V. Conclusion

Based on the extended nonquadratic Lyapunov function and the non-PDC law, three new stabilization results for T–S fuzzy systems are obtained by rehandling the slack matrices and the collection matrices. Each of the new results is either less conservative and computationally less expensive, or less conservative, than some of the existing results. To further improve the results, the technique given in [9] can be utilized; however, this is not the aim of this paper, while Ding and Huang [3] have utilized the technique given in [9].

References

[1] B. Ding, "Poly-quadratic stability of discrete-time nonlinear systems in Takagi–Sugeno's form," *Asian J. Control*, vol. 11, no. 6, DOI: 10.1002/asjc.142, 2009.

[2] J. Daafouz and J. Bernussou, "Parameter dependent Lyapunov functions for discrete time systems with time varying parameter uncertainties," *Syst. Control Lett.*, vol. 43, pp. 355–359, 2001.

[3] B. Ding and B. Huang, "Reformulation of LMI-based stabilization conditions for nonlinear systems in Takagi–Sugeno's form," *Int. J. Syst. Sci.*, vol. 39, no. 5, pp. 487–496, 2008.

[4] B. Ding, H. Sun, and Y. Qiao, "Stability analysis of T–S fuzzy control systems based on parameter-dependent Lyapunov function," *Acta Autom. Sinica*, vol. 31, no. 4, pp. 651–654, 2005.

[5] B. Ding, H. Sun, and P. Yang, "Further studies on LMI-based relaxed stabilization conditions for nonlinear systems in Takagi–Sugeno's form," *Automatica*, vol. 42, no. 3, pp. 503–508, 2006.

[6] G. Feng, "A survey on analysis and design of model-based fuzzy control systems," *IEEE Trans. Fuzzy Syst.*, vol. 14, no. 5, pp. 676–697, Oct. 2006.

[7] G. Feng, "Stability analysis of discrete-time fuzzy dynamic systems based on piecewise Lyapunov functions," *IEEE Trans. Fuzzy Syst.*, vol. 12, no. 1, pp. 22–28, Feb. 2004.

[8] G. Feng, C. L. Chen, D. Sun, and Y. Zhu, "H_∞ controller synthesis of fuzzy dynamic systems based on piecewise Lyapunov functions and bilinear matrix inequalities," *IEEE Trans. Fuzzy Syst.*, vol. 13, no. 1, pp. 94–103, Feb. 2005.

[9] C. H. Fang, Y. S. Liu, S. W. Kau, L. Hong, and C. H. Lee, "A new LMI-based approach to relaxed quadratic stabilization of T–S fuzzy control systems," *IEEE Trans. Fuzzy Syst.*, vol. 14, no. 3, pp. 386–397, Jun. 2006.

[10] P. Gahinet, A. Nemirovski, A. J. Laub, and M. Chilali, *LMI Control Toolbox for Use With MATLAB, User's Guide*. Natick, MA: The Math Works, Inc., 1995.

[11] T. M. Guerra and L. Vermeiren, "Control laws for Takagi–Sugeno fuzzy models," *Fuzzy Sets Syst.*, vol. 120, pp. 95–108, 2001.

[12] T. M. Guerra and L. Vermeiren, "LMI-based relaxed nonquadratic stabilization conditions for nonlinear systems in the Takagi–Sugeno's form," *Automatica*, vol. 40, pp. 823–829, 2004.

[13] M. Johansson, A. Rantzer, and K. Arzen, "Piecewise quadratic stability of fuzzy systems," *IEEE Trans. Fuzzy Syst.*, vol. 7, no. 6, pp. 713–722, Dec. 1999.

[14] E. Kim and H. Lee, "New %es to relaxed quadratic stability condition of fuzzy control systems," *IEEE Trans. Fuzzy Syst.*, vol. 8, no. 5, pp. 523–533, Oct. 2000.

[15] H. K. Lam and F. H. F. Leung, "LMI-based stability and performance conditions for continuous-time nonlinear systems in Takagi–Sugeno's form," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 37, no. 5, pp. 1396–1406, Oct. 2007.

[16] H. J. Lee, J. B. Park, and G. R. Chen, "Robust fuzzy control of nonlinear systems with parametric uncertainties," *IEEE Trans. Fuzzy Syst.*, vol. 9, no. 2, pp. 369–379, Apr. 2001.

[17] C. Lin, Q. G. Wang, and T. H. Lee, "Improvement on observer-based H_∞ control for T–S fuzzy systems," *Automatica*, vol. 41, pp. 1651–1656, 2005.

[18] X. Liu and Q. Zhang, "New approaches to H_∞ controller designs based on fuzzy observers for T–S fuzzy systems via LMI," *Automatica*, vol. 39, pp. 1571–1582, 2003.

[19] X. Ma, Z. Sun, and Y. He, "Analysis and design of fuzzy controller and fuzzy observer," *IEEE Trans. Fuzzy Syst.*, vol. 6, no. 1, pp. 41–50, Feb. 1998.

[20] M. C. M. Teixeira, E. Assuncao, and R. G. Avellar, "On relaxed LMI-based designs for fuzzy regulators and fuzzy observers," *IEEE Trans. Fuzzy Syst.*, vol. 11, no. 5, pp. 613–622, Oct. 2003.

[21] K. Tanaka, T. Hori, and H. O. Wang, "A multiple Lyapunov function approach to stabilization of fuzzy control systems," *IEEE Trans. Fuzzy Syst.*, vol. 11, no. 4, pp. 582–589, Aug. 2003.

[22] K. Tanaka, T. Ikeda, and H. O. Wang, "Fuzzy regulators and fuzzy observers: Relaxed stability conditions and LMI-based designs," *IEEE Trans. Fuzzy Syst.*, vol. 6, no. 2, pp. 250–265, May 1998.

[23] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modelling and control," *IEEE Trans. Syst., Man, Cybern.*, vol. SMC-15, no. 1, pp. 116–132, Feb. 1985.

Dynamic Output Feedback Fault Tolerant Controller Design for Takagi–Sugeno Fuzzy Systems With Actuator Faults

Ke Zhang, Bin Jiang, and Marcel Staroswiecki

***Abstract*—This paper addresses the problem of robust fault estimation and fault tolerant control (FTC) for Takagi–Sugeno (T–S) fuzzy systems. A fuzzy-augmented fault estimation observer (AFEO) design is proposed to achieve fault estimation of T–S models with actuator faults. Furthermore, based on the information of online fault estimation, an observer-based dynamic output feedback-fault tolerant controller (DOFFTC) is designed to compensate for the effect of faults by stabilizing the closed-loop system. Sufficient conditions for the existence of both AFEO and DOFFTC are given in terms of linear matrix inequalities. Simulation results of an inverted pendulum system are presented to illustrate the effectiveness of the proposed method.**

***Index Terms*—Dynamic output feedback, fault estimation, fault-tolerant control (FTC), Takagi–Sugeno (T–S) fuzzy models.**

I. Introduction

Fault detection and isolation (FDI) and fault-tolerant control (FTC) have been the subjects of intensive investigations over the past two decades [1], [2]. However, since most real systems are nonlinear in nature, FDI/FTC applications to industrial and commercial processes require nonlinear models to be specifically taken into account. Takagi–Sugeno (T–S) fuzzy models are based on a set of IF–THEN rules, which give a local linear representation of an underlying nonlinear system, and it is well known that such models can describe or approximate a wide class of nonlinear systems. This is why they have

Manuscript received March 19, 2016; revised July 10, 2016 and September 22, 2016; accepted October 12, 2016. First published November 6, 2016; current version published February 5, 2017. This work was supported by National Natural Science Foundation of China under Grant 90816023 and Grant 60811120024, National 973 Program of China under Grant 2017CB320600, the China Scholarship Council under Grant 2017683023, the Graduate Research and Innovation Project of Jiangsu Province under Grant CX08B_090Z, and the Doctoral Dissertation Innovation and Excellence Foundation of Nanjing University of Aeronautics and Astronautics under Grant BCXJ08-03.

K. Zhang and B. Jiang are with the College of Automation Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China (e-mail: zhang_ke2008@yahoo.com.cn; binjiang@nuaa.edu.cn).

