2016 American Control Conference
Marriott Waterfront, Baltimore, MD, USA
June 30-July 02, 2016

Robust Iterative Learning Control for Output Tracking via Second-order Sliding Mode Technique

Wen Chen* and Yang-Quan Chen

Abstract— **This paper provides a new design of robust Iterative Learning Control (ILC) for the purpose of output tracking using second-order sliding mode technique. The main feature of the design is that the controller signal is continuous; therefore it is chattering-free compared with the robust ILC using classical first-order sliding mode technique. The continuous and non-chattering control signals can prevent actuators from being damaged. This can not be realized by the discontinuous and chattering control inputs if the conventional sliding-mode control is applied. The robust ILC is suggested and the convergence of output-tracking error is proven. An illustrative example is presented to demonstrate the effectiveness of the proposed second-order continuous sliding mode ILC.**

I. INTRODUCTION

Iterative Learning Control (ILC) is to use the repetitive actions to improve the system performance, particularly output tracking performance without seeking accurate system model knowledge. Conventionally, ILC is to steer system output to track the desired one while rejecting periodic disturbances via repetitive trails. In other words, ILC is a robust control scheme that can suppress the effect of disturbances on system performance.

The classical research regarding robust ILC is presented in [1] where the effect of state disturbances, initial errors and output noise on a class of learning algorithms are investigated. The presented learning algorithm exhibits the bounds on asymptotic trajectory errors for the learned input and the corresponding state and output trajectories.

In recent years, various robust ILC schemes have been addressed. A nonlinear learning control scheme was developed in [2] by integrating iterative learning and adaptive robust control schemes for nonlinear systems. The main purpose of the paper [3] is to provide ILC designers with guidelines to select the learning gains to achieve arbitrarily high precision of output tracking regardless of measurement errors. In [4], a robust ILC problem for a class of nonlinear systems with structured periodic and unstructured aperiodic uncertainties is addressed. The backstepping idea is proposed to design the robust ILC systems. More research papers related to this topic can be found in [5]- [13], just to name a few.

Particularly, a robust ILC synthesizing learning control and sliding mode technique with the help of Lyapunov direct method is proposed in [14]. The learning control is applied to the structured uncertainties while the variable structure scheme is to handle the unknown unstructured uncertainties to ensure the global asymptotic stability. Another similar work is suggested in [15] where a learning variable structure control is formalized by combining variable structure control, as the robust part, and learning control, as the intelligent part. The proposed LVSC system achieves both uniform convergence of the tracking-error sequences to zero and the convergence of the learning control sequences to the equivalent control.

In the aforementioned two research papers, to avoid the undesired chattering of a traditional Sliding Mode Control (SMC), continuous approximations, using saturation functions, is employed to reduce the chattering caused by the signum function. The problem is that once the error signals excess the designated boundary layer, a signum function is back to be in charge of the control action. Hence, the saturation function itself can reduce the chattering to an extent that when the tracking-error signal is within the boundary, the control signal is continuous. In short, the saturation function can not eliminate the chattering completely. Therefore, the adopted variable structure schemes are essentially first-order sliding mode measures that inevitably causes chattering though a saturation function has been used to replace the signum function. The actuators, however, cannot respond sometimes to the chattering input signals, let alone the possible damage caused by the discontinuous and chattering input signals.

Higher-order sliding mode is a quality idea to hide the discontinuity of control in its higher derivatives. It turns out to be of enhanced accuracy and robustness to disturbances [16]–[20]. In other words, compared with the saturation approximation of the traditional SMC, the continuous SMC via second-order sliding mode technique will require less amount of control to maintain the operation on the region of convergence due to noises and disturbances.

This paper is to suggest a continuous robust ILC using second-order sliding mode technique so that the chattering of control signals can be eliminated eventually; thus, the continuous robust ILC can be applied broadly without damaging actuation devices.

This paper is organized as follows: in Section II, the considered nonlinear system is illustrated and the objective of this paper is also addressed. The switching surface and the controller design are described in Section III. The convergence of the output-tracking error is also proven using Lyapunov direct method in the same section. An illustrative example is employed to demonstrate the effectiveness of the proposed chattering-free robust ILC. At last, concluding

*W. Chen is with Division of Engineering Technology, Wayne State University, Detroit, MI, USA, 48202. Corresponding author: chen@eng.wayne.edu.

Y.-Q. Chen is with the Department of Electrical Engineering, Utah State University, 4120 Old Main Hill, Logan, UT 84322-4120, USA. yqchen@engineering.usu.edu.

978-1-4244-7427-1/10/$26.00 ©2010 AACC

remarks are made in Section V.

