A Simple Adaptive Control Approach for Trajectory Tracking of Electrically Driven Nonholonomic Mobile Robots

Bong Seok Park, Sung Jin Yoo, Jin Bae Park, and Yoon Ho Choi

***Abstract*—Almost all existing controllers for nonholonomic mobile robots are designed without considering the actuator dynamics. This is because the presence of the actuator dynamics increases the complexity of the system dynamics, and makes difficult the design of the controller. In this paper, we propose a simple adaptive control approach for path tracking of uncertain nonholonomic mobile robots incorporating actuator dynamics. All parameters of robot kinematics, robot dynamics, and actuator dynamics are assumed to be uncertain. For the simple controller design, the dynamic surface control methodology is applied and extended to mobile robots that the number of inputs and outputs is different. We also adopt the adaptive control technique to treat all uncertainties and derive adaptation laws from the Lyapunov stability theory. Finally, simulation results demonstrate the effectiveness of the proposed controller.**

***Index Terms*—Actuator dynamics, adaptive control, dynamic surface design, nonholonomic mobile robots, robot dynamics, robot kinematics.**

I. INTRODUCTION

OVER THE past 20 years, the control of mobile robots has been regarded as the attractive problem due to the nature of nonholonomic constraints. Many efforts have been devoted to the tracking control of nonholonomic mobile robots [1]–[8]. Most of these schemes have ignored the dynamics coming from electric motors which should be required to implement the mobile robots in the real environment, that is, the mobile robot model at the kinematics level or at the dynamics level has been only considered. It has been well-known that the actuator dynamics is an important part for the design of the complete robot dynamics, especially in the case of high-velocity movement and highly varying loads [9], [10]. Thus, some results were reported for mobile robots incorporating the actuator dynamics [11]–[15]. However, these works do not consider all parametric uncertainties for mobile robots at the actuator level, that is, the uncertainties of the robot dynamics were only considered in [11], [12], the uncertainties of the robot dynamics and the actuator dynamics were only considered in [13], and in [14], [15], any uncertainty was not considered. The controller design problem would become extremely difficult as the complexity of the system dynamics increases and when the mobile robot model includes the uncertainties of the actuator dynamics as well as the uncertainties of the robot kinematics and dynamics. In this paper, we would like to present the simple solution to this challenging problem.

The backstepping technique has been widely used as one of the representative methods for controlling nonholonomic mobile robots at the dynamics level or at the actuator level [13]–[21]. However, the backstepping design procedure has the "explosion of complexity" problem caused by the repeated differentiations of virtual controllers. That is, the complexity of the controller grows drastically as the order n of the system increases. When the model of the electrically driven mobile robots is considered, this problem of the backstepping design would become more serious due to the increase of the controller design procedure. Swaroop *et al.* [22] proposed a dynamic surface control (DSC) technique to solve this problem by introducing a first-order filtering of the synthesized virtual control law at each step of the backstepping design procedure. The DSC idea was extended to uncertain single-input single-output [23], [24] and multi-input multi-output systems [25]. Despite these efforts using the DSC technique, the DSC method is still not applied to mobile robots that have more degrees-of-freedom (DOFs) than the number of inputs under nonholonomic constraints.

Accordingly, we propose a simple adaptive controller for path tracking of uncertain nonholonomic mobile robots incorporating actuator dynamics. It is assumed that all parameters of robot kinematics and dynamics including actuator dynamics, and the external disturbances are unknown. For the simple control system design, we apply the DSC technique to electrically driven nonholonomic mobile robots, which have more DOFs than the number of inputs under nonholonomic constraints. All parametric uncertainties and external disturbances are compensated by the adaptive technique. In addition, the simplified parameter estimation technique is presented to reduce the number of tuning parameters. Based on the Lyapunov stability theorem, we also prove that all of the signals in the closed-loop system are semi-globally uniformly ultimately bounded and the tracking errors can be made arbitrarily small by adjusting the design parameters.