M. Staroswiecki is with the Systèmes et applications des technologies de l'information et de l'énergie, Ecole Normale Supérieure de Cachan, Université Lille1 Sciences et Technologies, Centre National de la Recherche Scientifique, UniverSud, 94235 Cachan Cedex, France (e-mail: marcel.staroswiecki@univ-lille1.fr).

Digital Object Identifier 10.1109/TFUZZ.2017.2036005

attracted considerable attention and given rise to many important results [3]–[8]. However, most address stability analysis and feedback stabilization, and very few address the issue of fault estimation and FTC.

Based on passive FTC idea in [9] and [10], a reliable fault-tolerant controller using T–S fuzzy models was developed against actuator faults, while issues of fault detection and estimation were not involved. Robust fault detection for T–S fuzzy systems was studied in [11]–[13], but the issue of fault estimation was not included. The problem of robust fault estimation for time-delay T–S fuzzy models was dealt with in [14], but under a restrictive assumption on the faults, i.e., $f(t) \in L_2[0, \infty)$. A sliding-mode observer (SMO) and an adaptive observer (AO) were proposed to achieve fault estimation in [6] and [15], but their design needed very restrictive conditions to be satisfied.

Based on the aforementioned works, this paper further investigates the issue of robust fault estimation and FTC for T–S fuzzy systems. Restrictive constraints are relaxed through a general observer-based dynamic output feedback-fault tolerant controller (DOFFTC) design for a class of T–S fuzzy systems under actuator faults. First, a multiobjective fuzzy augmented fault-estimation observer (AFEO), including a regional pole placement and a H_∞ performance level, is proposed, not only to guarantee the convergence speed of fault estimation but to restrict the influence of disturbances as much as possible as well. Then, using the online fault estimate, a fuzzy DOFFTC is designed to guarantee the system stability in the presence of actuator faults. In the design process, the AFEO and the DOFFTC are independently designed, and their performances are considered simultaneously, which is convenient for calculating the design parameters and can avoid the coupling problem generated by the observer-based state feedback control.

II. System Description

The T–S fuzzy model is described by fuzzy IF–THEN rules, whose collection represent the approximation of the nonlinear system. The ith rule of the T–S fuzzy model is of the following form.

Plant Rule i:

IF $z_1(t)$ is μ_{i1} and, ..., and $z_s(t)$ is μ_{is}, THEN

$$\dot{x}(t) = A_i x(t) + B_i\big(u(t) + f(t)\big) + D_{1i}\omega(t) \quad (1)$$

$$y(t) = C_i x(t) + D_{2i}\omega(t) \quad (2)$$

where $x(t) \in R^n$ is the state, $u(t) \in R^m$ is the input, $y(t) \in R^p$ is the output, $f(t) \in R^m$ represents the additive actuator fault, and $\omega(t) \in R^d$ is the disturbance, which is assumed to belong to $L_2[0, \infty)$. A_i, B_i, C_i, D_{1i}, and D_{2i} are constant real matrices of appropriate dimensions. It is supposed that matrices B_i are of full column rank, i.e., rank$(B_i) = m$, the pairs (A_i, B_i) are controllable, and the pairs (A_i, C_i) are observable. $z_j(t) (j = 1, \ldots, s)$ are the premise variables, $\mu_{ij} (i = 1, \ldots, q; j = 1, \ldots, s)$ are the fuzzy sets that are characterized by membership function, q is the number of IF–THEN rules, and s is the number of the premise variables. The overall fuzzy model achieved by fuzzy-blending of each individual plant rule (local model) is given by

$$\dot{x}(t) = \sum_{i=1}^{q} h_i(z(t))[A_i x(t) + B_i\big(u(t) + f(t)\big) + D_{1i}\omega(t)] \quad (3)$$

$$y(t) = \sum_{i=1}^{q} h_i(z(t))[C_i x(t) + D_{2i}\omega(t)] \quad (4)$$

where $z(t) = [z_1(t), \ldots, z_s(t)]$

$$h_i(z(t)) = \frac{\sigma_i(z(t))}{\sum_{i=1}^{q} \sigma_i(z(t))}, \quad \sigma_i(z(t)) = \prod_{j=1}^{s} \mu_{ij}(z_j(t))$$

and $\mu_{ij}(\cdot)$ is the grade of the membership function of μ_{ij}. We assume

$$\sigma_i(z(t)) \geq 0, \quad i = 1, \ldots, q, \quad \sum_{i=1}^{q} \sigma_i(z(t)) > 0 \quad (5)$$

for any $z(t)$. Hence, $h_i(z(t))$ satisfies

$$h_i(z(t)) \geq 0, \quad i = 1, \ldots, q, \quad \sum_{i=1}^{q} h_i(z(t)) = 1 \quad (6)$$

for any $z(t)$.

For simplicity, we introduce the following notations:

$$h_i = h_i(z(t)),\ A(h) = \sum_{i=1}^{q} h_i A_i,\ B(h) = \sum_{i=1}^{q} h_i B_i$$

$$C(h) = \sum_{i=1}^{q} h_i C_i,\ D_1(h) = \sum_{i=1}^{q} h_i D_{1i},\ \text{and}\ D_2(h) = \sum_{i=1}^{q} h_i D_{2i}.$$

Then, the T–S fuzzy model (3) and (4) can be rewritten as

$$\dot{x}(t) = A(h)x(t) + B(h)\big(u(t) + f(t)\big) + D_1(h)\omega(t) \quad (7)$$

$$y(t) = C(h)x(t) + D_2(h)\omega(t). \quad (8)$$

Before ending this section, a lemma related to quadratic d-stabilizability is presented [16].

Lemma 1: For a given matrix $\mathcal{A} \in R^{n\times n}$, the eigenvalues of $\mathcal{A}$ belong to the circular region $D(\alpha, r)$ with center $\alpha + j0$ and radius r, if and only if there exists a symmetric positive definite matrix $\mathcal{P} \in R^{n\times n}$ such that

$$\begin{bmatrix} -\mathcal{P} & \mathcal{P}(\mathcal{A} - \alpha I_n) \\ * & -r^2\mathcal{P} \end{bmatrix} < 0 \quad (9)$$

where here and everywhere in the sequel, $*$ denotes the symmetric elements in a symmetric matrix.

III. Main Results

A. Fuzzy AFEO Design

Now, we are ready to express our main results. In order to detect and estimate faults, the following fault estimation observer is constructed:

$$\dot{\hat{x}}(t) = A(h)\hat{x}(t) + B(h)\big(u(t) + \hat{f}(t)\big) - L(h)(\hat{y}(t) - y(t)) \quad (10)$$

$$\hat{y}(t) = C(h)\hat{x}(t) \quad (11)$$

$$\dot{\hat{f}}(t) = -F(h)(\hat{y}(t) - y(t)) \quad (12)$$

where $\hat{x}(t) \in R^n$ is the observer state, $\hat{y}(t) \in R^p$ is the observer output, and $\hat{f}(t) \in R^r$ is an estimate of the fault $f(t)$. $L(h) \in R^{n\times p}$ and $F(h) \in R^{r\times p}$ are the gain matrices to be designed $L(h) = \sum_{i=1}^{q} h_i L_i$, $F(h) = \sum_{i=1}^{q} h_i F_i$.

Denote $e_x(t) = \hat{x}(t) - x(t)$, $e_y(t) = \hat{y}(t) - y(t)$, and $e_f(t) = \hat{f}(t) - f(t)$; then, the error dynamics is given by

$$\dot{\bar{e}}(t) = (\bar{A}(h) - \bar{L}(h)\bar{C}(h))\bar{e}(t) + (\bar{L}(h)\bar{D}_2(h) - \bar{D}_1(h))\nu(t) \quad (13)$$

$$e_y(t) = \bar{C}(h)\bar{e}(t) - \bar{D}_2(h)\nu(t) \quad (14)$$

where

$$\bar{e}(t) = \begin{bmatrix} e_x(t) \\ e_f(t) \end{bmatrix}, \quad \nu(t) = \begin{bmatrix} \omega(t) \\ \dot{f}(t) \end{bmatrix}$$

$$\bar{A}(h) = \begin{bmatrix} A(h) & B(h) \\ 0 & 0 \end{bmatrix}, \quad \bar{L}(h) = \begin{bmatrix} L(h) \\ F(h) \end{bmatrix}$$

$$\bar{C}(h) = [\, C(h) \quad 0\,], \quad \bar{D}_1(h) = \begin{bmatrix} D_1(h) & 0 \\ 0 & I_m \end{bmatrix}$$

$$\bar{D}_2(h) = [\, D_2(h) \quad 0\,].$$

Assumption 1: $\dot{f}(t)$ belongs to $L_2[0, \infty)$.