II. Preliminaries

Consider the following higher-order single-input and single-output nonlinear dynamical system described by

$$\begin{aligned} \dot{x}_i(t) &= x_{i+1}(t), \quad i = 1, \cdots, m-1 \\ \dot{x}_m(t) &= \theta^\top(x,t)\xi(x,t) + b(x,t)u(t) + d(t) \\ y(t) &= x_1(t), \quad t \in [0,\ T] \end{aligned} \tag{1}$$

where the measurable system state $x(t) = [x_1,\ x_2,\ \cdots, x_m]^\top$, $u(t)$ and $y(t)$ are the control input and system output, respectively, $b(x,t)$ is a known non-zero function, $\theta(x,t)$ is a $p \times 1$ unknown and time-varying function to be learnt, $\xi(x,t)$ is a known vector-valued function with dimension of $p \times 1$. The variable $d(t)$ represents the unknown disturbance.

Assumption 1: The desired output trajectory $y_d(t)$ is differentiable with respect to time t up to the mth order on a finite time interval $[0,\ T]$, and all of the higher-order derivatives are available.

Assumption 2: The unknown disturbance variable $d(t)$ is bounded such that

$$|d(t)| \le b_d,\ \forall t \in [0,\ T],$$

where b_d is a known constant.

Assumption 3: The initial condition $e(0) = \dot{e}(0) = \ddot{e}(0) = \cdots = e^{(m)}(0) = 0$ at any iteration $\forall t \in [0,\ T]$, such that the switching surface $\sigma(0) = 0$, where $e(t)$ is the output tracking error that is defined as $e(t) = y_d(t) - y(t)$.

The control objective is to design a continuous second-order sliding-mode iterative learning controller $u(t)$ for the uncertain nonlinear system (1) such that system output can follow a desired one with a prescribed accuracy ϵ as follows:

$$\forall t \in [0,\ T],\ |y_d(t) - y(t)| \le \epsilon.$$

III. Main Results

The underlying robust ILC is to learn and approach the unknown state-dependent function and leave the remaining unknown function to the robust control. The global asymptotic convergence with respect to iteration is established by Lyapunov direct method.

A. Switching Surface

For the considered system (1), a switching surface is defined as follows:

$$\sigma(t) = c_1 e(t) + c_2 \dot{e}(t) + \cdots + c_m e^{(m-1)} = \sum_{i=1}^{m} c_i e^{(i-1)} \tag{2}$$

where $c_m = 1$, $c_i's$ $(i = 1, \cdots, m-1)$ are coefficients of a Hurwitz polynomial, and $e(t) = y_d(t) - y(t) = y_d(t) - x_1(t)$.

Taking derivatives with respect to time t on both sides of (2), it is obtained:

$$\dot{\sigma}(t) = c_1 \dot{e}(t) + c_2 \ddot{e}(t) + \cdots + c_m e^{(m)} = \sum_{i=1}^{m} c_i e^{(i)}. \tag{3}$$

Considering the fact that $e(t) = y_d(t) - x_1(t)$, the above equation can be further expanded:

$$\begin{aligned} \dot{\sigma}(t) &= c_1 [\dot{y}_d(t) - x_2(t)] + c_2 [\ddot{y}_d(t) - x_3(t)] + \cdots \\ &\quad + \left[y_d^{(m)}(t) - \theta^\top(x,t)\xi(x,t) - b(x,t)u(t) - d(t)\right] \\ &= \sum_{i=1}^{m} c_i y_d^{(i)} - \sum_{i=1}^{m-1} c_i x_{i+1} - \theta^\top(x,t)\xi(x,t) \\ &\quad - b(x,t)u(t) - d(t). \end{aligned} \tag{4}$$

The above equation can be further interpreted as the sliding variable dynamics. The condition, $\sigma(t) = 0$, defines the system motion on the sliding surface. The control signal, $u(t)$, is to be designed as an iterative and continuous control input signal. The task of this work is to design such a continuous and iterative control input to steer the sliding surface to be zero in finite time interval.

