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Modeling of synchronous machines with magnetic saturation

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Abstract

This paper deals with a method to derive multiple models of saturated round rotor synchronous machines, based on different selections of state-space variables. By considering the machine currents and fluxes as space vectors, possible d–q models are discussed and adequately numbered. As a result several novel models are found and presented. It is shown that the total number of d–q models for a synchronous machine, with basic dampers, is 64 and therefore much higher than known. Found models are classified into three families: current, flux and mixed models. These latter, the mixed ones, constitute the major part (52) and hence offer a large choice.

Regarding magnetic saturation, the paper also presents a method to account for whatever the choice of state-space variables. The approach consists of just elaborating the saturation model with winding currents as main variables and deriving all the other models from it, by ordinary mathematical manipulations. The paper emphasizes the ability of the proposed approach to develop any existing model without exception.

An application to prove the validity of the method and the equivalence between all developed models is reported.

Keywords: Synchronous machine; Modeling; State-space vectors; Main flux saturation

1. Introduction

When magnetic saturation in ac machines is evolved, the theory of main flux saturation in d–q axes remains the best [1–8]. Because of its simplicity, it is the most used in either motoring or generating mode for synchronous or asynchronous machines.

Although, it is considered as a global way of introducing the iron saturation, compared to other methods, today, its fidelity has no contest in predicting complex ac machine operations. This can be worth checked for example in [9–14]. Of course, it reaches its limits when local phenomena have to be investigated like in some cases of diagnosis. For that purpose, the 'abc' frame and the primary equations must be handled with the known difficulties [15–17]. For all these reasons, the theory of main flux saturation in d–q axes deserves more attention.

The present paper deals mainly with a method to develop various ac machine d–q models with main flux saturation. It may be considered as an alternative to that used in [18] with the following features:

(1) It utilizes uniquely the primitive d–p equations in conjunction with the winding currents model incorporating magnetic saturation which is relatively the most known and used in the literature [1].
(2) It can be used either for induction or synchronous machines.
(3) For a given ac machine, induction or synchronous, it can be applied to describe any possible model whatever the state-space variables, unlike in [18] where some existing models cannot be obtained with the method used.

As indicated by the title, the present work is entirely devoted to modeling saturated round rotor synchronous machines in d–q axes, regardless of the choice of the state variables. In a first part, the number of models is revised and found to be much more than known [18]. Further, by assuming the machine currents and fluxes to space vector*s*, it will be shown that the number of models for a synchronous machine with basic dampers is 64. Determination of such models is adequately detailed and deeply

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discussed. Depending on the nature of state variables, found models are arranged into three families: current (6), flux (6) and mixed models (52).

In a second part, the proposed approach for developing any possible model including magnetic saturation, is presented and applied to synchronous machines. It consists of just elaborating the saturation model with stator and rotor winding currents as main variables and deriving all the other models from it, by ordinary mathematical manipulations. To practice the method evolved, simulation was carried out with a saturated synchronous machine to prove the validity of the proposed models as well as the equivalence between them.

The study encompasses the following sections. In Section 2, the basic d–q equations of a round rotor synchronous machine and related hypothesis are provided. The number of models is immediately discussed in Section 3. After that, possible models are grouped and classified into families (Section 4). The winding currents model incorporating saturation is elaborated and followed by a description of the method for deriving the remaining models, in Section 5. Derivation of some selected models is the object of Section 6. The last Sections 7–9 constitute the application, the discussion and the conclusion.

2. Basic d–q equations and hypothesis

With the following assumptions:

(1) No magnetic hysteresis, magnetizing currents and fluxes are in phase.
(2) No skin effect, winding resistances are frequency not dependent.
(3) Space higher harmonics are neglected; flux and mmfs are sinusoidal in space.
(4) Damper windings are the same in both axes.
(5) Through each winding, the linkage flux is the sum of an appropriate leakage flux independent of saturation and a main flux subject to saturation.

The space vector electrical equations of a smooth air gap synchronous machine, in the rotor reference frame, are:

$$\bar{v}_s = R_s\bar{i}_s + \frac{d\bar{\lambda}_s}{dt} + jw\bar{\lambda}_s \quad (1)$$