II. PROBLEM STATEMENT

We consider a mobile robot with two actuated wheels as shown in Fig. 1. The kinematics and dynamics of nonholo-

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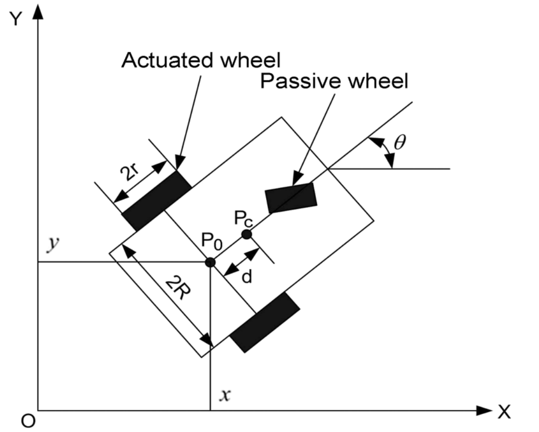

Fig. 1. Mobile robot with two actuated wheels.

nomic mobile robots are described by the following differential equations [21]:

$$\dot{q} = J(q)z = 0.5r \begin{bmatrix} \cos\theta & \cos\theta \\ \sin\theta & \sin\theta \\ R^{-1} & -R^{-1} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad (1)$$

$$\tau = M\dot{z} + C(\dot{q})z + Dz + \tau_d \quad (2)$$

where $q = [x, y, \theta]^T \in \mathbb{R}^3$; x, y are the coordinates of P_0, and θ is the heading angle of the mobile robot, $z = [v_1, v_2]^T \in \mathbb{R}^2$; v_1 and v_2 represent the angular velocities of right and left wheels. R is the half of the width of the mobile robot and r is the radius of the wheel

$$\begin{aligned} M &= \begin{bmatrix} m_{11} & m_{12} \\ m_{12} & m_{11} \end{bmatrix} \\ C(\dot{q}) &= 0.5R^{-1}r^2 m_c d \begin{bmatrix} 0 & \dot{\theta} \\ -\dot{\theta} & 0 \end{bmatrix} \\ D &= \begin{bmatrix} d_{11} & 0 \\ 0 & d_{22} \end{bmatrix} \\ m_{11} &= 0.25R^{-2}r^2(mR^2 + I) + I_w \\ m_{12} &= 0.25R^{-2}r^2(mR^2 - I) \\ m &= m_c + 2m_w \\ I &= m_c d^2 + 2m_w R^2 + I_c + 2I_m \\ \tau &= [\tau_1, \tau_2]^T \\ \tau_d &= [\tau_{d1}, \tau_{d2}]^T. \end{aligned}$$

In these expressions, d is the distance from the center of mass P_c of the mobile robot to the middle point P_0 between the right and left driving wheels. m_c and m_w are the mass of the body and wheel with a motor, respectively. I_c, I_w, and I_m are the moment of inertia of the body about the vertical axis through P_c, the wheel with a motor about the wheel axis, and the wheel with a motor about the wheel diameter, respectively. The positive terms d_{ii}, $i = 1, 2$, are the damping coefficients. $\tau \in \mathbb{R}^2$ is the control torque applied to the wheels of the robot. $\tau_d \in \mathbb{R}^2$ is a vector of disturbances including unmodeled dynamics.

Property: [26] The inertia matrix M is symmetric and positive definite.

Assumption 1: The disturbances are bounded so that $|\tau_{di}| \le d_{mi}$, $i = 1, 2$.

In addition, the dynamic model of dc motors can be represented as follows [11]:

$$\begin{cases} \tau_m = K_T i_a, \\ u = R_a i_a + L_a \dot{i}_a + K_E \dot{\theta}_m \end{cases} \quad (3)$$

where $\tau_m = [r_{m_1}, r_{m_2}]^T$ is the torque generated by dc motor, $K_T = \mathrm{diag}[k_{t_1}, k_{t_2}]$ is the motor torque constant, $i_a \in \mathbb{R}^2$ is the current, $u \in \mathbb{R}^2$ is the input voltage, $R_a = \mathrm{diag}[r_{a_1}, r_{a_2}]$ is the resistance, $L_a = \mathrm{diag}[l_{a_1}, l_{a_2}]$ is the inductance, $K_E = \mathrm{diag}[k_{e_1}, k_{e_2}]$ is the back electromotive force coefficient, and $\dot{\theta}_m = [\dot{\theta}_{m_1}, \dot{\theta}_{m_2}]^T$ is the angular velocity of the dc motor. Here, $\mathrm{diag}[\cdot]$ denotes the diagonal matrix.

The relationship between the dc motor and the mobile robot wheel can be written as

$$n_j = \frac{\dot{\theta}_{m_j}}{v_j} = \frac{\tau_j}{\tau_{m_j}} \quad (4)$$

where n_j, $j = 1, 2$, is the gear ratio. Using (4), the dynamic model of dc motors (3) can be rewritten as

$$\begin{cases} \tau = NK_T i_a \\ u = R_a i_a + L_a \dot{i}_a + NK_E z \end{cases} \quad (5)$$

where $N = \mathrm{diag}[n_1, n_2]$.

