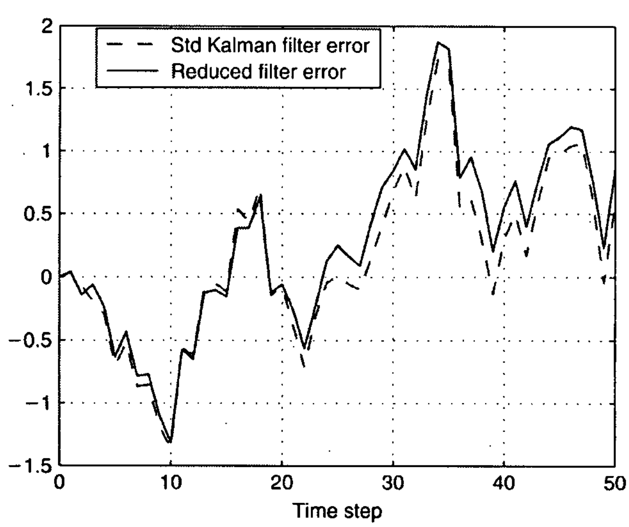

Figure 2. Error of estimate of first state element.

then the reduced order filter gain does not converge to a steady state value. Perhaps not coincidentally, the state transition matrix of the system in this case is unstable. This may provide a clue to further work in analytical determination of reduced order filter convergence

3.2 Results for a Higher Order System

In this section, we present simulation results for a more realistic higher order system. The problem that we consider is health parameter estimation for an aircraft turbofan engine. Health parameters represent engine component efficiencies and flow capacities. The performance of gas turbine engines deteriorates over time. This deterioration reduces the fuel economy of the engine. Airlines periodically collect engine data to evaluate the health of the engine and its components. The health evaluation is then used to determine maintenance schedules. Reliable health evaluations are used to anticipate future maintenance needs. This offers the benefits of improved safety and reduced operating costs. The money-saving potential of such health evaluations is substantial, but only if the evaluations are reliable. The data used to perform health evaluations are typically collected during flight and later transferred to ground-based computers for post-flight analysis. Data are collected for each flight at the same engine operating point and corrected to account for variability in ambient conditions. Further details about the aircraft turbofan health parameter estimation problem can be found in [23].

The simulation used in this section is a gas turbine engine simulation software package called MAPSS (Modular Aero Propulsion System Simulation) [24]. MAPSS is written using Matlab Simulink. The MAPSS engine model is based on a low frequency, transient, performance model of a high-pressure ratio, dual-spool, low-bypass, military-type, variable cycle, turbofan engine with a digital controller. The controller update rate is 50 Hz, and the component level model balances the mass/energy equations of the system at a rate of 2500 Hz. The three state variables used in MAPSS are low-pressure rotor speed, high-pressure rotor speed, and the average hot section metal temperature (measured from aft of the combustor to the high-pressure turbine). The 10-element health parameter vector consists of airflow and efficiency at the fan, booster tip, booster hub, high-pressure turbine, and lower-pressure turbine. The 11 measurements consist of the pressures at the low-pressure turbine exit, bypass duct, fan exit, booster inlet, and high-pressure compressor exit; the temperatures at the low-pressure turbine exit, high-pressure compressor inlet, high-pressure compressor exit, and low-pressure turbine blade; and the speeds of the low-pressure rotor and core rotor.

Although the engine model is nonlinear, it is linearized in this simulation so that Kalman filter theory can be applied. Also, even though the health parameters are not state variables of the model, the dynamic model is augmented in such a way that a Kalman filter can estimate

Table 1
Health parameter estimation errors (%), along with standard deviations, based on 20 Monte Carlo simulations. The estimation error is measured as $|(p-\hat{p})/p_f|$, where p is the true health parameter value, $\hat{p}$ is the estimated health parameter value, and p_f is the health parameter value at the end of the 50-flight simulation.