$$\bar{v}_r = R_r\bar{i}_r + \frac{d\bar{\lambda}_r}{dt} \quad (2)$$

$$\bar{v}_f = R_f\bar{i}_f + \frac{d\bar{\lambda}_f}{dt} \quad (3)$$

where

$$\bar{\lambda}_s = l_s\bar{i}_s + \bar{\lambda}_m \quad (4)$$

$$\bar{\lambda}_r = l_r\bar{i}_r + \bar{\lambda}_m \quad (5)$$

$$\bar{\lambda}_f = l_f\bar{i}_f + \bar{\lambda}_m \quad (6)$$

or

$$\bar{\lambda}_s = L_s\bar{i}_s + L_m(\bar{i}_r + \bar{i}_f) \quad (4a)$$

$$\bar{\lambda}_r = L_r\bar{i}_r + L_m(\bar{i}_s + \bar{i}_f) \quad (5a)$$

$$\bar{\lambda}_f = L_f\bar{i}_f + L_m(\bar{i}_s + \bar{i}_r) \quad (6a)$$

with

$$\bar{\lambda}_m = L_m\bar{i}_m \quad (7)$$

$$\bar{i}_m = \bar{i}_s + \bar{i}_r + \bar{i}_f \quad (8)$$

and

$$L_s = l_s + L_m, \qquad L_r = l_r + L_m, \qquad L_f = l_f + L_m \quad (9)$$

Definition of symbols and subscripts are given in Appendix A. All the rotor quantities are referred to the stator. Eq. (2) is that of damper windings, consisting of a kind of short-circuited cage, hence $\bar{v}_r = 0$. Due to the absence of the q-axis excitation winding, the number of equations in the d and q axes is not the same. Thus, it is more convenient to separate the space vector equations in d–q ones.

The d-axis equations are:

$$v_{ds} = R_s i_{ds} - w\lambda_{qs} + \frac{d\lambda_{ds}}{dt} \quad (10)$$

$$0 = R_r i_{dr} + \frac{d\lambda_{dr}}{dt} \quad (11)$$

$$v_f = R_f i_f + \frac{d\lambda_f}{dt} \quad (12)$$

with

$$\lambda_{ds} = L_s i_{ds} + L_m(i_{dr} + i_f) \quad (13)$$

$$\lambda_{dr} = L_r i_{dr} + L_m(i_{ds} + i_f) \quad (14)$$

$$\lambda_f = L_f i_f + L_m(i_{ds} + i_{dr}) \quad (15)$$

and

$$i_{dm} = i_{ds} + i_{dr} + i_f \quad (16)$$

$$\lambda_{dm} = L_m i_{dm} \quad (17)$$

Equations of q-axis are:

$$v_{qs} = R_s i_{qs} + w\lambda_{ds} + \frac{d\lambda_{qs}}{dt} \quad (18)$$

$$0 = R_r i_{qr} + \frac{d\lambda_{qr}}{dt} \quad (19)$$

with

$$\lambda_{qs} = L_s i_{qs} + L_m i_{qr} \quad (20)$$

$$\lambda_{qr} = L_r i_{qr} + L_m i_{qs} \quad (21)$$

and

$$i_{qm} = i_{qs} + i_{qr} \quad (22)$$

$$\lambda_{qm} = L_m i_{qm} \quad (23)$$

3. Number of possible models

According to (1)–(7) the possible state-space vectors are $\bar{i}_s, \bar{i}_r, \bar{i}_m, \bar{\lambda}_s, \bar{\lambda}_r, \bar{\lambda}_m, \bar{i}_f$ and $\bar{\lambda}_f$. This set can be divided into two parts, (i_f, λ_f) having only d-components and the remaining $(\bar{i}_s, \bar{i}_r, \bar{i}_m, \bar{\lambda}_s, \bar{\lambda}_r, \bar{\lambda}_m, \bar{i}_f)$ having both d and q components.

Evidently, resolving the main Eqs. (1)–(3) of a synchronous machine requires the selection of three vectors. But, according to (10)–(12), (18) and (19) three d-components are needed for the d-axis equations whereas two q-components are sufficient for the q-axis. Therefore, determination of possible combinations can be achieved as follows. Choose two vectors among $(\bar{i}_s, \bar{i}_r, \bar{i}_m, \bar{\lambda}_s, \bar{\lambda}_r, \bar{\lambda}_m)$ and complete each pair by i_f or λ_f. Because of the relation (7), the pair $(\bar{i}_m, \bar{\lambda}_m)$ has to be omitted. This group contains $(C_6^2 - 1) \times 2 = 28$ models. Or simply choose three vectors among the six having d and q components, except eight cases that are: $(\bar{i}_m, \bar{\lambda}_m, \bar{\lambda}_s)$, $(\bar{i}_m, \bar{\lambda}_m, \bar{i}_r)$, $(\bar{i}_m, \bar{\lambda}_m, \bar{\lambda}_s)$, $(\bar{i}_m, \bar{\lambda}_m, \bar{\lambda}_r)$ where the pair $(i_m, \bar{\lambda}_m)$ exists and $(\bar{i}_s, \bar{\lambda}_s, \bar{i}_m)$, $(\bar{i}_s, \bar{\lambda}_s, \bar{\lambda}_m)$, $(\bar{i}_r, \bar{\lambda}_r, \bar{i}_m)$, $(\bar{i}_r, \bar{\lambda}_r, \bar{\lambda}_m)$ because of the dependency formulated in (4) and (5). Also, it is to be noted that each combination of three vectors generates three possible cases and thus three different models in terms of space vectors. As an example, with $(\bar{i}_s, \bar{i}_r, \bar{\lambda}_s)$ we can form $(\bar{i}_s, \bar{i}_r, \lambda_{ds})$, $(\bar{i}_s, i_{dr}, \bar{\lambda}_s)$ and $(i_{ds}, \bar{i}_r, \bar{\lambda}_s)$. This second group encompasses $(C_6^3 - 8) \times 3 = 36$ models.