Assumption 2: All parameters of robot kinematics (1), robot dynamics (2), and actuator dynamics (5) are constants but unknown, and lie in a compact set.

Let us define the state variables as $x_1 = q$, $x_2 = z$, and $x_3 = i_a$. Then, (1), (2), and (5) can be expressed in the following state-space form:

$$\dot{x}_1 = J(x_1)x_2 \quad (6)$$

$$\dot{x}_2 = M^{-1}\left(-C(\dot{x}_1)x_2 - Dx_2 - \tau_d + NK_T x_3\right) \quad (7)$$

$$\dot{x}_3 = L_a^{-1}(u - R_a x_3 - NK_E x_2) \quad (8)$$

where $x_1 = [x_{11}, x_{12}, x_{13}]^T$, $x_2 = [x_{21}, x_{22}]^T$, and $x_3 = [x_{31}, x_{32}]^T$.

The control objective is to design a simple adaptive control law u for electrically driven nonholonomic mobile robots (6)–(8) to track the desired trajectory generated by the following reference robot:

$$\begin{cases} \dot{x}_r = v_r \cos\theta_r \\ \dot{y}_r = v_r \sin\theta_r \\ \dot{\theta}_r = \omega_r \end{cases} \quad (9)$$

where x_r, y_r, and θ_r are the position and orientation of the reference robot. v_r and ω_r are the linear and angular velocities of the reference robot, respectively.

Assumption 3: The reference signal $z_r = [v_r, \omega_r]^T$ is bounded, and $v_r > 0$.

Remark 1: In Assumption 3, $v_r > 0$ means that this paper only focuses on a simple controller design for *the trajectory tracking problem* of mobile robots incorporating actuator dynamics. That is, the case of $v_r = 0$ is not considered.

III. Main Results

A. Adaptive Controller Design

In this section, we develop a simple control system for electrically driven nonholonomic mobile robots. To design the adaptive control system using the DSC technique, we proceed step by step.

Step 1: Consider the robot kinematics (6). The first error surface is defined as follows:

$$\begin{bmatrix} S_{11} \\ S_{12} \\ \bar{S}_{13} \end{bmatrix} = \begin{bmatrix} \cos x_{13} & \sin x_{13} & 0 \\ -\sin x_{13} & \cos x_{13} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x_{11} \\ y_r - x_{12} \\ \theta_r - x_{13} \end{bmatrix}. \tag{10}$$

Differentiating (10) yields

$$\begin{aligned} \dot{S}_{11} &= \frac{r}{2R}(x_{21} - x_{22})S_{12} - \frac{r}{2}(x_{21} + x_{22}) + v_r \cos \bar{S}_{13} \\ \dot{S}_{12} &= -\frac{r}{2R}(x_{21} - x_{22})S_{11} + v_r \sin \bar{S}_{13} \\ \dot{\bar{S}}_{13} &= \omega_r - \frac{r}{2R}(x_{21} - x_{22}). \end{aligned} \tag{11}$$

In the tracking error model (11), S_{12} cannot be directly controlled. To overcome this problem, we introduce the error variable based on [27] as follows:

$$S_{13} = \bar{S}_{13} + \arctan(k_1 S_{12} v_r) \tag{12}$$

where k_1 is a positive constant. Using (12), $\dot{\bar{S}}_{13}$ in (11) is transformed into

$$\dot{S}_{13} = \omega_r - \frac{r(x_{21} - x_{22})}{2R}\left(1 + \frac{k_1 v_r S_{11}}{1 + (k_1 S_{12} v_r)^2}\right) + \alpha_1 \tag{13}$$

where $\alpha_1 = (k_1 v_r^2 \sin(\bar{S}_{13}) + k_1 S_{12} \dot{v}_r)/(1 + (k_1 S_{12} v_r)^2)$.