B. Robust ILC using Continuous Second-order Sliding Mode

In [17] and [18], the second-order sliding-mode concept is originated. It is further developed in [19]. In reference to these work, the proposed continuous second-order sliding-mode ILC at kth iteration is designed as follows:

$$\begin{aligned} u_k(t) &= b^{-1}(x_k,t)\Bigg(\sum_{i=1}^{m} c_i y_d^{(i)}(t) - \sum_{i=1}^{m-1} c_i x_{i+1,k} \\ &\quad - \hat{\theta}_k^\top(t)\xi(x_k,t) - v_k(t) + \alpha_1 |\sigma_k|^{\frac{2}{3}} sgn(\sigma_k) \\ &\quad + \alpha_3 \sigma_k(t)\Bigg) \end{aligned} \tag{5}$$

where k indicates the number of iterations, $x_k(t) = [x_{1,k},\ x_{2,k},\ \cdots, x_{m,k}]^\top$, α_1, α_2, and α_3 are positive constants, $|\cdot|$ is the absolute value, sgn is the signum function, $\hat{\theta}(t)$ is the recursive control part that is used to learn the unknown function $\theta(x,t)$ and generated by the following update law

$$\hat{\theta}_k(t) = \hat{\theta}_{k-1}(t) - q\xi(x_k,t)\left(\frac{4\eta}{3}|\sigma_k|^{\frac{1}{3}} sgn(\sigma_k) + \gamma\sigma_k(t)\right) \tag{6}$$

where q, η and γ are positive constants.

The variable $v(t)$ is an integral term that is defined below:

$$\dot{v}_k(t) = -\beta_1 \sigma_k(t) - \beta_2 |\sigma_k|^{\frac{1}{3}} sgn(\sigma_k) \tag{7}$$

where β_1 and β_2 are positive constants.

Controller (5), together with (6) and (7), defines the continuous second-order sliding-mode ILC.

Inserting the ILC law (5) into the sliding surface dynamics (4) yields:

$$\dot{\sigma}_k(t) = -\alpha_3\sigma_k(t) + \tilde{\theta}_k^\top(t)\xi(x_k,t) + v_k(t) - d_k(t) - \alpha_1|\sigma_k|^{\frac{2}{3}}sgn(\sigma_k) \tag{8}$$

where $\tilde{\theta}_k(t) = \hat{\theta}_k(t) - \theta_k(t)$.

The integral term $v(t)$ is used to attenuate the effect of the unknown disturbance $d(t)$.

Remark 1: According to [20], the second-order SMC is more robust to noises and disturbances than the saturation approximation of the traditional SMC because the control amount required to maintain the regision of convergence is less.

Theorem 1: The robust ILC law using second-order continuous sliding surface proposed in (5), (6), and (7) can guarantee that the system output of the considered system (1) follows the desired trajectory asymptotically while all system state variables are bounded.

Proof: To evaluate the convergence property of the output tracking error $e(t)$, we define the following composite energy function at kth iteration:

$$V_k(t) = V_k^1(t) + V_k^2(t) + V_k^3(t) + V_k^4(t) \tag{9}$$

where $V_k^1(t) = \dfrac{v_k(t)v_k(t)}{2}$, $V_k^2(t) = \eta|\sigma_k(t)|^{\frac{4}{3}}$, $V_k^3(t) = \gamma\dfrac{\sigma_k(t)\sigma_k(t)}{2}$, and $V_k^4(t) = \dfrac{1}{2q}\displaystyle\int_0^t \Phi_k^\top(\tau)\Phi_k(\tau)d\tau$, where $\Phi(t) = \hat{\theta}_k(t) - \hat{\theta}_{k-1}(t)$.

The proof consists two parts. The first part is to derive the difference of the energy function between two iterations; meanwhile the other part is to evaluate the convergence of the output tracking error.

1) Differences of the Energy Function: In the following derivations, the reset condition shown in Assumption 3 will be used.

The difference of the first energy function between kth and $(k-1)$th iterations is represented by $\Delta V_k^1(t) = V_k^1(t) - V_{k-1}^1(t)$ and has the following form:

$$\begin{aligned}\Delta V_k^1(t) &= \frac{v_k(t)v_k(t)}{2} - \frac{v_{k-1}(t)v_{k-1}(t)}{2} \\ &= \int_0^t v_k(\tau)\dot{v}_k(\tau)d\tau - \frac{v_{k-1}(t)v_{k-1}(t)}{2}.\end{aligned} \tag{10}$$

Substituting the integral term $v(t)$ proposed in (7) into (10), it is obtained:

$$\begin{aligned}\Delta V_k^1 = &-\beta_1\int_0^t v_k(\tau)\sigma_k(\tau)d\tau \\ &-\beta_2\int_0^t v_k(\tau)|\sigma_k|^{\frac{1}{3}}sgn(\sigma_k)d\tau \\ &-\frac{v_{k-1}(t)v_{k-1}(t)}{2}.\end{aligned} \tag{11}$$

The difference of the second energy function between kth and $(k-1)$th iterations can be expressed as:

$$\Delta V_k^2(t) = \eta|\sigma_k(t)|^{\frac{4}{3}} - \eta|\sigma_{k-1}(t)|^{\frac{4}{3}}. \tag{12}$$

