

Enhancement of combined heat and power economic dispatch using self adaptive real-coded genetic algorithm

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ABSTRACT

In this paper, a self adaptive real-coded genetic algorithm (SARGA) is implemented to solve the combined heat and power economic dispatch (CHPED) problem. The self adaptation is achieved by means of tournament selection along with simulated binary crossover (SBX). The selection process has a powerful exploration capability by creating tournaments between two solutions. The better solution is chosen and placed in the mating pool leading to better convergence and reduced computational burden. The SARGA integrates penalty parameterless constraint handling strategy and simultaneously handles equality and inequality constraints. The population diversity is introduced by making use of distribution index in SBX operator to create a better offspring. This leads to a high diversity in population which can increase the probability towards the global optimum and prevent premature convergence. The SARGA is applied to solve CHPED problem with bounded feasible operating region which has large number of local minima. The numerical results demonstrate that the proposed method can find a solution towards the global optimum and compares favourably with other recent methods in terms of solution quality, handling constraints and computation time.

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1. Introduction

Increasing demand for power and heat resulted in the existence of co-generation units which simultaneously produce power and heat. The objective of economic dispatch (ED) problem in a conventional power plant is to find the optimal point for the power production such that the total demand matches the generation with minimum fuel cost. However, the objective of combined heat and power economic dispatch (CHPED) is to find the optimal point of power and heat generation with minimum fuel cost such that both heat and power demands are met while the combined heat and power units are operated in a bounded heat versus power plane. The mutual dependencies of heat and power generation introduce a complication in the integration of co-generation units into the power system economic dispatch. A technique was developed in [1] to solve the CHPED problem using separability of the cost function and constraints. Here two-level strategy is adopted; the lower

level solves economic dispatch for the values of power and heat lambdas and the upper level updates the lambda's sensitivity coefficients. The procedure is repeated until the heat and power demands are met. It was claimed in [2] that the Lagrangian Relaxation method cannot deal with discontinuous and/or non-monotonic input/output model for generator fuel cost characteristics. However, a two-layer Lagrangian relaxation algorithm was developed and solved the CHPED problem [3], and a customized branch-and-bound algorithm was also developed and solved the CHPED problem [4,5].

Alternatives to the traditional mathematical approaches: evolutionary computation techniques such as genetic algorithm (GA) [6,7], evolutionary programming (EP) [2], multi-objective particle swarm optimization (MPSO) [8], harmony search (HS) [9], fuzzy decision making (FDM) [10] and improved ant colony search algorithm (ACSA) [11] have been successfully applied to CHPED problem. In [6], improved genetic algorithm with multiplier updating (IGAMU) approach is implemented to solve the CHPED problem using penalty based constraint handling method. However, certain drawbacks regarding values of penalty parameters have been reported in [6,7]. The real-coded GAs are more suitable for large dimensional search spaces than binary-coded GAs since they are more consistent, precise and lead to faster convergence [12–14]. In this paper, ability of real-coded GAs to make pair-wise comparison in tournament selection are exploited to devise a penalty

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function approach that does not require any penalty parameter and such an approach is implemented to solve CHPED problem. The equality and bounded heat versus power plane constraints are effectively handled by penalty parameterless approach. The performance of the SARGA is compared with those of Lagrangian Relaxation method, branch-and-bound algorithms, ACSA, GA based penalty function (GA_PF), PSO, EP, IGAMU and HS. The simulation results show that SARGA integrated with penalty parameterless approach performs better than these methods, in terms of solution quality, handling constraints and computation time.

Section 2 describes the characteristics of a co-generation unit and formulation of the CHPED problem, Section 3 deals with self adaptive real-coded genetic algorithm, Section 4 describes implementation of SARGA to CHPED problem, and Section 5 presents numerical results based on a simple co-generation system.

2. Characteristics and formulation of the CHPED problem

Combined heat and power generation is an established and mature technology, which has higher energy efficiency and less green house gas emission as compared with the other forms of energy supply. The basic difference between conventional condensing plant and combined heat and power units is in the type of the power obtained and the overall efficiency of each plant. In conventional condensing plants the energy from the fuel is used to produce electrical power only, while in combined heat and power (CHP) systems, the energy from the fuel is used to produce both electrical and thermal power thus increasing its efficiency. The conventional condensing plant delivers power at an efficiency of 35–55%. Using efficient flue gas condensation, the total efficiency of CHP unit is found to be in the range of 80–111% (lower heating value base) [15–17]. The heat production depends on power generation and vice versa. This introduces complexity due to the non-separable nature of electrical power and heat in the CHP units.