Finally, the total number of possible models for a damped synchronous machine, in terms of space vectors, is 64.

4. Classification of found models

As mentioned previously, all found models are classified into three families: current, flux and mixed models.

4.1. Current models

They are determined by choosing a pair of vectors belonging to $(\bar{i}_s, \bar{i}_r, \bar{i}_m)$ to be completed with i_f, or simply consider the set $(\bar{i}_s, \bar{i}_r, \bar{i}_m)$ which generates three cases $(\bar{i}_s, \bar{i}_r, i_{dm})$, $(\bar{i}_s, i_{dr}, \bar{i}_m)$ and $(i_{ds}, \bar{i}_r, \bar{i}_m)$. Thus, there are six pure current models.

4.2. Flux models

With $(\bar{\lambda}_s, \bar{\lambda}_r, \bar{\lambda}_m)$ and λ_f six flux models are obtained by proceeding similarly as with the currents. They are summarised at the end of the section.

4.3. Mixed current–flux models

The dissimetry observed in the d–q equations of synchronous machines complicates relatively the problem and allows the definition of four categories of current–flux models:

- models based on two current vectors and one flux d-component,
- models based on two flux vectors and one current d-component,
- models based on one current vector, one flux vector and a current d-component,
- models based on one current vector, one flux vector and a flux d-component.

4.4. Recapitulation

Resulting combinations of possible state-space variables can be recognized in the following recapitulation:

Current models : $(\bar{i}_s, \bar{i}_r, i_f), (\bar{i}_s, \bar{i}_m, i_f), (\bar{i}_r, \bar{i}_m, i_f), (\bar{i}_s, \bar{i}_r, i_{dm}),$
$(\bar{i}_s, i_{dr}, \bar{i}_m), (i_{ds}, \bar{i}_r, \bar{i}_m)$

Flux models : $(\bar{\lambda}_s, \bar{\lambda}_r, \lambda_f), (\bar{\lambda}_s, \bar{\lambda}_m, \lambda_f), (\bar{\lambda}_r, \bar{\lambda}_m, \lambda_f),$
$(\bar{\lambda}_s, \bar{\lambda}_r, \lambda_{dm}), (\bar{\lambda}_s, \lambda_{dr}, \bar{\lambda}_m), (\lambda_{ds}, \bar{\lambda}_r, \bar{\lambda}_m)$