Remark 1: In [14] and [17], the fault estimation filter is designed under the assumption $f(t) \in L_2[0, \infty)$. In general, SMO-based fault estimation requires the preliminary knowledge of the upper bound of $f(t)$ [6], [18]. However, in many practical systems, there is a transient period during which the fault establishes itself, after which, it remains more or less constant, meaning that the derivatives of the faults are energy-bounded, i.e., $\dot{f}(t) \in L_2[0, \infty)$. This is stated by Assumption 1, which is more general than those used in the aforementioned design methods.

Remark 2: From the augmented systems (13) and (14), it can be seen that $\bar{A}(h)$, $\bar{C}(h)$, $\bar{D}_1(h)$, and $\bar{D}_2(h)$ are known matrices, while $\bar{L}(h)$ contains the two gains $L(h)$ and $F(h)$ to be designed. Therefore, we can see that a necessary condition for the existence of fuzzy AFEO is that the pairs $(\bar{A}_i, \bar{C}_i)$ are observable, and globally robust stability of fuzzy AFEO can be guaranteed by the following Theorem 1.

Next, a multiobjective AFEO design method under regional pole placement and H_∞ performance specifications is proposed to achieve robust fault estimation.

Theorem 1: For two given positive scalars α and r, the eigenvalues of $(\bar{A}(h) - \bar{L}(h)\bar{C}(h))$ belong to $D(\alpha, r)$, and the error dynamics (13) satisfy the H_∞ performance index $\|e_f(t)\|_2 < \gamma\|\nu(t)\|_2$, if there exists a symmetric positive definite matrix $\bar{P} \in R^{(n+m)\times(n+m)}$ and matrices $\bar{Y}_i \in R^{(n+m)\times p}$ such that

$$\min \gamma \text{ subject to}$$

$$\Psi_{ii} < 0, \qquad i = 1, \ldots, q \tag{15}$$

$$\Psi_{ij} + \Psi_{ji} < 0, \qquad 1 \le i < j \le q \tag{16}$$

$$\Phi_{ii} < 0, \qquad i = 1, \ldots, q \tag{17}$$

$$\Phi_{ij} + \Phi_{ji} < 0, \qquad 1 \le i < j \le q \tag{18}$$

where

$$\Psi_{ij} = \begin{bmatrix} -\bar{P} & \bar{P}\bar{A}_i - \bar{Y}_i\bar{C}_j - \alpha\bar{P} \\ * & -r^2\bar{P} \end{bmatrix}, \quad \bar{I}_m = \begin{bmatrix} 0 \\ I_m \end{bmatrix}$$

$$\Phi_{ij} = \begin{bmatrix} \phi_{11} & \bar{Y}_i\bar{D}_{2j} - \bar{P}\bar{D}_{1i} & \bar{I}_m \\ * & -\gamma I_{d+m} & 0 \\ * & * & -\gamma I_m \end{bmatrix}, \quad Y_i = PL_i$$

$$\phi_{11} = \bar{P}\bar{A}_i + \bar{A}_i^T\bar{P} - \bar{Y}_i\bar{C}_j - \bar{C}_j^T\bar{Y}_i^T.$$

Proof: Constraints (15) and (16): Setting $(\bar{A}(h) - \bar{L}(h)\bar{C}(h)) \to \mathcal{A}$ and $\bar{P} \to \mathcal{P}$ in Lemma 1, one gets

$$\Psi := \begin{bmatrix} -\bar{P} & \bar{P}\left(\bar{A}(h) - \bar{L}(h)\bar{C}(h)\right) - \alpha\bar{P} \\ * & -r^2\bar{P} \end{bmatrix} < 0 \tag{19}$$

which can be rewritten as

$$\Psi = \sum_{i=1}^{q} h_i^2\Psi_{ii} + \sum_{i=1}^{q}\sum_{i<j}^{q} h_ih_j\left(\Psi_{ij} + \Psi_{ji}\right) < 0. \tag{20}$$

Therefore, if (15) and (16) hold, then the eigenvalues of $(\bar{A}(h) - \bar{L}(h)\bar{C}(h))$ belong to $D(\alpha, r)$.

For constraints (17) and (18), consider the following Lyapunov function:

$$V(t) = \bar{e}^T(t)\bar{P}\bar{e}(t). \tag{21}$$

Its derivative with respect to time is

$$\begin{aligned}\dot{V}(t) = \bar{e}^T(t)\Big(&\bar{P}\big(\bar{A}(h) - \bar{L}(h)\bar{C}(h)\big) \\ &+ \big(\bar{A}(h) - \bar{L}(h)\bar{C}(h)\big)^T\bar{P}\Big)\bar{e}(t) \\ &+ 2\bar{e}^T(t)\bar{P}\Big(\bar{L}(h)\bar{D}_2(h) - \bar{D}_1(h)\Big)\nu(t).\end{aligned} \tag{22}$$

Now, let us define

$$\begin{aligned}J &= \int_{t_f}^{\infty}\left[\frac{1}{\gamma}e_f^T(t)e_f(t) - \gamma\nu^T(t)\nu(t)\right]dt \\ &= \int_{t_f}^{\infty}\left[\frac{1}{\gamma}\bar{e}^T(t)\bar{I}_m\bar{I}_m^T\bar{e}(t) - \gamma\nu^T(t)\nu(t)\right]dt\end{aligned} \tag{23}$$

where t_f denotes the fault occurrence time. Then, under zero initial condition at time t_f, it can be shown that

$$J \le \int_{t_f}^{\infty}\left[\dot{V}(t) + \frac{1}{\gamma}\bar{e}^T(t)\bar{I}_m\bar{I}_m^T\bar{e}(t) - \gamma\nu^T(t)\nu(t)\right]dt. \tag{24}$$

Substituting (22) into (24), one gets

$$\begin{aligned}&\dot{V}(t) + \frac{1}{\gamma}\bar{e}^T(t)\bar{I}_m\bar{I}_m^T\bar{e}(t) - \gamma\nu^T(t)\nu(t) \\ &= \bar{e}^T(t)(\bar{P}(\bar{A}(h) - \bar{L}(h)\bar{C}(h)) \\ &\quad + (\bar{A}(h) - \bar{L}(h)\bar{C}(h))^T\bar{P})\bar{e}(t) \\ &\quad + 2\bar{e}^T(t)\bar{P}(\bar{L}(h)\bar{D}_2(h) - \bar{D}_1(h))\nu(t) \\ &\quad + \frac{1}{\gamma}\bar{e}^T(t)\bar{I}_m\bar{I}_m^T\bar{e}(t) - \gamma\nu^T(t)\nu(t) \\ &= \zeta^T(t)\Omega\zeta(t) \\ &= \sum_{i=1}^{q}\sum_{j=1}^{q} h_ih_j\zeta^T(t)\Omega_{ij}\zeta(t) \\ &= \sum_{i=1}^{q} h_i^2\zeta^T(t)\Omega_{ii}\zeta(t) + \sum_{i=1}^{q}\sum_{i<j}^{q} h_ih_j\zeta^T(t)(\Omega_{ij} + \Omega_{ji})\zeta(t)\end{aligned} \tag{25}$$

where

$$\zeta(t) = \begin{bmatrix} \bar{e}(t) \\ \nu(t) \end{bmatrix}, \; \Omega = \begin{bmatrix} \eta_{11} & \bar{P}(\bar{L}(h)\bar{D}_2(h) - \bar{D}_1(h)) \\ * & -\gamma I_{d+m} \end{bmatrix}$$

$$\Omega_{ij} = \begin{bmatrix} \phi_{11} + \frac{1}{\gamma}\bar{I}_m\bar{I}_m^T & \bar{Y}_i\bar{D}_{2j} - \bar{P}\bar{D}_{1i} \\ * & -\gamma I_{d+m} \end{bmatrix}$$

$$\begin{aligned}\eta_{11} &= \bar{P}\big(\bar{A}(h) - \bar{L}(h)\bar{C}(h)\big) \\ &\quad + \big(\bar{A}(h) - \bar{L}(h)\bar{C}(h)\big)^T\bar{P} + \frac{1}{\gamma}(\bar{I}_m\bar{I}_m^T).\end{aligned}$$