The combined heat and power economic dispatch (CHPED) problem of a system is to determine the unit heat and power production, so that the system production cost is minimized, while the heat and power demands and other constraints are met. Fig. 1 shows the feasible bounded region in the heat versus power plane of a combined cycle co-generation unit. The feasible operating region is enclosed by the boundary curve MNOPQR. The upper and lower bounds of power and heat units are restricted by their own generation limits. The primary objective of the CHPED is to

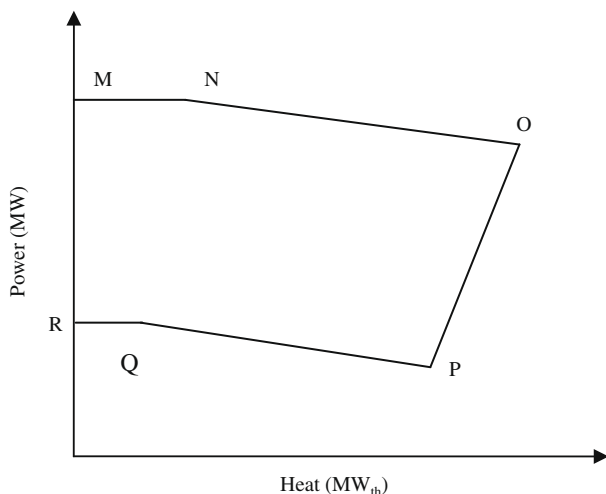


Fig. 1. Feasible operating region of a co-generation unit.

determine the most economic loading points of the combined heat and power generation units such that both the heat and power demands can be met and operated within the bounded region in the heat versus power plane.

The objective function of the CHPED problem is given by

$$\min f_{\text{cost}} = \sum_{i=1}^{N_p} C_i(P_i) + \sum_{j=1}^{N_c} C_j(O_j, H_j) + \sum_{k=1}^{N_h} C_k(T_k) \quad (1)$$

subjected to the equality constraints

$$\sum_{i=1}^{N_p} P_i + \sum_{j=1}^{N_c} O_j = P_d \quad (2)$$

$$\sum_{j=1}^{N_c} H_j + \sum_{k=1}^{N_h} T_k = H_d \quad (3)$$

and inequality constraints

$$P_i^{\min} \leq P_i \leq P_i^{\max}, \quad i = 1, \dots, N_p, \quad (4)$$

$$O_j^{\min}(H_j) \leq O_j \leq O_j^{\max}(H_j), \quad j = 1, \dots, N_c \quad (5)$$

$$H_j^{\min}(O_j) \leq H_j \leq H_j^{\max}(O_j), \quad j = 1, \dots, N_c \quad (6)$$

$$T_k^{\min} \leq T_k \leq T_k^{\max}, \quad k = 1, \dots, N_h \quad (7)$$

with

$$C_i(P_i) = a_i + b_i P_i + c_i P_i^2 \quad (8)$$

$$C_j(O_j, H_j) = a_j + b_j O_j + c_j O_j^2 + d_j H_j + e_j H_j^2 + f_j O_j H_j \quad (9)$$

$$C_k(T_k) = a_k + b_k T_k + c_k T_k^2 \quad (10)$$

where $\min f_{\text{cost}}$ is the total minimum fuel cost; C_i , C_j and C_k are the unit production costs of the conventional power, co-generation and heat-alone units, respectively; a_i , b_i and c_i are fuel cost coefficients of the i th conventional unit; a_j , b_j , c_j , d_j , e_j and f_j are fuel cost coefficients of the j th co-generation unit; a_k , b_k and c_k are fuel cost coefficients of the k th heat-alone unit; P_i and O_j are power generations of conventional power and co-generation units; H_j and T_k are heat generation of co-generation and heat-alone units; H_d and P_d are heat and power demands; N_p , N_c and N_h denote the number of conventional power, co-generation and heat-alone units, respectively; P_i^{\min} and P_i^{\max} are the minimum and maximum power generation limits of the conventional units; O_j^{\min} and O_j^{\max} are the minimum and maximum power generation limits of the co-generation units; H_j^{\min} and H_j^{\max} are the minimum and maximum heat generation limits of the co-generation units; T_k^{\min} and T_k^{\max} are the minimum and maximum heat generation limits of the heat-alone units.

The mutual dependencies of heat and power generations from (5) and (6) introduce a complication in the integration of co-generation units. Therefore, the optimization problem of the CHPED is non-linear and highly constrained in nature.

3. Self adaptive real-coded genetic algorithm

Self adaptation is a phenomenon which makes the genetic algorithms flexible and solves the CHPED problem with feasible operating region. SARGA [18,19] involves two critical issues: evolutionary search direction and population diversity. As the evolutionary direction is effective in searching, the strong evolutionary direction can reduce the computational burden and increase the probability of rapidly finding an optimal solution. Moreover, increase in population diversity creates the genotype of the offspring that differs more from the parents. Accordingly, a highly diverse population can increase the probability of exploring the global optimum and prevent the premature convergence to a local

optimum. In the next section, SARGA implementation to CHPED problem is presented.

3.1. Generation of initial population

In SARGA, each chromosome is encoded as a vector of floating-point numbers, with the same length as the vector of decision variables. The real-coded representation of genetic algorithm is accurate and efficient because it is closest to the real design space. For convenience, $(x_1, x_2, \dots, x_i, \dots, x_N)$ is represented as a vector of chromosome to the solution of the CHPED problem. Initialization of M individual population is generated using

$$x_i = x_i^u + \sigma_i(x_i^u - x_i^l) \quad (11)$$

where x_i^u and x_i^l are the domain of x_i and σ_i is a random number in the range of 0–1.