Mixed models : $(\bar{i}_s, \bar{i}_r, \lambda_{ds}), (\bar{i}_s, \bar{i}_r, \lambda_{dr}), (\bar{i}_s, \bar{i}_r, \lambda_{dm}), (\bar{i}_s, \bar{i}_r, \lambda_f),$
$(\bar{i}_s, \bar{i}_m, \lambda_{dr}), (\bar{i}_s, \bar{i}_m, \lambda_f), (\bar{i}_r, \bar{i}_m, \lambda_{ds}), (\bar{i}_r, \bar{i}_m, \lambda_f);$
$(\bar{\lambda}_s, \bar{\lambda}_r, i_{ds}), (\bar{\lambda}_s, \bar{\lambda}_r, i_{dr}), (\bar{\lambda}_s, \bar{\lambda}_r, i_{dm}), (\bar{\lambda}_s, \bar{\lambda}_r, i_f), (\bar{\lambda}_s, \bar{\lambda}_m, i_{dr}),$
$(\bar{\lambda}_s, \bar{\lambda}_m, i_f), (\bar{\lambda}_r, \bar{\lambda}_m, i_{ds}), (\bar{\lambda}_r, \bar{\lambda}_m, i_f);$ $(\bar{i}_r, \bar{\lambda}_r, i_{ds}), (\bar{i}_r, \bar{\lambda}_r, i_f),$
$(\bar{i}_m, \bar{\lambda}_s, i_{dr}), (\bar{i}_m, \bar{\lambda}_s, i_f), (\bar{i}_m, \bar{\lambda}_r, i_{ds}), (\bar{i}_m, \bar{\lambda}_r, i_f), (\bar{i}_s, \bar{\lambda}_r, i_{dr}),$
$(\bar{i}_s, \bar{\lambda}_r, i_{dm}), (\bar{i}_s, \bar{\lambda}_r, i_f), (\bar{i}_r, \bar{\lambda}_s, i_{ds}), (\bar{i}_r, \bar{\lambda}_s, i_{dm}), (\bar{i}_r, \bar{\lambda}_s, i_f),$
$(\bar{i}_r, \bar{\lambda}_m, i_{ds}), (\bar{i}_r, \bar{\lambda}_m, i_f);$ $(\bar{i}_s, \bar{\lambda}_s, \lambda_{dr}), (\bar{i}_s, \bar{\lambda}_s, \lambda_f),$
$(\bar{i}_s, \bar{\lambda}_m, \lambda_{dr}), (\bar{i}_s, \bar{\lambda}_m, \lambda_f), (\bar{i}_r, \bar{\lambda}_r, \lambda_{ds}), (\bar{i}_r, \bar{\lambda}_r, \lambda_f), (\bar{i}_m, \bar{\lambda}_s, \lambda_{dr}),$
$(\bar{i}_m, \bar{\lambda}_s, \lambda_f), (\bar{i}_m, \bar{\lambda}_r, \lambda_{ds}), (\bar{i}_m, \bar{\lambda}_r, \lambda_f), (\bar{i}_s, \bar{\lambda}_r, \lambda_{dr}),$
$(\bar{i}_s, \bar{\lambda}_r, \lambda_{dm}), (\bar{i}_s, \bar{\lambda}_r, \lambda_f), (\bar{i}_r, \bar{\lambda}_s, \lambda_{ds}), (\bar{i}_r, \bar{\lambda}_s, \lambda_{dm}),$
$(\bar{i}_r, \bar{\lambda}_s, \lambda_f), (\bar{i}_r, \bar{\lambda}_m, \lambda_{ds}), (\bar{i}_r, \bar{\lambda}_m, \lambda_f)$

5. Method description

As already stated, the proposed method is mainly based on the knowledge of the winding currents model. For that reason, a procedure showing how to obtain such model is following.

Since $(i_{ds}, i_{qs}, i_{dr}, i_{qr}, i_f)$ constitute the selected state variables, linkage fluxes and their time derivatives in the primitive d–q Eqs. (1)–(3) must be written as functions of them. The d–q components of $(\bar{\lambda}_s, \bar{\lambda}_r, \lambda_f)$ are naturally written in terms of the winding currents in (13)–(15), (20) and (21). Regarding their derivatives, a brief description is going to be exposed showing how to proceed for that.

Deriving the linkage fluxes expressed by (13)–(15), (20) and (21) leads to the time derivative of the magnetizing inductance L_m. The leakage inductances are supposed to be invariable. Now let's write:

$$\frac{dL_m}{dt} = \frac{dL_m}{di_m}\frac{di_m}{dt} \tag{24}$$

From (7), $L_m = \frac{\lambda_m}{i_m}$ and after deriving $i_m^2 = i_{dm}^2 + i_{qm}^2$ we get, respectively, (25) and (26):

$$\frac{dL_m}{di_m} = \frac{1}{i_m}(L_{mdy} - L_m), \quad L_{mdy} = \frac{d\lambda_m}{di_m} \tag{25}$$

$$\frac{di_m}{dt} = \frac{i_{dm}}{i_m}\frac{di_{dm}}{dt} + \frac{i_{qm}}{i_m}\frac{di_{qm}}{dt} = \cos\alpha\frac{di_{ds}}{dt} + \sin\alpha\frac{di_{qs}}{dt} + \cos\alpha\frac{di_{dr}}{dt} + \sin\alpha\frac{di_{qr}}{dt} + \cos\alpha\frac{di_f}{dt} \quad (26)$$