Health Parameter	Estimation Error (%)		
	Full Order Filter	Keller's Filter	Proposed Filter
Fan airflow	1.02 ± 0.09	3.23 ± 0.10	4.85 ± 0.10
Fan efficiency	1.07 ± 0.20	4.66 ± 0.12	3.11 ± 0.12
Booster tip airflow	0.62 ± 0.05	2.37 ± 0.11	6.58 ± 0.11
Booster tip efficiency*	NA	NA	NA
Booster hub airflow	3.90 ± 0.07	6.70 ± 0.07	0.84 ± 0.03
Booster hub efficiency	0.86 ± 0.06	4.81 ± 0.03	3.52 ± 0.03
High-pressure turbine airflow	0.70 ± 0.08	3.34 ± 0.09	6.92 ± 0.06
High-pressure turbine efficiency	4.42 ± 0.16	7.88 ± 0.90	3.48 ± 0.09
Low-pressure turbine airflow	12.34 ± 0.20	4.34 ± 1.02	7.43 ± 0.06
Low-pressure turbine efficiency	11.71 ± 0.49	5.01 ± 0.07	7.13 ± 0.06
Average	4.07 ± 0.05	4.70 ± 0.20	4.87 ± 0.04

*Booster tip efficiency is not yet implemented in MAPSS.

the health parameters. This gives an augmented state vector of dimension 13 (the three original states plus the 10 health parameters).

Sensor dynamics are assumed to be high enough bandwidth that they can be ignored in the dynamic equations. We simulated the methods discussed in this paper using Matlab. We measured a steady state 3 second burst of open loop engine data at 100 Hz during each flight. These routine data collections were performed over 50 flights at a single operating point, except the engine's health parameters deteriorated a small amount each flight. We simulated a linear-plus-exponential degradation of the 10 health parameters over 50 flights with open loop control.

Since we are interested only in health parameter estimation (not state estimation), we can design a reduced order filter to estimate only the 10 health parameter components of the augmented state vector while ignoring the original three state elements. Table 1 shows the root-mean-square (RMS) estimation error, averaged over 20 simulations, of the standard Kalman filter, Keller's reduced order filter [21], and the reduced order filter proposed in this paper. Keller's reduced order filter has a couple of restrictions that make it less flexible than the filter proposed in this paper. For example, Keller's filter is limited to time invariant systems, and the number of measurements must be greater than the number of unestimated states. However, those restrictions are not limiting factors in this example, so we can use Keller's filter for this problem.

For each filter shown in Table 1 the steady state filter gains were used. It is seen from the table that the full order filter performs better (on average) than the reduced order filters. However, for some health parameters the reduced order filters actually perform better than the full order filter. It is difficult to make any guarantees about performance for this example as it is a nonlinear system. Nevertheless these results show that the proposed filter works well and is competitive with other reduced order filters.

4. Conclusion

An approach has been presented for obtaining an optimal, unbiased, reduced order state estimator for stochastic dynamic systems. The reduced order estimator may be attractive in cases where the computational effort of the state estimator is an important consideration. Simulation results show the effectiveness of the proposed approach.

Previous to this paper, reduced order filters have been proposed by a number of different approaches. Reducing the order of the system model results in a reduced order filter, but this is at the expense of a loss of model information. Our approach is derived on the basis of the full order system model.

Bernstein presents an elegant approach that guarantees filter stability [18], but his approach applies only to steady state filtering and requires the solution of simultaneous Riccati equations, which may be mathematically difficult. Our approach does not guarantee stability, but it is mathematically simpler and can be used to derive time-varying filters as well as steady state filters.

Nagpal [19] and Keller [20, 21] present approaches that optimize the transition matrix of the filter as well as the gain of the filter. As our approach does not optimize the filter's transition matrix, it is expected that their approaches should yield better results than our approach. However, their approaches have restrictions on the number of states to be estimated and appear to be more complicated mathematically with the inclusion of matrix inverse operations. In our approach, like Napgal's approach, convergence and stability cannot be guaranteed without numerical calculations. However, Keller's approaches do include convergence and stability results.

This paper provides a new tool for reduced order state estimation. In some cases it may be more attractive than the previous approaches, while in other situations one of the previous approaches may be more attractive. The choice of which approach to use depends on the problem at hand. The advantages of the reduced order filter proposed here include mathematical simplicity and applicability to time-varying systems. For future work it would be important to analyze the convergence and stability of the filter proposed here.

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Biography

Dan Simon received his B.S., M.S., and Ph.D. from Arizona State University, the University of Washington, and Syracuse University, all in Electrical Engineering. He worked for 14 years in the aerospace, automotive, biomedical, process control, and software engineering fields. He is presently an Associate Professor at Cleveland State University, where he has been since 1999. His teaching and research are in the areas of control theory, state estimation, computational intelligence, and embedded systems. He has over 40 publications in refereed journals and conference proceedings. He is the author of the text *Optimal state estimation* (John Wiley & Sons, 2006).