Repeat (11) N times and produce the vector $x_1, x_2, \dots, x_i, \dots, x_N$. Repeat the above procedure M times to create the M uniformly distributed individuals as initial feasible solutions in the search space. The fitness of an individual is a measure of how close the solution is to the global optimum.

3.2. Tournament selection

In this selection, tournaments are played between two parents randomly chosen from initial feasible solution and better parent is selected and placed in the mating pool. Two other parents are picked up again and another slot in the mating pool is filled with better parents. In this manner, each parent can be made to participate in exactly two tournaments. The best parents in a population will win both times, thereby making two copies of them in the new population. Using this, worst parents will lose in both tournaments and will be eliminated from the population. In this way, any parents in a population will have zero, one or two copies in the new population. This leads to better convergence in terms of solution quality and computational time.

3.3. Simulated binary crossover

This crossover operator works with two parent solutions and creates two offsprings. SBX simulates the working principle of the single-point crossover operator on binary strings. During this operation common interval schemata between the parents are preserved in the offsprings. The procedure of computing the offsprings $x_i^{(1,t+1)}$ and $x_i^{(2,t+1)}$ from the parents $x_i^{(1,t)}$ and $x_i^{(2,t)}$ is described as follows: a spread factor β_{si} is obtained as the ratio of the absolute difference in the offspring values to that of the parents:

$$\beta_{si} = \left| \frac{x_i^{(2,t+1)} - x_i^{(1,t+1)}}{x_i^{(2,t)} - x_i^{(1,t)}} \right| \quad (12)$$

First, a random number u_i between 0 and 1 is created. Thereafter, from a specified probability distribution function, the ordinate β_{qi} is found so that the area under the probability curve from 0 to β_{qi} is equal to the chosen random number u_i . The probability distribution used to create an offspring is derived to have a similar search power to that in a single-point crossover in binary-coded GAs and is given as follows:

$$P(\beta_{si}) = \begin{cases} 0.5(\eta_c + 1)\beta_{si}^{\eta_c}, & \beta_{si} \leq 1 \\ 0.5(\eta_c + 1)\frac{1}{\beta_{si}^{\eta_c+2}}, & \beta_{si} > 1 \end{cases} \quad (13)$$

In (13), a large value of the distribution index η_c gives a higher probability for creating offsprings closer to parents and a smaller value of η_c creates offspring distant from the parents. Using (13) β_{qi} is calculated by equating the area under the probability curve equal to u_i as follows:

$$\beta_{qi} = \begin{cases} (2u_i)^{\frac{1}{\eta_c+1}}, & u_i \leq 0.5 \\ \left(\frac{1}{2(1-u_i)}\right)^{\frac{1}{\eta_c+1}}, & u_i > 0.5 \end{cases} \quad (14)$$

After obtaining the value β_{qi} from (14), the offsprings are calculated as follows:

$$x_i^{(1,t+1)} = 0.5 \left[(1 + \beta_{qi})x_i^{(1,t)} + (1 - \beta_{qi})x_i^{(2,t)} \right] \quad (15)$$

$$x_i^{(2,t+1)} = 0.5 \left[(1 - \beta_{qi})x_i^{(1,t)} + (1 + \beta_{qi})x_i^{(2,t)} \right] \quad (16)$$

3.4. Polynomial mutation

The polynomial mutation is similar to the non-uniform mutation operator using polynomial probability distribution instead of a normal distribution. Here, the probability of creating an offspring closer to the parents is more than the probability of creating one away from it. As the generation i proceeds, this probability of creating offspring closer to the parents gets higher and higher, and the offsprings created are given as follows:

$$y_i^{(1,t+1)} = x_i^{(1,t+1)} + (x_i^u - x_i^l)\bar{\delta}_i \quad (17)$$

where the parameter $\bar{\delta}_i$ is calculated from the polynomial probability distribution

$$P(\delta_i) = 0.5(\eta_m + 1)(1 - |\delta_i|)^{\eta_m} \quad (18)$$

$$\bar{\delta}_i = \begin{cases} (2r_i)^{\frac{1}{\eta_m+1}} - 1, & r_i < 0.5 \\ 1 - [2(1 - r_i)]^{\frac{1}{\eta_m+1}}, & r_i \geq 0.5 \end{cases} \quad (19)$$

In (18) and (19), η_m the mutation constant is any non-negative real number and r_i is a random number between 0 and 1; η_m produces a perturbation of the order $\left(\frac{1}{\eta_m}\right)$ in the normalized decision variable. For handling bounded decision variables, the mutation operator is modified for two regions. i.e., $[x_i^l, x_i]$ and $[x_i, x_i^u]$, which is similar to non-uniform mutation. The shape of the probability distribution is directly controlled by an external parameter η_m and the distribution is not dynamically changed with generations. This leads to slight perturbation and prevents the individuals from premature convergence.