Choose the virtual control law $\bar{x}_2$ as follows:

$$\bar{x}_2 = [\bar{x}_{21}, \bar{x}_{22}]^T = [h_1 + h_2, h_1 - h_2]^T \tag{14}$$

where

$$\begin{aligned} h_1 &= \hat{a}_1 v_r \cos(\bar{S}_{13}) + k_2 S_{11} + \frac{1}{2}(k_1 v_r)^4 S_{11}^3 \\ h_2 &= \left(1 + \frac{k_1 v_r S_{11}}{1 + (k_1 S_{12} v_r)^2}\right)^{-1} \\ &\quad \times \left(\hat{a}_1 \alpha_2 v_r S_{12} + \hat{a}_2(\omega_r + \alpha_1) + k_3 S_{13} + \frac{1}{2} S_{13}^3\right) \\ \alpha_2 &= \int_0^1 \cos\left(-\arctan(k_1 S_{12} v_r) + \eta S_{13}\right) d\eta \end{aligned}$$

in which $a_1 = 1/r$, $a_2 = R/r$, k_2, and k_3 are positive constants. $\hat{a}_i$ is the estimate of a_i, $i = 1, 2$. $\hat{a}_1$ and $\hat{a}_2$ are updated as follows:

$$\begin{aligned} \dot{\hat{a}}_1 &= \gamma_1\left(v_r S_{11} \cos(\bar{S}_{13}) + \alpha_2 v_r S_{12} S_{13}\right) - \sigma_1 \gamma_1 \hat{a}_1 \\ \dot{\hat{a}}_2 &= \gamma_2(\omega_r + \alpha_1) S_{13} - \sigma_2 \gamma_2 \hat{a}_2 \end{aligned} \tag{15}$$

with the initial estimates $\hat{a}_1(0) = \hat{a}_2(0) = 0$, the tuning gains $\gamma_1, \gamma_2 > 0$, and small gains $\sigma_1, \sigma_2 > 0$ for the σ-modification [28]. Then, to obtain the filtered virtual control $x_{2f} = [x_{2f_1}, x_{2f_2}]^T$, we pass $\bar{x}_2$ through the first-order filter

$$\tau_2 \dot{x}_{2f} + x_{2f} = \bar{x}_2, \quad x_{2f}(0) = \bar{x}_2(0) \tag{16}$$

with a time constant $\tau_2 > 0$.

Step 2: Consider the robot dynamics (7). Define the second error surface S_2 as

$$S_2 = x_2 - x_{2f}. \tag{17}$$

Then its derivative is

$$\begin{aligned} \dot{S}_2 &= \dot{x}_2 - \dot{x}_{2f} \\ &= M^{-1}\left(-C(\dot{x}_1)x_2 - Dx_2 - \tau_d + NK_T x_3\right) - \dot{x}_{2f} \\ &\le M^{-1} NK_T(\Phi_1 W_1 + x_3). \end{aligned} \tag{18}$$

Here Φ_1 and W_1 are defined in (19) as shown at the bottom of the page.

Choose the virtual control law $\bar{x}_3 = [\bar{x}_{31}, \bar{x}_{32}]^T$ as follows:

$$\bar{x}_3 = -k_4 S_2 - \frac{\hat{a}_3 \Phi_1 \Phi_1^T S_2}{2\delta_1^2} \tag{21}$$

$$\begin{aligned} \Phi_1 &= \begin{bmatrix} -(x_{21} - x_{22})x_{22} & 0 & -x_{21} & 0 & -\dot{x}_{2f_1} & -\dot{x}_{2f_2} & 0 & 0 & 1 & 0 \\ 0 & (x_{21} - x_{22})x_{21} & 0 & -x_{22} & 0 & 0 & -\dot{x}_{2f_2} & -\dot{x}_{2f_1} & 0 & 1 \end{bmatrix} \\ W_1 &= \left[\frac{r^3 m_c d}{4R^2 n_1 k_{t_1}}, \frac{r^3 m_c d}{4R^2 n_2 k_{t_2}}, \frac{d_{11}}{n_1 k_{t_1}}, \frac{d_{22}}{n_2 k_{t_2}}, \frac{m_{11}}{n_1 k_{t_1}}, \frac{m_{12}}{n_1 k_{t_1}}, \frac{m_{11}}{n_2 k_{t_2}}, \frac{m_{12}}{n_2 k_{t_2}}, \frac{d_{m1}}{n_1 k_{t_1}}, \frac{d_{m2}}{n_2 k_{t_2}}\right]^T \\ \dot{x}_{2f_1} &= \left(\bar{x}_{21} - x_{2f_1}\right)/\tau_2, \ \dot{x}_{2f_2} = \left(\bar{x}_{22} - x_{2f_2}\right)/\tau_2. \end{aligned} \tag{19}$$