By the Schur complement, (17) and (18) are equivalent to $\Omega_{ii} < 0$ and $\Omega_{ij} + \Omega_{ji} < 0$. It follows that the error dynamics (13) are robustly stable with a H_∞ performance index $\|e_f(t)\|_2 < \gamma\|\nu(t)\|_2$, provided (17) and (18) hold true. □

Remark 3: In Theorem 1, the purpose of introducing the disk constraints (15) and (16) is to improve the transient performance of fault estimation. Note that, other pole-placement constraints, such as α-stability, vertical strips, sectors, and the intersection thereof can also be considered [19], [20].

Remark 4: Achieving the estimation of actuator faults by the SMO-based and AO-based approaches proposed in [6] and [15] would require that the constraints $\text{rank}(C_iB_i) = \text{rank}(B_i)$ and $C_i(sI - A_i)^{-1}B_i$ be minimum phase to be satisfied. Details on the conservativeness of these constraints can be found in [21] and [18]. It is seen that no equality constraint is used in the design process of the proposed fuzzy AFEO, which resumes in finding matrices $\bar{L}(h)$ such that (13) is robustly stable.

Remark 5: Note that a fault-estimation filter followed from the design idea in [14] and [17] is constructed as

$$\dot{x}_f(t) = A_f(h)x_f(t) + B_f(h)y(t) \tag{26}$$

$$\hat{f}(t) = C_f(h)x_f(t) + D_f(h)y(t) \tag{27}$$

where $x_f(t) \in R^{n_f}$ is the filter state, and $\hat{f}(t) \in R^m$ is an estimate of the fault $f(t)$. $A_f(h)$, $B_f(h)$, $C_f(h)$, and $D_f(h)$ are the filter gain matrices of appropriate dimensions to be designed. Then, substituting (7) and (8) into (26) and (27), we obtain

$$\dot{\bar{x}}_f(t) = \bar{A}_f(h)\bar{x}_f(t) + \bar{B}_f(h)\rho(t) \tag{28}$$

$$e_f(t) = \bar{C}_f(h)\bar{x}_f(t) + \bar{D}_f(h)\rho(t) \tag{29}$$

where

$$\bar{x}_f(t) = \begin{bmatrix} x(t) \\ x_f(t) \end{bmatrix}, \quad \rho(t) = \begin{bmatrix} u(t) \\ f(t) \\ \omega(t) \end{bmatrix}$$

$$\bar{A}_f(h) = \begin{bmatrix} A(h) & 0 \\ B_f(h)C(h) & A_f(h) \end{bmatrix}$$

$$\bar{B}_f(h) = \begin{bmatrix} B(h) & B(h) & D_1(h) \\ 0 & 0 & B_f(h)D_2(h) \end{bmatrix}$$

$$\bar{C}_f(h) = [\, D_f(h)C(h) \quad C_f(h) \,]$$

$$\bar{D}_f(h) = [\, 0 \quad -I_m \quad D_f(h)D_2(h) \,].$$

Under the H_∞ performance $\|e_f(t)\|_2 < \gamma\|\rho(t)\|_2$, the fault-estimation filter design requires the assumption $f(t) \in L_2[0, \infty)$. From (28) to (29), it is shown that, since the occurred fault is viewed is as a "disturbance," it is impossible to realize asymptotical estimation of the constant fault (except for $f(t) = 0$ that is of no interest). Meanwhile, from system matrices $\bar{A}_f(h)$, it can be seen that the filter design is only suitable for open-loop stable systems. However, in practical situations, most systems are open-loop unstable, and therefore, such a constraint limits its application scopes.

B. *Fuzzy DOFFTC Design*

On the basis of the obtained online fault-estimation information, we design a fault-tolerant controller to guarantee stability in the presence of faults. Since the state $x(t)$ is unmeasurable, we use the fuzzy dynamical output feedback-controller scheme [8], [22] to construct the DOFFTC for T–S fuzzy models as

$$\dot{\xi}(t) = A_K(h,h)\xi(t) + B_K(h)y(t) \tag{30}$$

$$u(t) = C_K(h)\xi(t) + D_K y(t) - \hat{f}(t) \tag{31}$$

where $\xi(t)$ is the state, $A_K(h,h) \in R^{n\times n}$, $B_K(h) \in R^{n\times p}$, $C_K(h) \in R^{m\times n}$, and $D_K \in R^{m\times p}$ are the designed DOFFTC matrices, and $A_K(h,h) = \sum_{i=1}^q \sum_{j=1}^q h_i h_j A_{Kij}$, $B_K(h) = \sum_{i=1}^q h_i B_{Ki}$, $C_K(h) = \sum_{i=1}^q h_i C_{Ki}$.

Substituting (8) into (30) and (31), one obtains

$$\dot{\xi}(t) = A_K(h,h)\xi(t) + B_K(h)C(h)x(t) + B_K(h)D_2(h)\omega(t) \tag{32}$$

$$u(t) = C_K(h)\xi(t) + D_K C(h)x(t) + D_K D_2(h)\omega(t) - \hat{f}(t). \tag{33}$$

Then, one gets

$$\dot{\tilde{x}}(t) = \tilde{A}(h,h)\tilde{x}(t) + \tilde{D}_1(h,h)\mu(t) \tag{34}$$

$$y(t) = \tilde{C}(h)\tilde{x}(t) + \tilde{D}_2(h)\mu(t). \tag{35}$$

where

$$\tilde{x}(t) = \begin{bmatrix} x(t) \\ \xi(t) \end{bmatrix}, \quad \mu(t) = \begin{bmatrix} \omega(t) \\ e_f(t) \end{bmatrix}$$

$$\tilde{A}(h,h) = \begin{bmatrix} A(h) + B(h)D_K C(h) & B(h)C_K(h) \\ B_K(h)C(h) & A_K(h,h) \end{bmatrix}$$

$$\tilde{D}_1(h,h) = \begin{bmatrix} D_1(h) + B(h)D_K D_2(h) & -B(h) \\ B_K(h)D_2(h) & 0 \end{bmatrix}$$

$$\tilde{C}(h) = \bar{C}(h) = [\, C(h) \quad 0 \,]$$

$$\tilde{D}_2(h) = \bar{D}_2(h) = [\, D_2(h) \quad 0 \,].$$

Theorem 2: Assume a fault $f(t)$ occurs at some unknown time t_f, and let α and r be two positive scalars. The eigenvalues of $\tilde{A}(h,h)$ belong to $D(\alpha, r)$, and the system dynamics (34) and (35) satisfy the H_∞ performance index $\|y(t)\|_2 < \gamma\|\mu(t)\|_2$, if there exist two symmetric positive definite matrices $X, Y \in R^{n\times n}$ and matrices $\hat{A}_{ij} \in R^{n\times n}$, $\hat{B}_i \in R^{n\times p}$, $\hat{C}_i \in R^{m\times n}$, and $\hat{D} \in R^{m\times p}$ such that