3.5. Stopping rules

The algorithm stops when maximum number of generations is reached or it terminates early depending on the unsuccessful generation of the algorithm. Two rules for terminating the progress of the unsuccessful generations of the algorithm are used: (i) the best solution not changing for a prespecified interval of generations (ii) otherwise, the algorithm stops if the termination condition.

$$|f_{cost,i} - f_{cost,i-1}|/|f_{cost,i}| \leq 0.001 \quad \text{is satisfied} \quad (20)$$

where $f_{cost,i}$ and $f_{cost,i-1}$ are feasible solutions at generation i and $i - 1$, respectively.

3.6. Constraint handling strategy

In penalty parameter based method [20], an external penalty parameter which penalizes infeasible solutions is used. Based on the constraint violation concerning $g_p(x)$ or $Z_q(x)$ a bracket-operator penalty term is added to the objective function and a penalized function is formed:

$$F_{cost}(x) = f_{cost}(x) + \sum_{p=1}^P R_p(g_p(x)) + \sum_{q=1}^Q r_q|Z_q(x)| \quad (21)$$

where R_p and r_q are penalty parameter; P is the total number of inequality constraints and Q is the total number of equality

constraints. In (21), the bracket-operator $\langle \cdot \rangle$ denotes the absolute value of the operand, if the operand is negative; if the operand is non-negative, it returns value of zero. Since different constraints may take different orders of magnitude, it is essential to normalize all constraints before using the above equation. A constraint $g_p(x) \geq b_p$ can be normalized by using the following equation:

$$g'_p(x) = \frac{g_p(x)}{b_p} - 1 \geq 0 \quad (22)$$

Equality constraints $Z_q(x)$ can also be normalized similarly. Normalizing the constraints in this manner, has the advantage: all normalized constraint violations take more or less the same order of magnitude and hence, they all can be simply added as the overall constraint violation and thus only one penalty parameter R will be needed to make the overall constraint violation of the same order as the objective function:

$$F_{\text{cost}}(x) = f_{\text{cost}}(x) + R \left[\sum_{p=1}^P \langle g'_p(x) \rangle + \sum_{q=1}^Q |Z_q(x)| \right] \quad (23)$$

The optimal solution of $F_{\text{cost}}(x)$ depends on penalty parameter R . Users usually have to try different values of R to find which value would steer the search towards the feasible region. The most difficult aspect of the penalty function approach is to find appropriate penalty parameters needed to guide the search towards the optimal solution. This requires extensive experimentation to find any reasonable solution. In this paper, GA's population-based approach and its ability to make pair-wise comparison in tournament selection are exploited to devise a penalty function approach that does not require any penalty parameter and such an approach is given below:

$$F_{\text{cost}}(x) = f_{\text{cost}}(x), \quad \text{if } x \text{ is feasible} \\ = f_{\text{cost},\text{max}} + \sum_{p=1}^P \langle g'_p(x) \rangle + \sum_{q=1}^Q |Z_q(x)|, \quad \text{otherwise} \quad (24)$$

In (24), $f_{\text{cost},\text{max}}$ is the objective function value of the worst feasible solution in the population. The fundamental difference between this approach and that using the penalty parameter is that the objective function value is not computed for any infeasible solution. Since all feasible solution has zero constraint violation and all infeasible solutions are evaluated according to their constraint violation only, both objective function value and constraint violation are not combined in any solution in the population. Thus, there is no need to have any penalty parameter for this approach. The SARGA with this constraint handling approach has been implemented on CHPED problem. From the numerical results, the penalty parameterless approach has repeatedly found solution closer to the global optimal solution.

4. Implementation of SARGA to CHPED problem

The real-coded genetic algorithm combines the SBX along with the polynomial mutation. The tournament selection is inserted between initialization of population and SBX crossover. Then, the systematic reasoning ability is incorporated in the crossover operations to select the better genes for crossover, and consequently enhance the real-coded genetic algorithm. The steps of the SARGA approach are depicted in Fig. 2 and are described as follows:

- Step 1 Parameter setting.
- Input: population size M , crossover rate p_c , SBX crossover constant η_c , mutation rate p_m , mutation constant η_m and number of generations.