$$\begin{aligned} \Phi_2 &= \begin{bmatrix} -x_{31} & 0 & -x_{21} & 0 & -\dot{x}_{3f_1} & 0 \\ 0 & -x_{32} & 0 & -x_{22} & 0 & -\dot{x}_{3f_2} \end{bmatrix} \\ W_2 &= [r_{a_1}, r_{a_2}, n_1 k_{e_1}, n_2 k_{e_2}, l_{a_1}, l_{a_2}]^T \ \dot{x}_{3f_1} = \left(\bar{x}_{31} - x_{3f_1}\right)/\tau_3, \ \dot{x}_{3f_2} = \left(\bar{x}_{32} - x_{3f_2}\right)/\tau_3 \end{aligned} \tag{20}$$

where k_4 and δ_1 are positive constants, $\hat{a}_3$ is the estimate of the unknown parameter $a_3 = \|W_1\|^2$ and it is updated by

$$\dot{\hat{a}}_3 = \gamma_3 \frac{S_2^T \Phi_1 \Phi_1^T S_2}{2\delta_1^2} - \sigma_3 \gamma_3 \hat{a}_3 \tag{22}$$

with the initial estimates $\hat{a}_3(0) = 0$, the tuning gain $\gamma_3 > 0$, and the small gain $\sigma_3 > 0$. Then, $\bar{x}_3$ is passed through the first-order filter with a time constant $\tau_3 > 0$ to obtain $x_{3f} = [x_{3f_1}, x_{3f_2}]^T$

$$\tau_3 \dot{x}_{3f} + x_{3f} = \bar{x}_3, \quad x_{3f}(0) = \bar{x}_3(0). \tag{23}$$

Step 3: Consider the actuator dynamics (8). To design the actual control input law u, we define the third error surface S_3 as

$$S_3 = x_3 - x_{3f}. \tag{24}$$

The time derivative of S_3 is given by

$$\begin{aligned}\dot{S}_3 &= \dot{x}_3 - \dot{x}_{3f} \\ &= L_a^{-1}(u - R_a x_3 - N K_E x_2) - \dot{x}_{3f} \\ &= L_a^{-1}(u + \Phi_2 W_2)\end{aligned} \tag{25}$$

where $\Phi_2 W_2 = -R_a x_3 - N K_E x_2 - L_a \dot{x}_{3f}$. Here, Φ_2 and W_2 are defined in (20) shown at the bottom of the previous page.

We choose the actual control law u as follows:

$$u = -k_5 S_3 - \frac{\hat{a}_4 \Phi_2 \Phi_2^T S_3}{2\delta_2^2} \tag{26}$$

where k_5 and δ_2 are positive constants, $\hat{a}_4$ is the estimate of the unknown parameter $a_4 = \|W_2\|^2$, and is updated by

$$\dot{\hat{a}}_4 = \gamma_4 \frac{S_3^T \Phi_2 \Phi_2^T S_3}{2\delta_2^2} - \sigma_4 \gamma_4 \hat{a}_4 \tag{27}$$

with the initial estimates $\hat{a}_4(0) = 0$, the tuning gain $\gamma_4 > 0$, and the small gain $\sigma_4 > 0$.

Remark 2: There are totally 16 unknown parameters in W_1 and W_2. Estimating all these parameters may cause the degradation of the system performance and it is difficult to choose the tuning gains for these parameters due to the computational complexity. Therefore, we present the simplified parameter estimation technique to reduce the number of tuning parameters. That is, although the robot dynamics (7) and the actuator dynamics (8) includes many unknown parameters, the proposed controller (14), (21), and (26) requires only four tuning parameters.

Remark 3: Compared with the previous works [13]–[21] based on the backstepping technique, the proposed controller for nonholonomic mobile robots can overcome the "explosion of complexity" problem by using the first-order filters. That is, the proposed approach does not require the repeated derivatives of virtual controllers because they are computed easily by the first-order filter at each step of the controller design procedure (i.e., $\dot{x}_{2f_i} = (\bar{x}_{2i} - x_{2f_i})/\tau_2$ and $\dot{x}_{3f_i} = (\bar{x}_{3i} - x_{3f_i})/\tau_3$, $i = 1, 2$). This merit is more efficient when the dynamics of the mobile robot is extended to the actuator level. Thus, the proposed adaptive controller based on the DSC technique can be simpler than the adaptive backstepping controller reported in [13]–[21].