$$\min \gamma \text{ subject to}$$

$$\Pi_{ii} < 0, \qquad i = 1, \ldots, q \tag{36}$$

$$\Pi_{ij} + \Pi_{ji} < 0, \qquad 1 \le i < j \le q \tag{37}$$

$$\Xi_{ii} < 0, \qquad i = 1, \ldots, q \tag{38}$$

$$\Xi_{ij} + \Xi_{ji} < 0, \qquad 1 \le i < j \le q \tag{39}$$

where

$$\Pi_{ij} = \begin{bmatrix} -X & -I_n & \pi_{13} & A_i + B_i\hat{D}C_j - \alpha I_n \\ * & -Y & \hat{A}_{ij} - \alpha I_n & YA_i + \hat{B}_jC_i - \alpha Y \\ * & * & -r^2X & -r^2I_n \\ * & * & * & -r^2Y \end{bmatrix}$$

and
$$\Xi_{ij} = \begin{bmatrix} \chi_{11} & \chi_{12} & D_{1i} + B_i\hat{D}D_{2j} & -B_i & XC_i^T \\ * & \chi_{22} & YD_{1i} + \hat{B}_jD_{2i} & -YB_i & C_i^T \\ * & * & -\gamma I_d & 0 & D_{2i}^T \\ * & * & * & -\gamma I_m & 0 \\ * & * & * & * & -\gamma I_p \end{bmatrix}$$

with

$$\pi_{13} = A_iX + B_i\hat{C}_j - \alpha X$$

$$\chi_{11} = A_iX + XA_i^T + B_i\hat{C}_j + \hat{C}_j^T B_i^T$$

$$\chi_{12} = \hat{A}_{ij}^T + A_i + B_i\hat{D}C_j$$
$$\text{and}\quad \chi_{22} = YA_i + A_i^TY^T + \hat{B}_jC_i + C_i^T\hat{B}_j^T.$$

The parameter matrices of the DOFFTC are given by

$$\begin{aligned}
D_K &= \hat{D}\\
C_{Ki} &= (\hat{C}_i - D_KC_iX)M^{-T}\\
B_{Ki} &= N^{-1}(\hat{B}_i - YB_iD_K)\\
A_{Kij} &= N^{-1}\big(\hat{A}_{ij} - Y(A_i + B_iD_KC_j)X\big)M^{-T}\\
&\quad - B_{Kj}C_iXM^{-T} - N^{-1}YB_iC_{Kj}
\end{aligned}$$

where $M, N \in R^{n\times n}$ satisfy $MN^T = I_n - XY$.

Proof: We start with the proof of (38) and (39), and (36) and (37) will be considered subsequently. For constraints (38) and (39), consider the following Lyapunov function:

$$V(t) = \tilde{x}^T(t)\tilde{P}\tilde{x}(t). \tag{40}$$

Its derivative with respect to time is

$$\begin{aligned}
\dot{V}(t) &= \tilde{x}^T(t)\Big(\tilde{P}\tilde{A}(h,h) + \tilde{A}^T(h,h)\tilde{P}\Big)\tilde{x}(t)\\
&\quad + 2\tilde{x}^T(t)\tilde{P}\tilde{D}_1(h,h)\mu(t).
\end{aligned} \tag{41}$$

Let us introduce

$$J = \int_{t_f}^{\infty}\left[\frac{1}{\gamma}y^T(t)y(t) - \gamma\mu^T(t)\mu(t)\right]dt. \tag{42}$$

It can be shown that

$$J \le \int_{t_f}^{\infty}\left[\dot{V}(t) + \frac{1}{\gamma}y^T(t)y(t) - \gamma\mu^T(t)\mu(t)\right]dt. \tag{43}$$

Substituting (41) into (43), one obtains

$$\begin{aligned}
&\dot{V}(t) + \frac{1}{\gamma}y^T(t)y(t) - \gamma\mu^T(t)\mu(t)\\
&\quad = \tilde{x}^T(t)(\tilde{P}\tilde{A}(h,h) + \tilde{A}^T(h,h)\tilde{P})\tilde{x}(t) + 2\tilde{x}^T(t)\tilde{P}\tilde{D}_1(h,h)\mu(t)\\
&\quad + \frac{1}{\gamma}\tilde{x}^T(t)\tilde{C}^T(h)\tilde{C}(h)\tilde{x}(t) + \frac{2}{\gamma}\tilde{x}^T(t)\tilde{C}^T(h)\tilde{D}_2(h)\mu(t)\\
&\quad + \frac{1}{\gamma}\mu^T(t)\tilde{D}_2^T(h)\tilde{D}_2(h)\mu(t) - \gamma\mu^T(t)\mu(t) = \varsigma^T(t)\Theta\varsigma(t)
\end{aligned} \tag{44}$$

where

$$\varsigma(\tau) = \begin{bmatrix}\tilde{x}(t)\\ \mu(t)\end{bmatrix} \text{ and } \Theta = \begin{bmatrix}\theta_{11} & \theta_{12}\\ * & \theta_{22}\end{bmatrix}$$

with

$$\begin{aligned}
\theta_{11} &= \tilde{P}\tilde{A}(h,h) + \tilde{A}^T(h,h)\tilde{P} + \frac{1}{\gamma}\tilde{C}^T(h)\tilde{C}(h)\\
\theta_{12} &= \tilde{P}\tilde{D}_1(h,h) + \frac{1}{\gamma}\tilde{C}^T(h)\tilde{D}_2(h)\\
\text{and}\quad \theta_{22} &= \frac{1}{\gamma}\tilde{D}_2^T(h)\tilde{D}_2(h) - \gamma I_{d+m}.
\end{aligned}$$

Using the Schur complement, $\Theta < 0$ is equivalent to

$$\begin{bmatrix}\tilde{P}\tilde{A}(h,h) + \tilde{A}^T(h,h)\tilde{P} & \tilde{P}\tilde{D}_1(h,h) & \tilde{C}^T(h)\\ * & -\gamma I_{d+m} & \tilde{D}_2^T(h)\\ * & * & -\gamma I_p\end{bmatrix} < 0. \tag{45}$$

Then, we express the symmetric positive definite matrix $\tilde{P}$ and its inverse matrix $\tilde{P}^{-1}$ as $\tilde{P} = [\begin{smallmatrix}Y & N\\ N^T & W\end{smallmatrix}]$ and $\tilde{P}^{-1} = [\begin{smallmatrix}X & M\\ M^T & Z\end{smallmatrix}]$. Due to $\tilde{P}\tilde{P}^{-1} = I_{2n}$, one gets $\tilde{P}[\begin{smallmatrix}X\\ M^T\end{smallmatrix}] = [\begin{smallmatrix}I_n\\ 0_n\end{smallmatrix}]$ and $\tilde{P}[\begin{smallmatrix}X & I_n\\ M^T & 0_n\end{smallmatrix}] = [\begin{smallmatrix}I_n & Y\\ 0_n & N^T\end{smallmatrix}]$, respectively.

Denote $F_1 = [\begin{smallmatrix}X & I_n\\ M^T & 0_n\end{smallmatrix}]$ and $F_2 = [\begin{smallmatrix}I_n & Y\\ 0_n & N^T\end{smallmatrix}]$; then, it follows that $\tilde{P}F_1 = F_2$. Premultiplying and postmultiplying by $\text{diag}(F_1^T, I_{d+m}, I_p)$, and its transpose in (45), and denoting

$$\begin{aligned}
\hat{A}(h,h) &= Y(A(h) + B(h)D_KC(h))X + NB_K(h)C(h)X\\
&\quad + YB(h)C_K(h)M^T + NA_K(h,h)M^T\\
\hat{B}(h) &= YB(h)D_K + NB_K(h)\\
\hat{C}(h) &= D_KC(h)X + C_K(h)M^T, \text{ and } \hat{D} = D_K.
\end{aligned}$$

Then, (45) can be expressed as

$$\Xi := \begin{bmatrix}\varphi_{11} & \varphi_{12} & \varphi_{13} & -B(h) & XC^T(h)\\ * & \varphi_{22} & \varphi_{23} & -YB(h) & C^T(h)\\ * & * & -\gamma I_d & 0 & D_2^T(h)\\ * & * & * & -\gamma I_m & 0\\ * & * & * & * & -\gamma I_p\end{bmatrix} < 0 \tag{46}$$

where

$$\begin{aligned}
\varphi_{11} &= A(h)X + XA^T(h) + B(h)\hat{C}(h) + \hat{C}^T(h)B^T(h)\\
\varphi_{12} &= \hat{A}^T(h,h) + A(h) + B(h)\hat{D}C(h)\\
\varphi_{13} &= D_1(h) + B(h)\hat{D}D_2(h)\\
\varphi_{22} &= YA(h) + A^T(h)Y^T + \hat{B}(h)C(h) + C^T(h)\hat{B}^T(h)\\
\varphi_{23} &= YD_1(h) + \hat{B}(h)D_2(h).
\end{aligned}$$

Therefore, $\Xi < 0$ can be rewritten as

$$\begin{aligned}
\Xi &= \sum_{i=1}^{q}\sum_{j=1}^{q}h_ih_j\Xi_{ij}\\
&= \sum_{i=1}^{q}h_i^2\Xi_{ii} + \sum_{i=1}^{q}\sum_{i<j}^{q}h_ih_j(\Xi_{ij} + \Xi_{ji}) < 0.
\end{aligned} \tag{47}$$

Therefore, if (38) and (39) hold, the system dynamics (34) and (35) are robustly stable with a H_∞ performance index $\|y(t)\|_2 < \gamma\|\mu(t)\|_2$.