Argument α is the angular position of space vector $\bar{i}_m$ (or $\bar{\lambda}_m$) with respects to d-axis so that:

$$\alpha = \tan^{-1}\left(\frac{i_{qm}}{i_{dm}}\right) \quad (27)$$

Finally, we have:

$$\frac{d\lambda_{ds}}{dt} = (l_s + L_{d1})\frac{di_{ds}}{dt} + L_{dq1}\frac{di_{qs}}{dt} + L_{d1}\frac{di_{dr}}{dt} + L_{dq1}\frac{di_{qr}}{dt} + L_{d1}\frac{di_f}{dt} \quad (28)$$

$$\frac{d\lambda_{qs}}{dt} = L_{dq1}\frac{di_{ds}}{dt} + (l_s + L_{q1})\frac{di_{qs}}{dt} + L_{dq1}\frac{di_{dr}}{dt} + L_{q1}\frac{di_{qr}}{dt} + L_{dq1}\frac{di_f}{dt} \quad (29)$$

$$\frac{d\lambda_{dr}}{dt} = L_{d1}\frac{di_{ds}}{dt} + L_{dq1}\frac{di_{qs}}{dt} + (l_r + L_{d1})\frac{di_{dr}}{dt} + L_{dq1}\frac{di_{qr}}{dt} + L_{d1}\frac{di_f}{dt} \quad (30)$$

$$\frac{d\lambda_{qr}}{dt} = L_{dq1}\frac{di_{ds}}{dt} + L_{q1}\frac{di_{qs}}{dt} + L_{dq1}\frac{di_{dr}}{dt} + (l_r + L_{q1})\frac{di_{qr}}{dt} + L_{dq1}\frac{di_f}{dt} \quad (31)$$

$$\frac{d\lambda_f}{dt} = L_{d1}\frac{di_{ds}}{dt} + L_{dq1}\frac{di_{qs}}{dt} + L_{d1}\frac{di_{dr}}{dt} + L_{dq1}\frac{di_{qr}}{dt} + (l_f + L_{d1})\frac{di_f}{dt} \quad (32)$$

where

$$L_{dq1} = (L_{mdy} - L_m)\cos\alpha\sin\alpha \quad (33)$$

$$L_{d1} = L_m + L_{dq1}\cot\alpha, \qquad L_{q1} = L_m + L_{dq1}\tan\alpha \quad (34)$$

Parameters L_m and L_{mdy} stand for the static and dynamic magnetizing inductances, calculated after approximating by a mathematical function the well-known saturation curve $\lambda_m(i_m)$. Using the matrix form $[v] = [A][\dot{X}] + [B][X]$, with $[X]$ vector formed by d–q components of the winding currents and $[\dot{X}]$ its iime derivative, matrices $[A]$ and $[B]$ are:

$$[A] = \begin{bmatrix} l_s + L_{d1} & L_{dq1} & L_{d1} & L_{dq1} & L_{d1} \\ L_{dq1} & l_s + L_{q1} & L_{dq1} & L_{q1} & L_{dq1} \\ L_{d1} & L_{dq1} & l_r + L_{d1} & L_{dq1} & L_{d1} \\ L_{dq1} & L_{q1} & L_{dq1} & l_r + L_{q1} & L_{dq1} \\ L_{d1} & L_{dq1} & L_{d1} & L_{dq1} & l_f + L_{d1} \end{bmatrix} \quad (35)$$

$$[B] = \begin{bmatrix} R_s & -wL_s & 0 & -wL_m & 0 \\ wL_s & R_s & wL_m & 0 & wL_m \\ 0 & 0 & R_r & 0 & 0 \\ 0 & 0 & 0 & R_r & 0 \\ 0 & 0 & 0 & 0 & R_f \end{bmatrix} \quad (36)$$

It is clear that the winding currents model, which is the most known and used, is the heaviest one to compute. All 25 elements of its matrix $[A]$ are present and saturation dependent. Also, it contains all kinds of magnetic couplings along the d-axis (L_{d1}), the q-axis (L_{q1}), and the d–q axis (L_{dq1}). On the contrary, the winding currents model, not therefore advised, is exploited here to derive any other saturation model whatever the state-space variables.

The proposed approach consists of three stages. First, a combination of state-space vectors is chosen among the 63 remaining possibilities. Second, the d–q components of linkage fluxes and winding currents are described in terms of these selected variables using (13)–(17) and (20)–(23). Third, by ordinary manipulations of (28)–(32) time derivatives of the d–q components of the linkage fluxes are written as functions of the chosen variables.

6. Derivation of possible models

Obviously, it is not possible to report in one paper all results related to the remaining 63 models. In return, special interest will be done to novel models or to models cited in the literature but cannot be developed by previous works [18].