Take derivative on $|\sigma_k(t)|^{\frac{4}{3}}$ with respect to time t, we have:

$$\frac{d}{dt}\left(|\sigma_k(t)|^{\frac{4}{3}}\right) = \frac{4}{3}|\sigma_k(t)|^{\frac{1}{3}}sgn(\sigma_k)\dot{\sigma}_k(t). \tag{13}$$

Based on (13), (12) has an alternative form:

$$\Delta V_k^2(t) = \frac{4}{3}\eta\int_0^t |\sigma_k|^{\frac{1}{3}}sgn(\sigma_k)\dot{\sigma}_k(\tau)d\tau - \eta|\sigma_{k-1}|^{\frac{4}{3}}. \tag{14}$$

Combining (8) into (14) yields

$$\begin{aligned}\Delta V_k^2(t) = &-\frac{4\alpha_3\eta}{3}\int_0^t |\sigma_k|^{\frac{1}{3}}sgn(\sigma_k)\sigma_k d\tau \\ &+\frac{4\eta}{3}\int_0^t |\sigma_k|^{\frac{1}{3}}sgn(\sigma_k)\tilde{\theta}_k^\top(\tau)\xi(x_k,\tau)d\tau \\ &+\frac{4\eta}{3}\int_0^t |\sigma_k|^{\frac{1}{3}}sgn(\sigma_k)v_k(\tau)d\tau \\ &-\frac{4\eta}{3}\int_0^t |\sigma_k|^{\frac{1}{3}}sgn(\sigma_k)d_k(\tau)d\tau \\ &-\frac{4\alpha_1\eta}{3}\int_0^t |\sigma_k|^{\frac{1}{3}}sgn(\sigma_k)|\sigma_k|^{\frac{2}{3}}sgn(\sigma_k)d\tau \\ &-\eta|\sigma_{k-1}(t)|^{\frac{4}{3}}\end{aligned} \tag{15}$$

Considering the fact that

$$sgn(\sigma_k)\sigma_k = |\sigma_k|,\ |sgn(\sigma_k)| = 1, \tag{16}$$

and

$$sgn(\sigma_k)sgn(\sigma_k) = 1, \tag{17}$$

(15) can be simplified as:

$$\begin{aligned}\Delta V_k^2(t) \leq &-\eta|\sigma_{k-1}(t)|^{\frac{4}{3}} - \frac{4\alpha_3\eta}{3}\int_0^t |\sigma_k(\tau)|^{\frac{4}{3}}d\tau \\ &+\frac{4\eta}{3}\int_0^t |\sigma_k(\tau)|^{\frac{1}{3}}sgn(\sigma_k)\tilde{\theta}_k^\top(\tau)\xi(x_k,\tau)d\tau \\ &+\frac{4\eta}{3}\int_0^t |\sigma_k(\tau)|^{\frac{1}{3}}sgn(\sigma_k)v_k(\tau)d\tau \\ &+\frac{4b_d\eta}{3}\int_0^t |\sigma_k(\tau)|^{\frac{1}{3}}d\tau - \frac{4\alpha_1\eta}{3}\int_0^t |\sigma_k(\tau)|d\tau.\end{aligned} \tag{18}$$

It is worth noting that the upper bound, b_d, of the unknown disturbance is inserted into the above inequality.

The difference of the third energy function between kth and $(k-1)$th iterations has the following form:

$$\Delta V_k^3(t) = \gamma\frac{\sigma_k(t)\sigma_k(t)}{2} - \gamma\frac{\sigma_{k-1}(t)\sigma_{k-1}(t)}{2}$$
$$= \gamma\int_0^t \sigma_k(\tau)\dot{\sigma}_k(\tau)d\tau - \gamma\frac{\sigma_{k-1}(t)\sigma_{k-1}(t)}{2}. \quad (19)$$

Substituting (8) into the above equation and considering (16), the above equation can be rearranged as:

$$\begin{aligned}\Delta V_k^3(t) \le & -\gamma\frac{\sigma_{k-1}(t)\sigma_{k-1}(t)}{2} - \gamma\alpha_3\int_0^t \sigma_k(\tau)\sigma_k(\tau)d\tau \\ & +\gamma\int_0^t \sigma_k(\tau)\tilde{\theta}_k^\top(t)\xi(x_k,t)d\tau \\ & +\gamma\int_0^t \sigma_k(\tau)v_k(\tau)d\tau \\ & +b_d\gamma\int_0^t |\sigma_k(\tau)|d\tau - \gamma\alpha_1\int_0^t |\sigma_k(\tau)|^{\frac{5}{3}}d\tau. \end{aligned} \quad (20)$$