Constraints (36) and (37): Setting $\tilde{A}(h,h) \to \mathcal{A}$ and $\tilde{P} \to \mathcal{P}$ in Lemma 1, one has

$$\begin{bmatrix}-\tilde{P} & \tilde{P}\tilde{A}(h,h) - \alpha\tilde{P}\\ * & -r^2\tilde{P}\end{bmatrix} < 0. \tag{48}$$

Premultiplying and postmultiplying by $\text{diag}(F_1^T, F_1^T)$ and its transpose in (48) and then using the definition $\hat{A}(h,h)$, $\hat{B}(h)$, $\hat{C}(h)$, and $\hat{D}$, one gets

$$\Pi := \begin{bmatrix}-X & -I_n & \varpi_{13} & \varpi_{14}\\ * & -Y & \hat{A}(h,h) - \alpha I_n & \varpi_{24}\\ * & * & -r^2X & -r^2I_n\\ * & * & * & -r^2Y\end{bmatrix} < 0 \tag{49}$$

where

$$\begin{aligned}
\varpi_{13} &= A(h)X + B(h)\hat{C}(h) - \alpha X\\
\varpi_{14} &= A(h) + B(h)D_KC(h) - \alpha I_n\\
\varpi_{24} &= YA(h) + \hat{B}(h)C(h) - \alpha Y.
\end{aligned}$$

This can be rewritten as

$$\begin{aligned}\Pi &= \sum_{i=1}^{q}\sum_{j=1}^{q} h_i h_j \Pi_{ij} \\ &= \sum_{i=1}^{q} h_i^2 \Pi_{ii} + \sum_{i=1}^{q}\sum_{i<j}^{q} h_i h_j \left(\Pi_{ij} + \Pi_{ji}\right) < 0. \end{aligned} \tag{50}$$

Therefore, if (36) and (37) hold, then the eigenvalues of $\tilde{A}(h,h)$ belong to $D(\alpha, r)$. □

Remark 6: From the top left subblock $\left[\begin{smallmatrix} -X & -I_n \\ * & -Y \end{smallmatrix}\right]$ in (36), we can obtain $Y > 0$ and $X - Y^{-1} > 0$, which imply that $I_n - XY$ is nonsingular. Therefore, we can always find nonsingular matrices M and N satisfying $MN^T = I_n - XY$, and they can be calculated by the `qr` function of MATLAB toolbox.

Remark 7: In the whole design process, the AFEO and DOFFTC are designed separately, and their performances are considered simultaneously, which can avoid design difficulties caused by the coupling between the fault-diagnosis observer and the observer-based state feedback fault-tolerant controller. It is the coupling that makes the separate principle that does not hold [15], [23], [24].

Remark 8: It is noted that more fuzzy rules can approximate nonlinear systems more accurately. However, as the number of fuzzy rules increases, computation burden and design conservatism are unavoidable such that there might be no solution. Therefore, the two conditions are partially conflicting with each other, and a tradeoff between the number of fuzzy rules and design conservatism should be made.

IV. Simulation Results

Consider the problem of balancing and swing-up of an inverted pendulum on a cart [7], [22]. The equations of the pendulum motion are given by

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= \frac{\begin{bmatrix} g\sin(x_1(t)) - amlx_2^2(t)\sin(2x_1(t))/2- \\ a\cos(x_1(t))u(t) \end{bmatrix}}{4l/3 - aml\cos^2(x_1(t))} \\ y(t) &= x_1(t) \end{aligned}$$

where $x_1(t)$ is the angle (rad) of the pendulum from the vertical, and $x_2(t)$ is the angular velocity (rad/s). Note that, we assume only the first state to be measured, rather than all states [7], to achieve the actuator faults estimation, thus setting a more restrictive problem; $g = 9.8$ m/s is the gravity constant, m is the pendulum mass, M is the cart mass, $2l$ is the pendulum length, and $a = 1/(m + M)$. In all simulations, $m = 2.0$ kg, $M = 2.0$ kg, and $2l = 1.0$ m.

First, we represent the system by using a two-rule T–S fuzzy model.

Rule 1: IF $x_1(t)$ is about 0, THEN

$$\dot{x}(t) = A_1 x(t) + B_1 u(t), \quad y(t) = C_1 x(t).$$

Rule 2: IF $x_1(t)$ is about $\pm\frac{\pi}{2}$ $(|x_1(t)| < \frac{\pi}{2})$, THEN

$$\dot{x}(t) = A_2 x(t) + B_2 u(t), \quad y(t) = C_2 x(t).$$

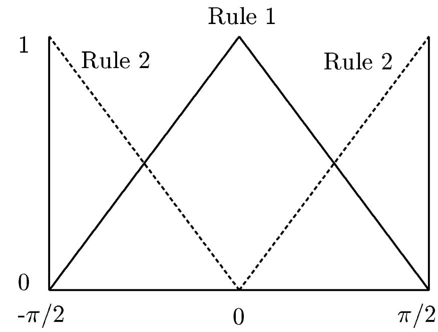

Fig. 1. Membership functions of the two-rule model.

where

$$A_1 = \begin{bmatrix} 0 & 1 \\ \dfrac{g}{(4l/3 - aml)} & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ -\dfrac{a}{(4l/3 - aml)} \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 \\ \dfrac{2g}{\pi(4l/3 - aml\beta^2)} & 0 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0 \\ -\dfrac{a\beta}{(4l/3 - aml\beta^2)} \end{bmatrix}$$

$$C_1 = [1 \quad 0], \quad C_2 = [1 \quad 0]$$

and $\beta = \cos(88^\circ)$ (notice that when $x_1(t) = \pm\pi/2$, the system is uncontrollable). Membership functions for Rules 1 and 2 are shown in Fig. 1. Meanwhile, disturbance distribution matrices are, respectively, chosen as $D_{11} = D_{12} = [0.01 \quad 0.01]^T$, and $D_{21} = D_{22} = [0.001 \quad 0.001]^T$. It is easy to verify that the pairs (A_i, B_i) are controllable and that the pairs (A_i, C_i) are observable.

It can be easily shown that $\text{rank}(C_i B_i) = 0$ and (A_i, B_i, C_i) do not possess any zeros; therefore, the SMO-based and AO-based methods cannot be used for this system to achieve fault estimation [6], [15]. Meanwhile, from system matrices A_i, it is shown that the system is open-loop unstable, and therefore, the fault-estimation filter design is also not suitable for such system [14], [17].