B. Stability Analysis

In this section, we show that all signals of the proposed control system are semi-globally uniformly ultimately bounded.

Define the boundary layer errors as

$$y_2 = x_{2f} - \bar{x}_2 \tag{28}$$

$$y_3 = x_{3f} - \bar{x}_3. \tag{29}$$

Then, the derivative of y_2 and y_3 are

$$\dot{y}_2 = \dot{x}_{2f} - \dot{\bar{x}}_2 = -\frac{y_2}{\tau_2} + \Xi_1(S_1, S_2, y_2, z_r, \hat{a}_1, \hat{a}_2) \tag{30}$$

$$\dot{y}_3 = \dot{x}_{3f} - \dot{\bar{x}}_3 = -\frac{y_3}{\tau_3} + \Xi_2(S_1, S_2, S_3, y_2, y_3, z_r, \hat{a}_3) \tag{31}$$

where $S_1 = [S_{11}, S_{12}, S_{13}]^T$. Here, Ξ_1 and Ξ_2 are defined in (32) as shown at the bottom of the page.

Consider the Lyapunov function candidate as follows:

$$V = V_1 + V_2 \tag{33}$$

where

$$V_1 = \frac{1}{2r} S_{11}^2 + \frac{1}{2r} S_{12}^2 + \frac{R}{2r} S_{13}^2 + \frac{1}{2\gamma_1}\tilde{a}_1^2 + \frac{1}{2\gamma_2}\tilde{a}_2^2 \tag{34}$$

$$\begin{aligned}V_2 = \frac{1}{2} \Big(& S_2^T (M^{-1} N K_T)^{-1} S_2 + S_3^T L_a S_3 + y_2^T y_2 \\ & + y_3^T y_3 + \frac{1}{\gamma_3}\tilde{a}_3^2 + \frac{1}{\gamma_4}\tilde{a}_4^2 \Big)\end{aligned} \tag{35}$$

with the estimation errors $\tilde{a}_j = a_j - \hat{a}_j$, $j = 1, \ldots, 4$.

Theorem 1: Consider the electrically driven nonholonomic mobile robot (6)–(8) with parametric uncertainties and disturbances controlled by the adaptive control law (26). If the proposed control system satisfies Assumptions 1–3 and the un-

$$\begin{aligned}\Xi_1(S_1, S_2, y_2, z_r, \hat{a}_1, \hat{a}_2) &= -\begin{bmatrix} \frac{\partial v}{\partial v_r}\dot{v}_r + \frac{\partial v}{\partial S_1}\dot{S}_1 + \frac{\partial v}{\partial \hat{a}_1}\dot{\hat{a}}_1 + \frac{\partial w}{\partial z_r}\dot{z}_r + \frac{\partial w}{\partial S_1}\dot{S}_1 + \frac{\partial w}{\partial \hat{a}_1}\dot{\hat{a}}_1 + \frac{\partial w}{\partial \hat{a}_2}\dot{\hat{a}}_2 \\ \frac{\partial v}{\partial v_r}\dot{v}_r + \frac{\partial v}{\partial S_1}\dot{S}_1 + \frac{\partial v}{\partial \hat{a}_1}\dot{\hat{a}}_1 - \frac{\partial w}{\partial z_r}\dot{z}_r - \frac{\partial w}{\partial S_1}\dot{S}_1 - \frac{\partial w}{\partial \hat{a}_1}\dot{\hat{a}}_1 - \frac{\partial w}{\partial \hat{a}_2}\dot{\hat{a}}_2 \end{bmatrix} \\ \Xi_2(S_1, S_2, S_3, y_2, y_3, z_r, \hat{a}_3) &= k_4 \dot{S}_2 + \frac{1}{2\delta_2^2}\left(\dot{\hat{a}}_3 \Phi_1 \Phi_1^T S_2 + \hat{a}_3 \dot{\Phi}_1 \Phi_1^T S_2 + \hat{a}_3 \Phi_1 \dot{\Phi}_1^T S_2 + \hat{a}_3 \Phi_1 \Phi_1^T \dot{S}_2\right)\end{aligned} \tag{32}$$

known parameters a_1, a_2, a_3, and a_4 are trained by the adaptation laws (15), (22), and (27), respectively, then for any initial conditions satisfying $V(0) \leq \mu$, where μ is any positive constant, there exists a set of gains $k_1, \ldots, k_5$, τ_{i+1}, γ_j, and σ_j, where $i = 1, 2$ and $j = 1, \ldots, 4$, such that all signals in the closed-loop system are semi-globally uniformly ultimately bounded and the tracking errors can be made arbitrarily small.