At last, the difference of the fourth energy function between kth and $(k-1)$th iterations is shown below:

$$\begin{aligned}\Delta V_k^4(t) = & \frac{1}{2q}\int_0^t \left(\Phi_k^\top(\tau)\Phi_k(\tau)\right)d\tau \\ & -\frac{1}{2q}\int_0^t \left(\Phi_{k-1}^\top(\tau)\Phi_{k-1}(\tau)\right)d\tau.\end{aligned} \quad (21)$$

According to the update law (6) and in reference to [21], the following relationship also holds:

$$\begin{aligned} & \frac{1}{2q}\left(\Phi_k^\top\Phi_k - \Phi_{k-1}^\top\Phi_{k-1}\right) \\ = & \frac{1}{2q}\left(\hat{\theta}_k - \hat{\theta}_{k-1}\right)^\top\left(\hat{\theta}_k + \hat{\theta}_{k-1} - 2\theta\right) \\ = & \frac{1}{q}\left(\hat{\theta}_k - \theta\right)^\top(\hat{\theta}_k - \hat{\theta}_{k-1}) \\ & -\frac{1}{2q}\left(\hat{\theta}_k - \hat{\theta}_{k-1}\right)^\top\left(\hat{\theta}_k - \hat{\theta}_{k-1}\right) \\ = & -\frac{1}{q}\tilde{\theta}_k^\top(t)\xi(x_k,t)\left(\frac{4q\eta}{3}|\sigma_k|^{\frac{1}{3}}sgn(\sigma_k) + q\gamma\sigma_k\right) \\ & -\frac{1}{2q}\left(\hat{\theta}_k - \hat{\theta}_{k-1}\right)^\top\left(\hat{\theta}_k - \hat{\theta}_{k-1}\right) \\ = & -\frac{4\eta}{3}|\sigma_k|^{\frac{1}{3}}sgn(\sigma_k)\tilde{\theta}_k^\top\xi(x_k,t) - \gamma\sigma_k\tilde{\theta}_k^\top\xi(x_k,t) \\ & -\frac{1}{2q}\left(\hat{\theta}_k - \hat{\theta}_{k-1}\right)^\top\left(\hat{\theta}_k - \hat{\theta}_{k-1}\right).\end{aligned} \quad (22)$$

Therefore, (21) is related to sliding surface dynamics in the following way:

$$\begin{aligned}\Delta V_k^4(t) = & -\frac{4\eta}{3}\int_0^t\left(|\sigma_k|^{\frac{1}{3}}sgn(\sigma_k)\tilde{\theta}_k^\top(\tau)\xi(x_k,\tau)\right)d\tau \\ & -\gamma\int_0^t\left(\sigma_k(\tau)\tilde{\theta}_k^\top(\tau)\xi(x_k,\tau)\right)d\tau \\ & -\frac{1}{2q}\left(\hat{\theta}_k - \hat{\theta}_{k-1}\right)^\top\left(\hat{\theta}_k - \hat{\theta}_{k-1}\right).\end{aligned} \quad (23)$$

On the basis of the difference between two iterations, we go further to second part to prove the convergence of the output tracking error.

2) Convergence of Output Tracking Error: Let $\gamma = \beta_1$, $\beta_2 = \frac{4\eta}{3}$, and $b_d\gamma = \frac{4\alpha_1\eta}{3}$, the sum of the differences of the total energy function can be obtained by adding all of them:

$$\begin{aligned}\Delta V_k(t) = & \Delta V_k^1(t) + \Delta V_k^2(t) + \Delta V_k^3(t) + \Delta V_k^4(t) \\ \le & -\frac{v_{k-1}(t)v_{k-1}(t)}{2} - \eta|\sigma_k(t)|^{\frac{4}{3}} \\ & -\frac{4\alpha_3\eta}{3}\int_0^t |\sigma_k(\tau)|^{\frac{4}{3}}d\tau \\ & -\gamma\frac{\sigma_{k-1}(t)\sigma_{k-1}(t)}{2} - \alpha_3\gamma\int_0^t \sigma_k(\tau)\sigma_k(\tau)d\tau \\ & +\frac{4b_d\eta}{3}\int_0^t |\sigma_k(\tau)|^{\frac{1}{3}}d\tau - \gamma\alpha_1\int_0^t |\sigma_k(\tau)|^{\frac{5}{3}}d\tau.\end{aligned} \quad (24)$$