6.1. Derivation of novel models

As explained before, with three state-space vectors other than i_f or λ_f, we can form three different models. For instance, let us take the case $(\bar{i}_s, \bar{i}_r, \bar{i}_m)$ which generates the following three independent models $(i_{ds}, \bar{i}_r, \bar{i}_m)$, $(\bar{i}_s, i_{dr}, \bar{i}_m)$ and $(\bar{i}_s, \bar{i}_r, i_{dm})$. By applying the method steps explained in Section 5, these kinds of current models where $\bar{i}_m$ is present can be obtained in a straightforward way.

Among the three cited models, the $(\bar{i}_s, \bar{i}_r, i_{dm})$ one is the easiest to derive from the winding currents model $(\bar{i}_s, \bar{i}_r, i_f)$ taken, of course, as a basis for the presented method. Indeed, since $[X] = \left[i_{ds}, i_{qs}, i_{dr}, i_{qr}, i_{dm}\right]^t$, in the primitive d–q equations only i_f has to be eliminated. For this purpose, Eq. (16) plays an important role. It permits the description of i_f in terms of the selected variables so that:

$$i_f = i_{dm} - i_{ds} - i_{dr} \quad (37)$$

Next, derivative of (37) when introduced in (28)–(32) gives immediately matrix $[A]$ with, in addition, some extra simplifications. Concerning the stator flux components, λ_{qs} is well described in (20) and λ_{ds} becomes

$$\lambda_{ds} = l_s i_{ds} + L_m i_{dm} \quad (38)$$

The resulting matrices [A] and [B] are:

$$[A]=\begin{bmatrix} l_s & L_{dq1} & 0 & L_{dq1} & L_{d1} \\ 0 & l_s+L_{q1} & 0 & L_{q1} & L_{dq1} \\ 0 & L_{dq1} & l_r & L_{dq1} & L_{d1} \\ 0 & L_{q1} & 0 & l_r+L_{q1} & L_{dq1} \\ -l_f & L_{dq1} & -l_f & L_{dq1} & l_f+L_{d1} \end{bmatrix} \tag{39}$$

$$[B]=\begin{bmatrix} R_s & -wL_s & 0 & -wL_m & 0 \\ wl_s & R_s & 0 & 0 & wL_m \\ 0 & 0 & R_r & 0 & 0 \\ 0 & 0 & 0 & R_r & 0 \\ -R_f & 0 & -R_f & 0 & R_f \end{bmatrix} \tag{40}$$

Following similar steps for $(i_{ds}, \bar{i}_r, \bar{i}_m)$ where $[X]=\left[i_{ds}, i_{dr}, i_{qr}, i_{dm}, i_{qm}\right]^t$, we get:

$$[A]=\begin{bmatrix} l_s & 0 & 0 & L_{d1} & L_{dq1} \\ 0 & 0 & -l_s & L_{dq1} & l_s+L_{q1} \\ 0 & l_r & 0 & L_{d1} & L_{dq1} \\ 0 & 0 & l_r & L_{dq1} & L_{q1} \\ -l_f & -l_f & 0 & l_f+L_{d1} & L_{dq1} \end{bmatrix} \tag{41}$$

$$[B]=\begin{bmatrix} R_s & 0 & wl_s & 0 & -wL_s \\ wl_s & 0 & -R_s & wL_m & R_s \\ 0 & R_r & 0 & 0 & 0 \\ 0 & 0 & R_r & 0 & 0 \\ -R_f & -R_f & 0 & R_f & 0 \end{bmatrix} \tag{42}$$

Unfortunately, because of the limited space the last possible model $(\bar{i}_s, i_{dr}, \bar{i}_m)$ is not reported here.

6.2. *Derivation of models can't be developed in previous works*

One of the principal objects of the paper is to show the ability of the proposed approach to derive any possible model with magnetic saturation; especially those cited but cannot be developed in previous works like the $(\bar{i}_s, \bar{i}_r, \lambda_f)$ model [18]. Curiously, this model not obtained in [18], although using a very interesting method, is going to be derived here without difficulty.

Relation (15) gives

$$i_f=-\frac{L_m}{L_f}i_{ds}-\frac{L_m}{L_f}i_{dr}+\frac{1}{L_f}\lambda_f \tag{43}$$