Proof: We first consider the Lyapunov function candidate V_1. Noting that $\sin(\bar{S}_{13}) = -\sin(\arctan(k_1 S_{12} v_r)) + S_{13}\alpha_2$, the time derivative of (34) along (11)–(15), (17), and (28) yields

$$\begin{aligned}\dot{V}_1 = & S_{11}\left[-\frac{x_{21}+x_{22}}{2} + a_1 v_r \cos(\bar{S}_{13})\right]\\ &+ S_{13}\left[a_2(\omega_r+\alpha_1) + a_1 v_r \alpha_2 S_{12}\right.\\ &\left.- \left(1+\frac{k_1 v_r S_{11}}{1+(k_1 S_{12} v_r)^2}\right)\left(\frac{x_{21}-x_{22}}{2}\right)\right]\\ &- \frac{1}{r}S_{12}v_r \sin\left(\arctan(k_1 S_{12} v_r)\right) - \frac{\tilde{a}_1\dot{\hat{a}}_1}{\gamma_1} - \frac{\tilde{a}_2\dot{\hat{a}}_2}{\gamma_2}\\ = & -k_2 S_{11}^2 - \frac{1}{r}S_{12}v_r \sin\left(\arctan(k_1 S_{12} v_r)\right) - k_3 S_{13}^2\\ &- \frac{1}{2}(k_1 v_r)^4 S_{11}^4 - \frac{1}{2}S_{13}^4 + \sigma_1\tilde{a}_1\hat{a}_1 + \sigma_2\tilde{a}_2\hat{a}_2\\ &- \left(\frac{S_{21}+S_{22}+y_{21}+y_{22}}{2}\right)S_{11}\\ &- \left(1+\frac{k_1 v_r S_{11}}{1+(k_1 S_{12} v_r)^2}\right)\\ &\times\left(\frac{S_{21}-S_{22}+y_{21}-y_{22}}{2}\right)S_{13}. \quad (36)\end{aligned}$$

Second, consider the Lyapunov function candidate V_2. The time derivative of (35) along (18), (25), (30), and (31) is given by

$$\begin{aligned}\dot{V}_2 \leq & S_2^T(x_3+\Phi_1 W_1) + S_3^T(u+\Phi_2 W_2) - y_2^T\left(\frac{y_2}{\tau_2} - \Xi_1\right)\\ &- y_3^T\left(\frac{y_3}{\tau_3} - \Xi_2\right) - \frac{1}{\gamma_3}\tilde{a}_3\dot{\hat{a}}_3 - \frac{1}{\gamma_4}\tilde{a}_4\dot{\hat{a}}_4\\ \leq & S_2^T\bar{x}_3 + \frac{\left(S_2^T\Phi_1 W_1\right)\left(S_2^T\Phi_1 W_1\right)^T}{2\delta_1^2} + \frac{\delta_1^2}{2} + S_3^T u\\ &+ \frac{\left(S_3^T\Phi_2 W_2\right)\left(S_3^T\Phi_2 W_2\right)^T}{2\delta_2^2} + \frac{\delta_2^2}{2} - \frac{1}{\tau_2}\|y_2\|^2\\ &- \frac{1}{\tau_3}\|y_3\|^2 + S_2^T S_3 + S_2^T y_3 + y_2^T\Xi_1 + y_3^T\Xi_2\\ &- \frac{1}{\gamma_3}\tilde{a}_3\dot{\hat{a}}_3 - \frac{1}{\gamma_4}\tilde{a}_4\dot{\hat{a}}_4\\ \leq & S_2^T\bar{x}_3 + \frac{a_3 S_2^T\Phi_1\Phi_1^T S_2}{2\delta_1^2} + S_3^T u + \frac{a_4 S_3^T\Phi_2\Phi_2^T S_3}{2\delta_2^2}\\ &+ \frac{\delta_1^2}{2} + \frac{\delta_2^2}{2} - \frac{1}{\tau_2}\|y_2\|^2 - \frac{1}{\tau_3}\|y_3\|^2 + S_2^T S_3 + S_2^T y_3\\ &+ y_2^T\Xi_1 + y_3^T\Xi_2 - \frac{1}{\gamma_3}\tilde{a}_3\dot{\hat{a}}_3 - \frac{1}{\gamma_4}\tilde{a}_4\dot{\hat{a}}_4. \quad (37)\end{aligned}$$