The above inequality can be further simplified as

$$\Delta V_k(t) \le -\alpha_3\gamma\int_0^t \sigma_k(\tau)\sigma_k(\tau)d\tau,\ \forall\ |\sigma_k(t)| > \frac{b_d}{\alpha_3} \quad (25)$$

which is negative definite when $\sigma_k(t) \neq 0,\ t \in [0,\ T]$. This concludes that the energy function $V_k(t)$ is convergent. In addition, positive definiteness of $V_k(t)$ can ensure the convergence of the sliding surface dynamics $\sigma_k(t)$ to the region of $|\sigma_k(t)| = \frac{b_d}{\alpha_3}$. Since the sliding surface dynamics (2) is selected to be Hurwitz, then, the output-tracking error is convergent asymptotically. This completes the proof of the Theorem 1.

■

Remark 2: By selecting a large α_3, the region of convergence can be made small.

Remark 3: The region of convergence of the robust ILC based on second-order SMC is greater than that for traditional SMC. The advantage is that the robust ILC is continuous and chattering-free for practical implementation.

Remark 4: Future research could be pursued to use estimated system states for the design of the robust ILC.

IV. An Illustrative Example

Consider the following robotic manipulator example borrowed from [21]:

$$\begin{aligned}\dot{x}_1(t) &= x_2(t)\\ \dot{x}_2(t) &= \frac{1}{ml^2+I}\left(u - gl\cos(x_1) + d(t)\right)\\ y(t) &= x_1(t)\end{aligned} \tag{26}$$

where x_1 is the joint angle, x_2 is the angular velocity, m is the mass, l is the length, I is the moment of inertia, u is the joint input and $d(t)$ is a disturbance being $0.2\sin(x_1x_2)$. Further, the system output is x_1 and the desired output trajectory is $sin^2(t)$. The parameters take the values: $m = 3kg$, $l = 1m$, and $I = 0.5kg.m^2$.

Inserting the parameter values, (26) has an alternative form:

$$\begin{aligned}\dot{x}_1(t) &= x_2(t)\\ \dot{x}_2(t) &= -2.8\cos(x_1) + 0.2857u(t) + 0.2\sin(x_1x_2)\\ y(t) &= x_1(t)\end{aligned} \tag{27}$$

Comparing the above system with the considered system (1), we obtain: $b = 0.2857$, $\xi(x,t) = -2.8$, and $\theta(x,t) = cos(x_1)$ which is assumed unknown and to be learnt.

Following (2), a switching surface is design as:

$$\sigma(t) = c_1e(t) + \dot{e}(t) \tag{28}$$

where $c_1 = 1$, and $e(t) = sin^2(t) - x_1(t)$. Taking derivative on the above switching surface yields:

$$\dot{\sigma}(t) = c_1\dot{e}(t) + \ddot{e}(t) \tag{29}$$

The proposed robust ILC can be designed as follows:

$$\begin{aligned}u_k(t) = \; & b^{-1}(x,t)\{2c_1\sin(t)\cos(t) - c_1x_2(t)\\ & +2[\cos(t)\cos(t) - \sin(t)\sin(t)]\\ & +2.8\hat{\theta}(t) - v(t) + \alpha_1|\sigma_k|^{2/3}sgn(\sigma_k)\\ & +\alpha_3\sigma_k(t)\}\end{aligned} \tag{30}$$

The parameters take the following values: $\alpha_1 = 0.15, \alpha_3 = 1, \beta_1 = 0.1, \eta = 0.1, \beta_2 = \eta \times \frac{4}{3}, q = 0.1, \gamma = \beta_1$.

We first of all test the proposed ILC with the number of trials being 50. The output-tracking errors at $5th$, $20th$, $30th$, and $50th$ trials are shown in Figs. 1 through 4 where the dashed lines are the desired trajectory while the solid lines are the actual system outputs. It is clearly demonstrated that the system output gradually converges to the desired trajectory. Figure 5 further shows the convergence of the output-tracking error.

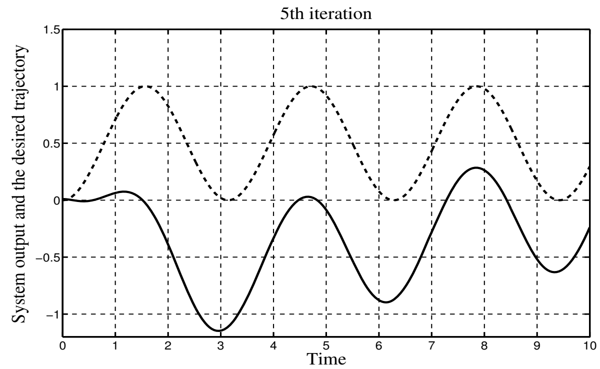

Fig. 1. System output and the desired trajectory at 5th iteration.