We first design the fuzzy AFEO. By solving the conditions in Theorem 1 with the circle region $D(30, 30)$ based on the `mincx` function of MATLAB toolbox, one obtains the minimum attenuation value $\gamma = 3.6104$ with

$$L_1 = \begin{bmatrix} 61.3775 \\ 911.1319 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 52.8509 \\ 611.4022 \end{bmatrix}$$

$$F_1 = -40383.7598, \text{ and } \quad F_2 = -32963.7904.$$

Next, we design the fuzzy DOFFTC. By solving the conditions in Theorem 2 with the circle region $D(30, 30)$, one obtains the minimum attenuation value $\gamma = 0.0388$ with

$$A_{K11} = \begin{bmatrix} -92.0199 & 0.2014 \\ -5919.5413 & 16.3694 \end{bmatrix}$$

$$A_{K12} = \begin{bmatrix} -22.8679 & -1.1238 \\ 1500.3734 & -152.4579 \end{bmatrix}$$

$$A_{K21} = \begin{bmatrix} -90.1229 & 0.6926 \\ -5694.2247 & 75.2751 \end{bmatrix}$$

$$A_{K22} = \begin{bmatrix} -18.7826 & -0.0260 \\ 1993.0790 & -18.9805 \end{bmatrix}$$

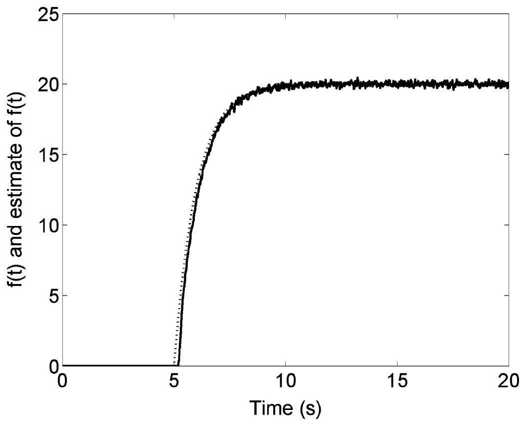

Fig. 2. Constant fault $f(t)$ (dotted) and its estimate $\hat{f}(t)$ (solid).

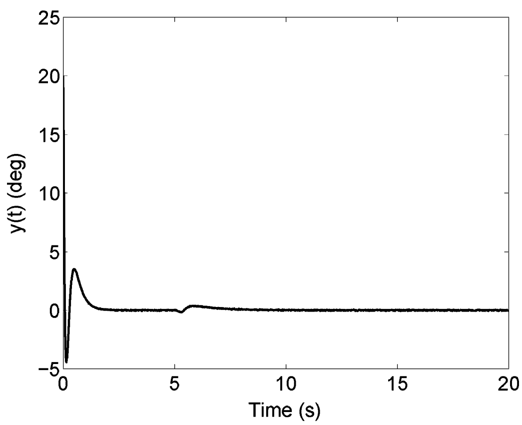

Fig. 3. For constant fault, output response under the fuzzy DOFFTC.

$$B_{K1} = \begin{bmatrix} -97.6313 \\ -6546.8816 \end{bmatrix}, \quad B_{K2} = \begin{bmatrix} -16.8077 \\ 2227.0462 \end{bmatrix}$$
$$C_{K1} = [\,7986.5346 \quad -24.4210\,]$$
$$C_{K2} = [\,8245.9040 \quad 47.1746\,], \quad D_K = 8856.8760.$$

First, it is assumed that a constant fault $f(t)$ is created as

$$f(t) = \begin{cases} 0, & 0 \le t < 5 \\ 20(1 - e^{-(t-5)}), & 5 \le t \le 20. \end{cases}$$

It is supposed that $\omega(t)$ are band-limited white noise with power 0.001 and sampling time 0.01 s. Under initial value $(20\pi/180,\ 0)$, simulation results are displayed as follows. In the following simulation results, we apply the proposed design method to the original nonlinear system rather than T–S fuzzy models, whose purpose is to verify the robustness of the proposed method with respect to modeling errors. Fig. 2 illustrates fault-estimation simulation results. Simulation results for the system output response are shown in Fig. 3 (a 0.2-s detection delay is considered).

Then, a time-varying fault is simulated as

$$f(t) = \begin{cases} 0, & 0 \le t < 5 \\ 10\sin(t-5), & 5 \le t \le 20. \end{cases}$$

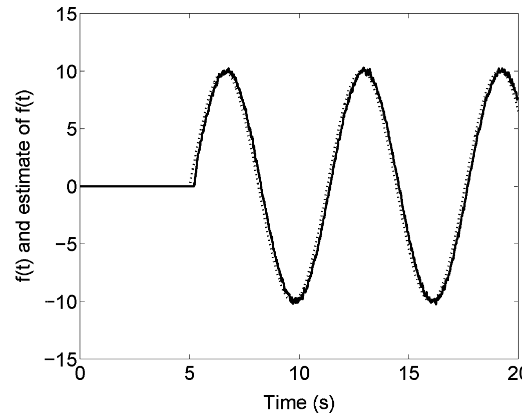

Fig. 4. (Dotted) Time-varying fault $f(t)$ and (solid) its estimate $\hat{f}(t)$.

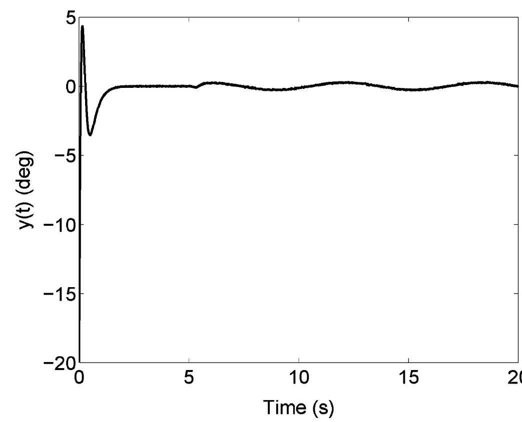

Fig. 5. For time-varying fault, output response under the fuzzy DOFFTC.

Under initial value $(-20\pi/180,\ 0)$, Fig. 4 illustrates the simulation result of fault estimation, while the system output response is shown in Fig. 5.

From the above simulation results, we can see that, despite the fact that $\mathrm{rank}(C_i B_i) = \mathrm{rank}(B_i)$ is not satisfied, and the open-loop system is unstable, the proposed design still achieves the performance under actuator faults, and the stability of the closed-loop system is guaranteed by the fuzzy DOFFTC. Note that, the proposed fuzzy AFEO design can achieve asymptotical estimation for constant fault, while for the time-varying fault, the fuzzy AFEO can almost realize accurate fault estimation.

V. Conclusion

In this paper, a detailed design framework for observer-based robust fault estimation and FTC is developed for a class of nonlinear systems described by a T–S fuzzy model. The framework includes a fuzzy AFEO and DOFFTC to guarantee given stability requirements, while limiting the influence of disturbances, in the presence of actuator faults. Simulation results of an inverted pendulum example are used to show the effectiveness of the obtained results. This paper focuses on fault-tolerant control design for T–S fuzzy systems with external disturbances, while approximation error of T–S fuzzy systems does exist [25]. Therefore, fault-tolerant control design for T–S fuzzy systems with approximation error and how to apply to practical nonlinear systems are meaningful and challenging issues, which will be studied in our future work.