To form matrix [B], i_f is substituted by (43) just in (12) and (13). In a similar manner, matrix [A] is formed by substituting to $\frac{di_f}{dt}$ in (28)–(31) its new expression drawn from (32). Hence:

$$[X]=\left[i_{ds}, i_{qs}, i_{dr}, i_{qr}, \lambda_f\right]^t \tag{44}$$

$$[A]=\begin{bmatrix} l_s+L_{d2} & L_{dq2} & L_{d2} & L_{dq2} & \frac{L_{d2}}{l_f} \\ L_{dq2} & l_s+L_{q2} & L_{dq2} & L_{q2} & \frac{L_{dq2}}{l_f} \\ L_{d2} & L_{dq2} & l_r+L_{d2} & L_{dq2} & \frac{L_{d2}}{l_f} \\ L_{dq2} & L_{q2} & L_{dq2} & l_r+L_{q2} & \frac{L_{dq2}}{l_f} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{45}$$

$$[B]=\begin{bmatrix} R_s & -wL_s & 0 & -wL_m & 0 \\ w(l_s+l_f\frac{L_m}{L_f}) & R_s & wl_f\frac{L_m}{L_f} & 0 & w\frac{L_m}{L_f} \\ 0 & 0 & R_r & 0 & 0 \\ 0 & 0 & 0 & R_r & 0 \\ -R_f\frac{L_m}{L_f} & 0 & -R_f\frac{L_m}{L_f} & 0 & \frac{R_f}{L_f} \end{bmatrix} \tag{46}$$

$$L_{dq2}=l_f\frac{(L_{mdy}-L_m)\cos\alpha\sin\ \alpha}{L_f+(L_{mdy}-L_m)\cos\alpha^2} \tag{47}$$

$$L_{d2}=l_f\frac{L_m}{L_f}+\frac{l_f}{L_f}L_{dq2}\cot\alpha, \qquad L_{q2}=L_m+\frac{L_f}{l_f}L_{dq2}\tan\alpha \tag{48}$$

7. Application

The typical example of transients following the excitation of an isolated alternator is performed here. The aim of such application is to verify the equivalence between models and the influence of magnetic saturation. Generally, build-up of phase voltages is established at no load, but in this case all stator currents are null and the set of main equations to be solved is reduced to (2) and (3). In order to make a complete verification of the developed models, a load is added and the stator equations are implied. The alternator, which parameters are given at the end of the section, is supposed driven at rated speed. Fig. 1 illustrates the build-up process of field current as well as terminal voltage and current of phase (a) for almost a half-loaded machine. Fig. 2 shows the stator phase voltage with and without magnetic saturation, in same conditions of excitation and load.

Moreover, Figs. 1 and 2 can be obtained using any of the 64 found models, especially with those fully developed in this paper. It is to be noted that there was no need to report results separately for each model because they gave exactly the same ones.

The generator per unit parameters are: $R_s=0.003$, $R_r=0.01334$, $R_f=0.000927$, $l_s=0.19$, $l_r=0.08129$, $l_f=0.1415$, $w=120\pi$.

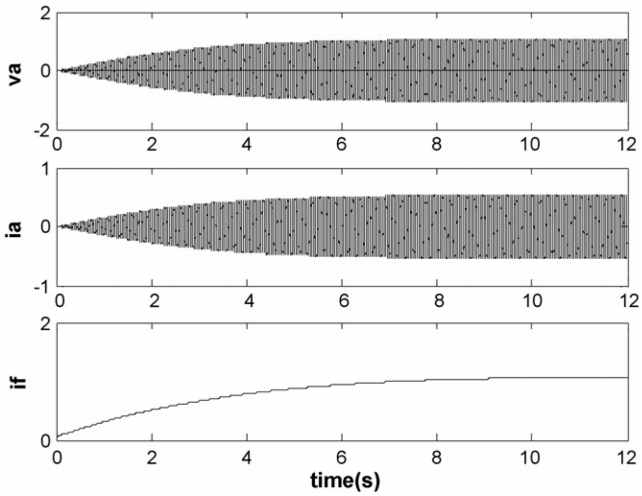

Fig. 1. Build-up of winding voltage, winding current and excitation current for an isolated alternator with 50% load and rated speed.

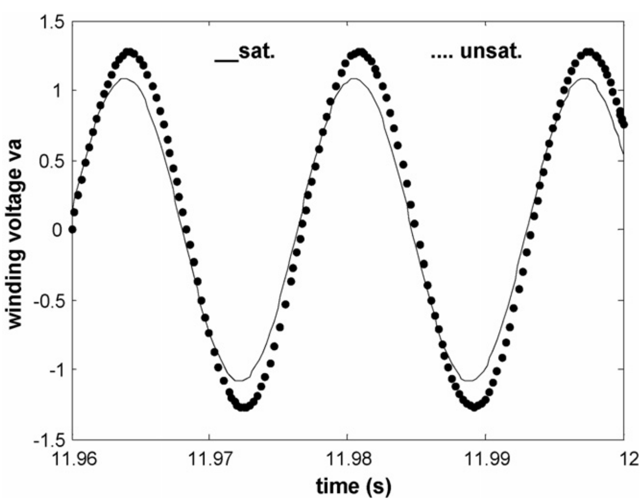

Fig. 2. Alternator winding voltage V_a with and without magnetic saturation for same conditions as in Fig. 1.