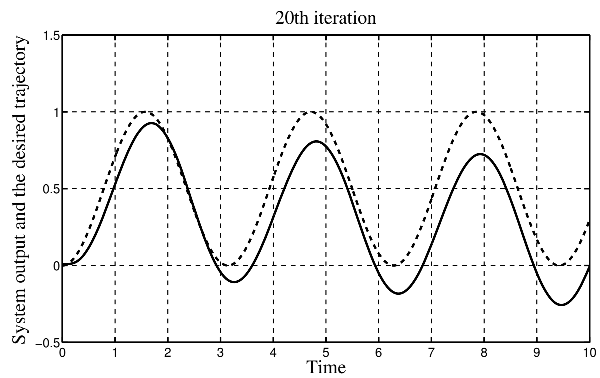

Fig. 2. System output and the desired trajectory at 20th iteration.

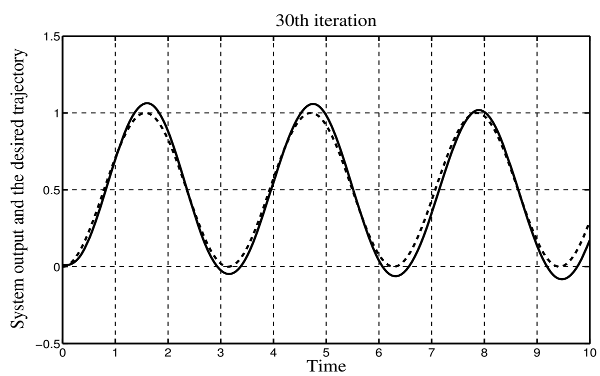

Fig. 3. System output and the desired trajectory at 30th iteration.

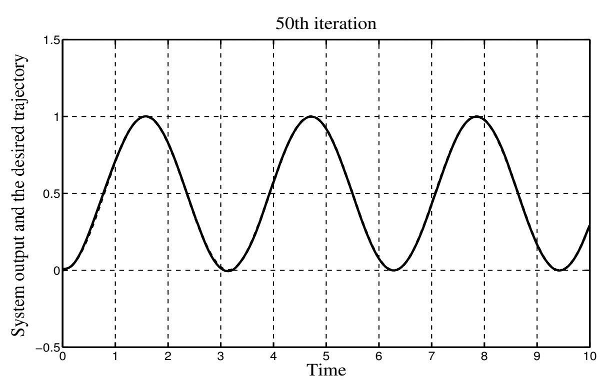

Fig. 4. System output and the desired trajectory at 50th iteration.

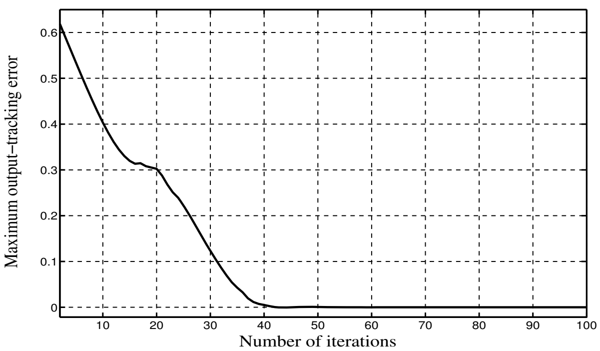

Fig. 5. Maximum output-tracking error for 100 trials.

V. Conclusions

This paper has proposed a continuous robust iterative learning control strategy for a class of nonlinear systems for the purpose of output tracking. The insight of the chattering-free design is the employment of the second-order sliding mode technique that can completely remove the discontinuity of the control signals. This continuous control signal is beneficial to actuation devices. In other words, possible damage of actuators resulted from the discontinuous control signals is avoided. The design process has shown that the robustness of the second-order sliding-mode control is maintained while the chattering is eliminated at the same time. The simulation example has clearly exhibited the excellent output-tracking performance by the continuous second-order sliding-mode-based robust iterative learning control.

References

[1] G. Heinzinger, D. Fenwick, B. Paden, and F. Miyazaki, "Stability of learning control with disturbances and uncertain initial conditions," *IEEE Trans Auto. Contr.*, vol. 31, no. 1, 1992, pp. 110-114.

[2] J. X. Xu and B. Viswanathan, "Adaptive robust itarative learning control with dead zone scheme," *Automatica*, vol. 36, 2000, pp. 91-99.