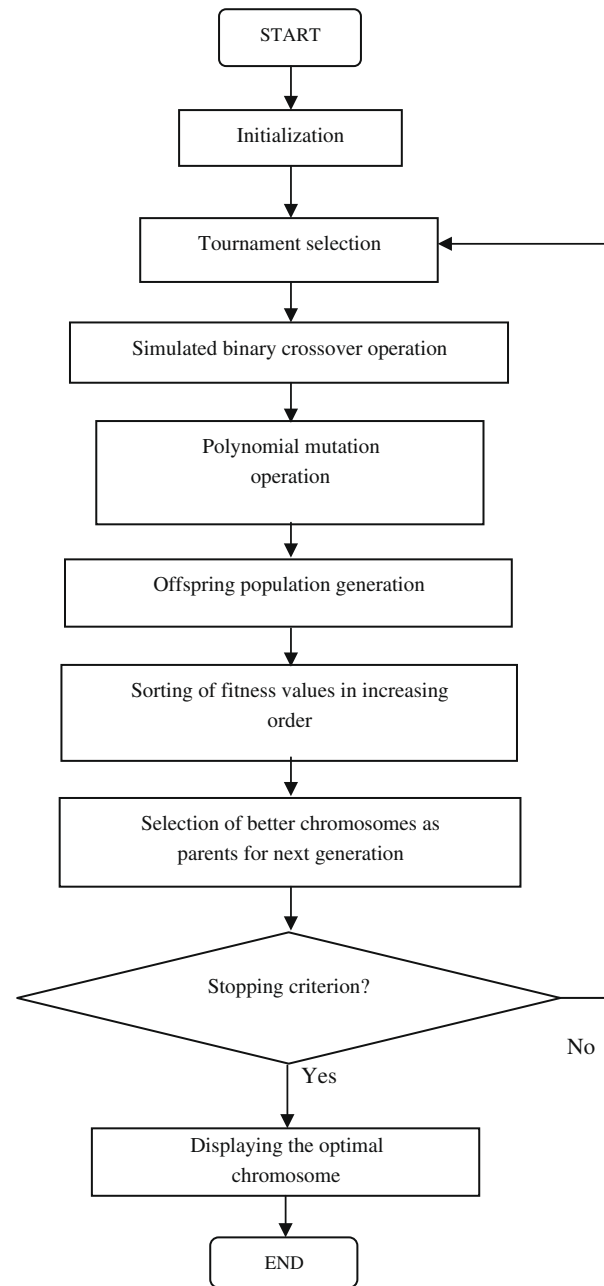


Fig. 2. Flowchart for implementation of SARGA.

- Step 2 Initial population is generated. The function values of the population are then calculated using f_{cost} .
- Step 3 Tournament selection operation is performed.
- Step 4 Crossover operations using simulated binary crossover. The probability of crossover is determined by crossover rate p_c .
- Step 5 Mutation operations using polynomial mutation. The probability of mutation is determined by mutation rate p_m .
- Step 6 Offspring population is generated.
- Step 7 Sort the fitness values in increasing order among the generated population.
- Step 8 Select the better M , chromosomes as parents of the next generations.
- Step 9 Check for the stopping criteria.
- Step 10 Display the optimal chromosome and the optimal fitness value.

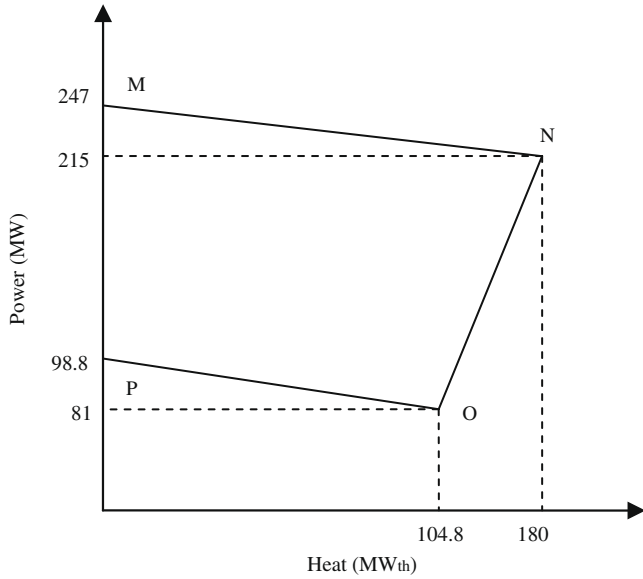


Fig. 3. Feasible operating regions of co-generation unit 1.

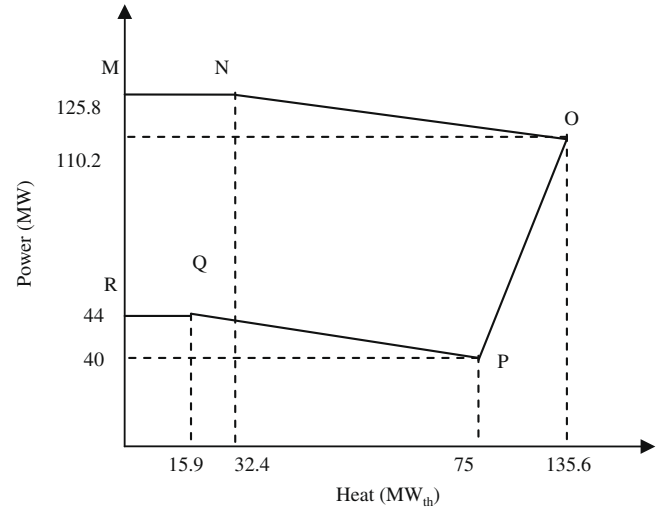


Fig. 4. Feasible operating regions of co-generation unit 2.