Magnetizing curve description: if $i_m \leq 0.484$ then $\lambda_m = 1.645 i_m$, if $i_m \geq 0.742$ then $\lambda_m = \frac{3.7393 i_m}{1+2.277 i_m}$, else $\lambda_m = \frac{2.5077 i_m}{1+1.0832 i_m}$.

8. Discussion

After numbering and classifying the possible models for synchronous machines, a procedure of saturation modeling is exposed. It relies on the knowledge of the well established and already available winding currents model including magnetic saturation. The mathematical developments indicate that, for each model, system matrix [A] is mainly formed by three inductances defined as L_d, L_q and L_{dq}. These parameters may vary from one model to another as in (33), (34), (47) and (48). They represent respectively the magnetic couplings along the d axis, the q-axis and between the d–q axes. As axes d and q are orthogonal, the cross-coupling coefficient L_{dq} appears only whether saturation is accounted for. This is verified when linear case is assumed and $L_{mdy} = L_m$. Moreover, the quantities L_d and L_q are written each as the sum of a main term which exists with or without saturation and an additional one proportional to the cross-coupling coefficient and thus disappears with it (34) and (48). In general, the main terms of L_d and L_q are identical for a given model, like in (34). Unfortunately, this is not the case of the models, object of the Section 6.2, where the relations expressing L_d and L_q are characterized by a specific asymmetry (48), even if saturation is neglected. This fact is accompanied by a supplementary complexity in expressing saturation factors L_{dq} (47), compared to the remaining models. Such asymmetry and resulting complexity could be the major reason for which the procedure used in [18] was unsuccessful with this kind of models.

It has to be emphasized that the accuracy of saturation representation in all 64 models is the same. Otherwise, all found models have exactly the same ability to describe any transient regime. This is well confirmed by the simulation study in the preceding section where all the saturated machine models yield the same results. Models differ only by the structure of their characteristic matrices, due to the change of state-space variables.

Generally, the simulation time longevity is mainly related to the shape of system matrix [A]. More it contains zeros more the simulation is short. At this basis the linkage fluxes model, which matrix [A] reduces to identity, is strongly advised. Note that this model is the unique one that does not imply any magnetic coupling between the orthogonal axes and therefore not concerned by the so-called “cross-saturation” phenemenon. In the opposite, the winding currents model is the heaviest to handle although it is historically the most known and widely used. All its matrix [A] elements are non-null and depend on magnetic saturation (35).

Logically, between these two limits are situated the remaining models. Specially interested by this point, the simulation performances which is not the main object of this paper, may find extra ligths in [19] where it is demonstrated that, for a particular class of mixed models cross-coupling terms may be omitted without significant loss of accuracy. Such simplifications lead obviously to lighter system matrices and consequently a reduction in simulation time.

9. Conclusion

The number of models based on the state-space variables choice, of a saturated round rotor synchronous machine with basic dampers, is reviewed and found to be 64. A simple method consisting of elaborating just the winding currents model, with magnetic saturation, and deriving all the other models from it, is presented. In this study special interest was particularly focused on either novel models or existing models cited in the literature but cannot be obtained by other approaches. In all cases, if the differential equations of the machine are formulated in terms of a set of variables other than the winding currents, a noticeable reduction in the size of equations may be obtained and consequently less time computing.

Although the study was restricted to the round rotor synchronous machine in the present paper, the proposed method is identically applicable to other known ac machines. It can be easily extended to induction machines. Additionally, it can be equally applied to salient pole synchronous machines, especially if magnetic saturation is introduced by means of a single saturation factor which permits conversion of the anisotropic machine to an isotropic one.

The approach seems to be able to derive any possible model whatever the state-space variables and the type of the ac machine and hence can be classified as a general approach.

Appendix A. List of symbols

d, q	indices for direct and quadrature axis
$\bar{i}$	space vector winding current
$\bar{i}_m$	space vector magnetizing current
l	winding leakage inductance
L	winding self-inductance
L_m	static magnetizing inductance
L_{mdy}	dynamic magnetizing inductance
R	winding resistance
s, r	indices for stator and rotor, respectively
$\bar{v}$	space vector winding voltage
w_a	frame speed
w_e	actual rotor electrical speed
w'	difference between frame and rotor speeds ($w' = w_a - w_e$).
$\bar{\lambda}$	space vector winding flux
$\bar{\lambda}_m$	space vector magnetizing flux

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