[3] S. S. Saab, "Selection of the Learning Gain Matrix of an Iterative Learning Control Algorithm in Presence of Measurement Noise," *IEEE Trans Auto. Contr.*, vol. 50, no. 11, 2015, pp. 1761-1774.

[4] Y.-P. Tian, X. H. Yu, "Robust learning control for a class of nonlinear systems with periodic and aperiodic uncertainties," *Automatica*, vol. 39, 2013, pp. 1957 1966.

[5] D. Wang, "Convergence and robustness of discrete time nonlinear systems with iterative learning control," *Automatica*, vol. 34, no. 11, 1998, pp. 14451448.

[6] J. X. Xu and Y. Tan, "Robust optimal design and convergence properties analysis of iterative learning control approaches," *Automatica*, vol. 38, 2012, pp. 1867 1880.

[7] D.-Y. Pi and K. Panaliappan, Robustness of discrete nonlinear systems with open-closed-loop iterative learning control, in *Proc. 2002 Int. Conf. Mach. Learning Cybern.*, Nov. 2002, pp. 12631266.

[8] A. Tayebi and J. X. Xu, Observer-based iterative learning control for a class of time-varying nonlinear systems, *IEEE Trans. Circuits Syst. IFundam. Theory Appl.*, vol. 50, no. 3, pp. 452455, 2013.

[9] A. Tayebi and M. B. Zaremba, Robust ILC design is straightforward for uncertain LTI systems satisfying the robust performance condition, *IFAC 15th World Congr.*, Barcelona, Spain, Jul. 2002.

[10] A. Tayebi and M. B. Zaremba, "Internal model-based robust iterative learning control for uncertain LTI systems," *Proceedings of the 39th IEEE Conference on Decision and Control,"* Sydney, Australia, December, 2000, pp.3439 - 3444.

[11] H.-S. Ahn, Y. Q. Chen and K. L. Moore, "Intermittent Iterative Learning Control," *Proceedings of the 2006 IEEE International Symposium on Intelligent Control*, Munich, Germany, October 4-6, 2006, pp. 832-837.

[12] H.-S. Ahn, Y. Q. Chen and K. L. Moore, " Stability analysis of discrete-time iterative learning control systems with interval uncertainty," *Automatica*, vol. 43, no. 5, 2007, pp. 892-902.

[13] J.-H. Moon, T.-Y. Doh and M. J. Chung, "A robust approach for iterative learning control design for uncertain systems," *Automatica*, vol. 34, no. 8, pp. 1001-1004.

[14] J. X. Xu and Z. Qu, "Robust Iterative Learning Control for a Class of Nonlinear Systems," *Automatica*, vol. 34, no. 8, 1998, pp. 983-988.

[15] J. X. Xu and W. J. Cao, "Learning variable structure control approaches for repeatable tracking control tasks," *Automatica*, vol. 37, no. 7, 2001, pp. 997-1006.

[16] G. Bartolini, A. Ferrara, and E. Usai, Chattering Avoidance by Second-Order Sliding Mode Control, *IEEE Trans. Automat. Contr.*, Vol. 43, No. 2, , 1998, pp. 241-246.

[17] A. Levant, Robust Exact Differentiation via Sliding Mode Technique, Automatica, Vol. 34, No. 3, 1998, pp. 379-384.

[18] A. Levant, Universal SISO Sliding-Mode Controllers with Finite-Time Convergence, *IEEE Trans. Automat. Contr.*, Vol. 46, No. 9, 2001, pp. 1447-1451.

[19] B. S. Yuri, I. A. Shkolnikov, and A. levant, " Guidance and Control of Missile Interceptor using Second-Order Sliding Modes," *IEEE Trans. Aero. Ele. Syst. ,* vol. 45, no. 1, 2009, pp. 110-124.

[20] M. D. J. Brown, Y. B. Shtessel and J. M. Buffington, "Finite reaching time continuous sliding mode control with enhanced robustness", *in AIAA Guidance, Navigation and Control Conference and Exhibit*, Aug. 14-17, Denver, CO, 2000, AIAA-2000-3964.

[21] J. X. Xu and J. Xu, "On iterative learning from different tracking tasks in the presence of time-varying uncertainties," *IEEE Trans. Sys., Man, Cyber*, vol. 34, 2014, pp. 589-597.

[22] H. Kwakernaak and R. Sivan, *Modern Signals and Systems*, Prentice Hall, Englewood Cliffs, NJ; 1991.

[23] D. Boley and R. Maier, "A Parallel QR Algorithm for the Non-Symmetric Eigenvalue Algorithm", *in Third SIAM Conference on Applied Linear Algebra*, Madison, WI, 1988, pp. A20.