5. Numerical results based on a simple co-generation system

This section considers a single area co-generation system to illustrate the effectiveness of the proposed SARGA in terms of quality of solution and computation time. The example [3] consists of one conventional power unit, two co-generation units and one heat-alone unit. The power generation limits of the power unit are 0 and 150 MW and heat generation limits of heat-alone units are 0 and 2695.2 MW_{th}. The feasible operating regions of the two co-generation units are given in Fig. 3 and 4. The system power demand P_d and the heat demand H_d are 200 MW and 115 MW_{th}, respectively.

The fuel cost characteristics of conventional, co-generation and heat-alone units are given in (26)–(29). The objective function of the CHPED problem is

$$\min f_{\text{cost}} = C_1(P_1) + \sum_{j=1}^2 C_j(O_j, H_j) + C_1(T_1) \tag{25}$$

where

$$C_1(P_1) = 50P_1 \tag{26}$$

$$C_1(O_1, H_1) = 2650 + 14.5O_1 + 0.0345O_1^2 + 4.2H_1 + 0.03H_1^2 + 0.031O_1H_1 \tag{27}$$

$$C_2(O_2, H_2) = 1250 + 36O_2 + 0.0435O_2^2 + 0.6H_2 + 0.027H_2^2 + 0.011O_2H_2 \tag{28}$$

$$C_1(T_1) = 23.4T_1 \tag{29}$$

subjected to the equality constraints:

$$Z_1 : P_1 + O_1 + O_2 = P_d$$

$$Z_2 : H_1 + H_2 + T_1 = H_d$$

and the inequality constraints:

$$g_1 : 1.781914894H_1 - O_1 - 105.7446809 \leq 0$$

$$g_2 : 0.177777778H_1 + O_1 - 247.0 \leq 0$$

$$g_3 : -0.169847328H_1 - O_1 + 98.8 \leq 0$$

$$g_4 : 1.158415842H_2 - O_2 - 46.88118818 \leq 0$$

$$g_5 : 0.151162791H_2 + O_2 - 130.6976744 \leq 0$$

$$g_6 : -0.067681895H_2 - O_2 + 45.07614213 \leq 0$$

$$g_7 : 0.0 - P_1 \leq 0$$

$$g_8 : P_1 - 150.0 \leq 0$$

$$g_9 : 0.0 - T_1 \leq 0 \text{ and}$$

$$g_{10} : T_1 - 2695.2 \leq 0$$

The system consists of six decision variables ($P_1, O_1, H_1, O_2, H_2, T_1$) power balance constraint, heat balance constraint and ten inequality constraints.

5.1. Simulation results and discussion

This section presents the simulation results of the chosen CHPED problem with focus on the comparison of SARGA with Lagrangian Relaxation [3], branch-and-bound algorithm [4,5], ACSA [11], GA_PF [7], PSO [8], EP [2], IGAMU [6] and HS [9]. All these methods are coded in MATLAB and executed using a Pentium

Table 1 Comparison of various methods for CHPED problem.

Power/heat	Lagrange relaxation techniques	Branch-and-bound algorithm	ACSA	GA_PF	PSO	EP	IGAMU	HS	SARGA
P_1 (MW)	0.00	0.00	0.08	0.00	0.05	0.00	0.00	0.00	0.00
O_1 (MW)	160.00	160.00	150.93	159.23	159.43	160.00	160.00	160.00	159.99
O_2 (MW)	40.00	40.00	49.00	40.77	40.57	40.00	40.00	40.00	40.01
H_1 (MW _{th})	40.00	40.00	48.84	39.94	39.97	40.00	39.99	40.00	39.99
H_2 (MW _{th})	75.00	75.00	65.79	75.06	75.03	75.00	75.00	75.00	75.00
T_1 (MW _{th})	0.00	0.00	0.37	0.00	0.00	0.00	0.00	0.00	0.00
P_d (MW)	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00
H_d (MW _{th})	115.00	115.00	115.00	115.00	115.00	115.00	115.00	115.00	115.00
Total cost (US\$)	9257.10	9257.10	9452.20	9267.28	9265.10	9257.10	9257.09	9257.07	9257.07
Execution time in seconds	3.98	4.27	5.26	4.32	3.09	7.96	5.53	4.21	3.76

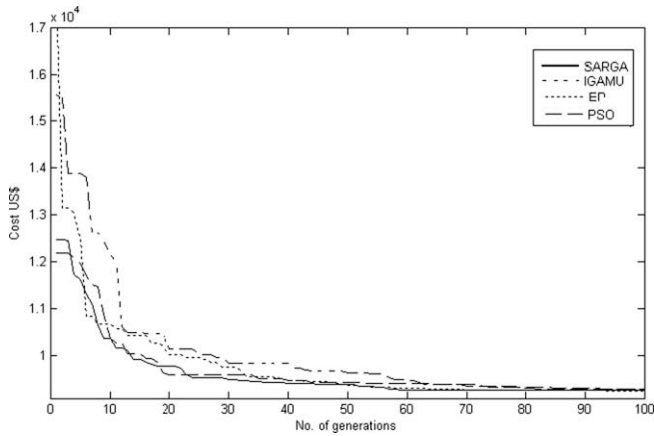


Fig. 5. Convergence characteristics of SARGA in CHPED problem.