References

[1] J. Chen and R. J. Patton, *Robust Model-Based Fault Diagnosis for Dynamic Systems*. Boston, MA: Kluwer, 1999.
[2] M. Blanke, M. Kinnaert, J. Lunze, and M. Staroswiecki, *Diagnosis and Fault-Tolerant Control*, 2nd ed. Berlin/Heidelberg, Germany: Springer-Verlag, 2016.
[3] X. J. Ma, Z. Q. Sun, and Y. Y. He, "Analysis and design of fuzzy controller and fuzzy observer," *IEEE Trans. Fuzzy Syst.*, vol. 6, no. 1, pp. 41–51, Feb. 1998.
[4] Y. Y. Cao and P. M. Frank, "Stability analysis and synthesis of nonlinear time-delay systems via linear Takagi-Sugeno fuzzy models," *Fuzzy Sets Syst.*, vol. 124, no. 2, pp. 213–229, 2001.
[5] S. K. Nguang and P. Shi, "H_∞ fuzzy output feedback control design for nonlinear systems: An LMI approach," *IEEE Trans. Fuzzy Syst.*, vol. 11, no. 3, pp. 331–340, Jun. 2013.
[6] H. H. Choi, "LMI-based nonlinear fuzzy observer-controller design for uncertain MIMO nonlinear systems," *IEEE Trans. Fuzzy Syst.*, vol. 15, no. 5, pp. 956–971, Oct. 2007.
[7] H. Gao and T. Chen, "Stabilization of nonlinear systems under variable sampling: A fuzzy control approach," *IEEE Trans. Fuzzy Syst.*, vol. 15, no. 5, pp. 972–983, Oct. 2007.
[8] J. Dong and G.-H. Yang, "Dynamic output feedback H_∞ control synthesis for discrete-time T–S fuzzy systems via switching fuzzy controllers," *Fuzzy Sets Syst.*, vol. 160, no. 4, pp. 482–499, 2009.
[9] T. Wang, S. Tong, and S. Tong, "Robust fault tolerant fuzzy control for nonlinear systems with actuator failures," in *Proc. 2nd Int. Conf. Innovative Comput., Inf. Control*, Kumamoto, Japan, 2007, pp. 44–44.
[10] S. Tong, T. Wang, and W. Zhang, "Fault tolerant control for uncertain fuzzy systems with actuator failures," *Int. J. Innovative Comput., Inf. Control*, vol. 4, no. 10, pp. 2461–2474, 2008.
[11] R. J. Patton, J. Chen, and C. J. Lopez-Toribio, "Fuzzy observers for non-linear dynamic systems fault diagnosis," in *Proc. 37th IEEE Conf. Decis. Control*, Tampa, FL, 1998, pp. 84–89.
[12] C. J. Lopez-Toribio and R. J. Patton, "Takagi-Sugeno fuzzy fault-tolerant control for a non-linear system," in *Proc. 38th IEEE Conf. Decis. Control*, Phoenix, AZ, 1999, pp. 4368–4373.
[13] S. K. Nguang, P. Shi, and S. X. Ding, "Fault detection for uncertain fuzzy systems: An LMI approach," *IEEE Trans. Fuzzy Syst.*, vol. 15, no. 6, pp. 1251–1262, Dec. 2007.
[14] S. K. Nguang, P. Shi, and S. X. Ding, "Delay-dependent fault estimation for uncertain time-delay nonlinear systems: An LMI approach," *Int. J. Robust Nonlinear Control*, vol. 16, no. 18, pp. 913–933, 2006.
[15] B. Jiang, M. Staroswiecki, and V. Cocquempot, "Fault accommodation for nonlinear dynamic systems," *IEEE Trans. Autom. Control*, vol. 51, no. 9, pp. 1578–1583, Sep. 2006.
[16] G. Garcia and J. Bernussou, "Pole assignment for uncertain systems in a specified disk by state feedback," *IEEE Trans. Autom. Control*, vol. 40, no. 1, pp. 184–190, Jan. 1995.
[17] E. G. Nobrega, M. O. Abdalla, and K. M. Grigoriadis, "Robust fault estimation of uncertain systems using an LMI-based approach," *Int. J. Robust Nonlinear Control*, vol. 18, no. 18, pp. 1657–1680, 2008.
[18] S. K. Spurgeon, "Sliding mode observers: A survey," *Int. J. Syst. Sci.*, vol. 39, no. 8, pp. 751–764, 2008.
[19] M. Chilali and P. Gahinet, "H_∞ design with pole placement constraints: An LMI approach," *IEEE Trans. Autom. Control*, vol. 41, no. 3, pp. 358–367, Aug. 1995.
[20] M. Chilali, P. Gahinet, and P. Apkarian, "Robust pole placement in LMI regions," *IEEE Trans. Automat. Control*, vol. 44, no. 12, pp. 2257–2270, Dec. 1999.
[21] C. Edwards, X. G. Yan, and S. K. Spurgeon, "On the solvability of the constrained Lyapunov problem," *IEEE Trans. Automat. Control*, vol. 52, no. 10, pp. 1982–1987, Oct. 2007.
[22] J. Li, H. O. Wang, D. Niemann, and K. Tanaka, "Dynamic parallel distributed compensation for Takagi-Sugeno fuzzy systems: An LMI approach," *Inf. Sci.*, vol. 123, no. 3–4, pp. 201–221, 2000.
[23] B. Jiang, J. L. Wang, and Y. C. Soh, "An adaptive technique for robust diagnosis of faults with independent effects on system outputs," *Int. J. Control*, vol. 75, no. 11, pp. 792–802, 2012.
[24] C. Liu and S. Hu, "Fault-Tolerant flight control based on iterative learning observer and T-S fuzzy model," *J. Nanjing Univ. Aeronaut. Astronaut.*, vol. 41, no. 1, pp. 48–53, 2009.
[25] W.-H. Ho, J.-T. Tsai, and J.-H. Chou, "Robust quadratic-optimal control of ts-fuzzy-model-based dynamic systems with both elemental parametric uncertainties and norm-bounded approximation error," *IEEE Trans. Fuzzy Syst.*, vol. 17, no. 3, pp. 518–531, Jun. 2009.

H_∞-Filter Design for a Class of Networked Control Systems Via T–S Fuzzy-Model Approach

Bin Jiang, Zehui Mao, and Peng Shi

***Abstract*—This paper is concerned with H_∞-design for a class of networked control systems (NCSs) with multiple state-delays via the Takagi–Sugeno (T–S) fuzzy model. The transfer delays and packet loss that are induced by the limited bandwidth of communication networks are considered. The focus of this paper is on the analysis and design of a full-order H_∞ filter, such that the filtering-error dynamics are stochastically stable, and a prescribed H_∞ attenuation level is guaranteed. Sufficient conditions are established for the existence of the desired filter in terms of linear-matrix inequalities (LMIs). An example is given to illustrate the effectiveness and applicability of the proposed design method.**

***Index Terms*—Filter, fuzzy systems, networked control systems (NCSs).**

I. Introduction

Networked control systems (NCSs) are the control systems in which controller and plant are connected via a communication channel. The defining feature of an NCS is that information (reference input, plant output, control input, etc.) is exchanged using a network among control-system components (sensors, controller, actuators, etc.). NCSs are applicable to many fields like dc motors, advanced aircraft, spacecraft automotive, and manufacturing processes. Therefore, increasing attention has been paid to the study of networked systems (see, for example, [1]–[5] and the references therein). It should be noted that the network itself is a dynamical system including some issues, such as data dropout, limited bandwidth, time delay, and quantization. Thus, conventional control theories for point-to-point control systems must be reevaluated before they can be applied to networked systems.

On the other hand, the existence of time delays is commonly encountered in many dynamic systems, and time delay has become one of the main causes of instability and poor performance of systems [1]. Therefore, the study of NCSs in the presence of network-induced delays has attracted great attention over the past few years. The delays of NCSs are generally described by stochastic process. Such systems have gained persistent attention in the last years, for which a lot of results can be found, such as [6] and [7]. Up until now, there have been three approaches to describe the data-dropout phenomenon, i.e., the Bernoulli process [8], the Markovian jump parameter [9], and the incompleteness matrix [10]. Corresponding literature can be found about filter design for NCSs under these approaches, for example, [3], [11], and [12]. It should be pointed out that most of the considered plants on the issues are linear systems, and to the best of our knowledge, H_∞ filtering problem for NCSs has not been fully investigated, and only a few results are available in the literature, which motivates the present study.

Manuscript received February 1, 2016; revised August 19, 2016; accepted October 20, 2016. First published November 20, 2016; current version published February 5, 2017. This work is supported in part by the National Natural Science Foundation of China under Grant 60874051, in part by the 973 Program of China under Grant 2009CB320600, and in part by the Engineering and Physical Sciences Research Council under EP/F029195, U.K.

B. Jiang and Z. Mao are with the College of Automation Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing, 210016, China (e-mail: binjiang@nuaa.edu.cn; zehuimao@nuaa.edu.cn).

P. Shi is with the Department of Computing and Mathematical Sciences, University of Glamorgan, Pontypridd, CF37 1DL, U.K., and also with the School of Science and Engineering, Victoria University, Melbourne, Vic. 8001, Australia. He is also with the School of Mathematics and Statistics, University of South Australia, Adelaide, S.A. 5095, Australia (e-mail: pshi@glam.ac.uk).

Digital Object Identifier 10.1109/TFUZZ.2017.2037009