IV based PC as the test platform. To verify the performance of the SARGA, the program is run a hundred times on the example. The resulting fuel costs and average CPU times are used to compare the performance of the SARGA with those of other methods. During this evolutionary process, the following parameter setting is used in SARGA: population size $M = 100$ crossover rate $p_c = 0.9$ mutation rate $p_m = 0.01$ crossover constant $\eta_c = 5$ and mutation $\eta_m = 1$. The parameter settings for ACSA, GA_PF, PSO, EP, IGAMU and HS are taken from [11,7,8,2,6,9], respectively. The best optimal solution obtained for this example by [3,21] is \$ 9257. The results obtained for this example using SARGA method are given in Table 1, and the results are compared with those of Lagrangian relaxation technique, branch-and-bound algorithm, ACSA, GA_PF, PSO, EP, IGAMU and HS. Among these algorithms, PSO, GA_PF and ACSA converge to a much higher cost due to their premature convergence. Even though PSO requires less computational time, it fails to approach near global optimal solution. Lagrangian relaxation technique, branch-and-bound algorithms, IGAMU and HS algorithms obtain better solution at the expense of computational time. From Table 1, it can be seen that the proposed method, SARGA obtains minimum cost with less computational time.

The convergence nature of the different algorithms is shown in Fig. 5 from which it is evident that SARGA and PSO have the fastest convergence. However, PSO fails to approach towards global optimal solution which has been shown also in Table 1.

In order to get more insights into the working of SARGA, the influence of the evolutionary parameters on the performance of SARGA is brought out as shown in Tables 2 and 3. The parameters: crossover constant η_c and mutation constant η_m in SARGA affect the solution quality and convergence. Influence of these parameters in CHPED problem is shown in Table 2. For $\eta_c = 5$ and $\eta_m = 1$ the solution exhibits better results in terms of standard deviation, mean solution and worst solution. Moreover, out of 100 trials the solution hits towards global optimal solution 100 times.

Table 2 Influence of η_m and η_c in SARGA for CHPED problem.

Population size 100	$p_c = 0.9, p_m = 0.01, \eta_c = 5$				$p_c = 0.9, p_m = 0.01, \eta_m = 1$			
	$\eta_m = 5$	$\eta_m = 10$	$\eta_m = 15$	$\eta_m = 20$	$\eta_c = 20$	$\eta_c = 15$	$\eta_c = 10$	$\eta_c = 5$
Best solution	9257.7	9257.4	9257.2	9257.2	9257.2	9257.2	9257.4	9257.07
Worst solution	10107.1	9352.9	9742.8	9430.5	10157.6	10107.3	9960.8	9301.2
Mean solution	9271.8	9264.6	9269.3	9274.1	9294.2	9283.3	9281.2	9265.0
Standard deviation	84.54	13.23	50.17	33.75	117.81	87.45	78.94	7.4390
No. of hits to global minimum	99	93	89	90	93	96	94	100
CPU time	3.87	3.79	3.79	3.82	3.74	3.78	3.80	3.76

Table 3 Influence of population size in SARGA for CHPED problem.

Population size	50	75	100
Best solution	9257.2	9257.4	9257.07
Worst solution	9962.1	9960.7	9301.2
Mean solution	9306.3	9275.7	9265.0
Standard deviation	94	58	7.44
No. of hits to global minimum	92	98	100
CPU time	2.89	3.01	3.76

Table 4 Performance of SARGA based on different constraint handling strategy.

Population size 100, $\eta_c = 5, \eta_m = 1$	With penalty parameter		Without penalty parameter	
	Penalty R	10^2	10^3	
Best solution	Infeasible	9258.2	9257.3	9257.07
Worst solution	Infeasible	9650.5	9724.8	9301.2
Mean solution	Infeasible	9283.1	9271.8	9265.0
Standard deviation	-NA-	61.89	52.43	7.43
No. of hits to global minimum	0	91	97	100
CPU time	6.83	6.70	6.95	3.76

The best solutions obtained by SARGA are summarized in Table 3 for different sizes of population. During this performance analysis, $p_c = 0.9, \eta_c = 5, p_m = 0.01, \eta_m = 1$ are fixed. When population size is small, the algorithm does not guarantee a 100% hit towards global optimal solution. Moreover, the standard deviation, worst and mean solutions obtained for smaller population size, exhibit larger deviation as shown in Table 3. For population size below 50 the algorithm may not approach towards global optimal solution.

During this simulation process, SARGA is implemented and tested for constraints handling strategy. The constraint handling abilities are evaluated next and the results are presented in Table 4. With $R = 10$ it is not able to find a single feasible solution in 100 trials. This happens because smaller values of R do not force the feasibility of the solution. With large penalty parameters, the pressure for the solution to become feasible is more and it hits 97 times towards global optimal solution resulting in smaller value of standard deviation.

In penalty parameterless approach, there is no need to have any penalty parameter: the objective function value is not computed for any infeasible solution. All feasible solutions and all infeasible solutions are handled according to (24).

SARGA searches from a population of points and hence, discovers the nearest global point and directly use the fitness function information in the search procedure. The search is based on the stochastic operations. In SBX crossover, the two offsprings are symmetric about the parent solutions thereby avoiding a bias towards any particular parent solution in a single crossover operation. Binary GA requires large population size for larger strings. The large population size increases the computational complexity. The

real-coded GA uses real parameters directly without any string coding thereby reducing the computational complexity.

6. Conclusion

This paper presents an approach for solving the combined heat and power economic dispatch problem (CHPED) considering the feasible operating region. SARGA has effectively provided the global optimal solution satisfying both equality and inequality constraints. For the chosen example, SARGA has superiority to the other algorithms viz. ACSA, GA_PF, PSO, EP, IGAMU and HS in terms of solution accuracy, handling constraints and computation time. Moreover, the results of SARGA method are very close to those of the conventional numerical methods. Combined with their relatively low computational requirements as well as their suitability for parallel implementation, the algorithm provides global optimal solution to a real world CHPED problem. Hence, SARGA has the merits of global exploration, fast convergence, robustness and statistical soundness.

References

- [1] Rooijers FJ, Van Amerongen RAM. Static economic dispatch for co-generation systems. *IEEE Trans Power Syst* 1994;9(3):1392–8.
- [2] Wong KP, Algie C. Evolutionary programming approach for combined heat and power dispatch. *Electr Power Syst Res* 2002;61:227–32.
- [3] Guo T, Henwood MI, Van Ooijen M. An algorithm for combined heat and power economic dispatch. *IEEE Trans Power Syst* 1996;11(4):1778–84.
- [4] Makkonen S, Lahdelma R. Non-convex power plant modeling in energy optimization. *Eur J Oper Res* 2006;171(3):1113–26.
- [5] Rong A, Lahdelma R. An efficient envelope-based branch-and-bound algorithm for non-convex combined heat and power production planning. *Eur J Oper Res* 2007;183(1):412–31.
- [6] Su CT, Chiang CL. An incorporated algorithm for combined heat and power economic dispatch. *Electr Power Syst Res* 2004;69(2–3):187–95.
- [7] Song YH, Xuan QY. Combined heat and power economic dispatch using genetic algorithm based penalty function method. *Electr Power Comp Syst* 1998;26(4):363–72.
- [8] Wang LF, Singh C. Stochastic combined heat and power dispatch based on multi-objective particle swarm optimization. *Int J Electr Power Energy Syst* 2008;30:226–34.
- [9] Vasebi A, Fesanghary M, Bathaee SMT. Combined heat and power economic dispatch by harmony search algorithm. *Int J Electr Power Energy Syst* 2007;29:713–9.
- [10] Chang CS, Fu W. Stochastic multiobjective generation dispatch of combined heat and power systems. *IEE Proc Gener Transm Distrib* 1998;145(5):583–91.
- [11] Song YH, Chou CS, Stonham TJ. Combined heat and power dispatch by improved ant colony search algorithm. *Electr Power Syst Res* 1999;52:115–21.
- [12] Baskar S, Subbaraj P, Chidambaram P. Application of genetic algorithms to unit commitment problem. *J Inst Eng (India)* 2001;81:195–9.
- [13] Baskar S, Subbaraj P, Rao MVC, Tamilselvi S. Genetic algorithms solution to generator maintenance scheduling with modified genetic operators. *IEE Proc Gener Transm Distrib* 2003;150(1):56–60.
- [14] Baskar S, Subbaraj P, Rao MVC. Performance of real coded genetic algorithms with new genetic operators. *J Inst Eng (India)* 2004;85:124–8.
- [15] Wahlund B, Yan J, Westermark M. Comparative assessment of biofuel – based combined heat and power generation plants in Sweden. In: *First world conference on biomass for energy and industry*, Sevilla, Spain; 2000. p. 5–9.
- [16] Wahlund B, Yan J, Westermark M. A total energy system of fuel upgrading by drying biomass feedstock for co-generation: a case study of Skelleftea bioenergy combine. *Biomass Bioenergy* 2002;23:271–81.
- [17] Wahlund B, Yan J, Westermark M. Increasing biomass utilization in energy systems: a comparative study of CO₂ reduction and cost for different bioenergy processing options. *Biomass Bioenergy* 2004;26:531–44.
- [18] Deb K, Goyal M. A combined genetic adaptive search (GeneAS) for engineering design. *Comput Sci Infor* 1996;26(4):30–45.
- [19] Deb K, Beyer HG. Self-adaptation in real parameter genetic algorithms with simulated binary crossover. *Proc Int Conf Genet Evol Comput* 1999:172–9.
- [20] Deb K. An efficient constraint handling method for genetic algorithms. *Comput Meth Appl Mech Eng* 2000;186:311–38.
- [21] Nagendra Rao PS. Combined heat and power economic dispatch: a direct solution. *Electr Power Comp Syst* 2006;34(9):1043–56.