Substituting (21), (22), (26), and (27) into (37) yields

$$\begin{aligned}\dot{V}_2 \leq & -k_4\|S_2\|^2 - k_5\|S_3\|^2 - \frac{1}{\tau_2}\|y_2\|^2 - \frac{1}{\tau_3}\|y_3\|^2 + S_2^T S_3\\ &+ S_2^T y_3 + y_2^T\Xi_1 + y_3^T\Xi_2 + \sigma_3\tilde{a}_3\hat{a}_3 + \sigma_4\tilde{a}_4\hat{a}_4 + \frac{\delta_1^2}{2} + \frac{\delta_2^2}{2}. \quad (38)\end{aligned}$$

Finally, consider the Lyapunov function candidate V. Substituting (36) and (38) into the time derivative of (33), we have

$$\begin{aligned}\dot{V} \leq & -k_2 S_{11}^2 - \frac{1}{r}S_{12}v_r\sin\left(\arctan(k_1 S_{12} v_r)\right) - k_3 S_{13}^2\\ &- \frac{1}{2}(k_1 v_r)^4 S_{11}^4 - \frac{1}{2}S_{13}^4 - k_4\|S_2\|^2 - k_5\|S_3\|^2\\ &- \frac{1}{\tau_2}\|y_2\|^2 - \frac{1}{\tau_3}\|y_3\|^2 + \|S_2\|\|S_3\|\\ &+ \|S_2\|\|y_3\| + \|y_2\|\|\Xi_1\| + \|y_3\|\|\Xi_2\| + \sum_{i=1}^{4}\sigma_i\tilde{a}_i\hat{a}_i\\ &+ \frac{\delta_1^2}{2} + \frac{\delta_2^2}{2} + \frac{1}{2}\left(|S_{21}|+|S_{22}|+|y_{21}|+|y_{22}|\right)|S_{11}|\\ &+ \left|1+\frac{k_1 v_r S_{11}}{1+(k_1 v_r S_{12})^2}\right|\\ &\times\frac{|S_{21}|+|S_{22}|+|y_{21}|+|y_{22}|}{2}|S_{13}|.\end{aligned}$$

Consider sets $A_1 := \{(1/r)S_{11}^2 + (1/r)S_{12}^2 + (R/r)S_{13}^2 + S_2^T(M^{-1}NK_T)^{-1}S_2 + y_2^T y_2 + (1/\gamma_1)\tilde{a}_1^2 + (1/\gamma_2)\tilde{a}_2^2 \leq 2\mu\}$ and $A_2 := \{(1/r)S_{11}^2 + (1/r)S_{12}^2 + (R/r)S_{13}^2 + S_2^T(M^{-1}NK_T)^{-1}S_2 + S_3^T L_a S_3 + \sum_{i=1}^{2} y_{i+1}^T y_{i+1} + \sum_{i=1}^{3}(1/\gamma_i)\tilde{a}_i^2 \leq 2\mu\}$. Since A_1 and A_2 are compact in $\mathbb{R}^9$ and $\mathbb{R}^{14}$, respectively, there exist positive constants p_1, p_2 such that $\|\Xi_1\| \leq p_1$ on A_1 and $\|\Xi_2\| \leq p_2$ on A_2. Using the fact $|k_1 v_r S_{11}/1+(k_1 v_r S_{12})^2| \leq |k_1 v_r S_{11}|$ and Young's inequality (i.e., $z_1 z_2 \leq (1/2)z_1^2 + (1/2)z_2^2$), we have

$$\begin{aligned}\dot{V} \leq & -(k_2-1)S_{11}^2\\ &- \frac{1}{r}S_{12}v_r\sin\left(\arctan(k_1 S_{12} v_r)\right) - (k_3-1)S_{13}^2\\ &- \left(k_4 - \frac{7}{4}\right)\|S_2\|^2 - \left(k_5 - \frac{1}{2}\right)\|S_3\|^2\\ &- \left(\frac{1}{\tau_2} - \frac{3}{4}\right)\|y_2\|^2 - \left(\frac{1}{\tau_3} - \frac{1}{2}\right)\|y_3\|^2 + \frac{\|y_2\|^2\|\Xi_1\|^2}{2\delta_3}\\ &+ \frac{\|y_3\|^2\|\Xi_2\|^2}{2\delta_4} + \frac{\delta_3}{2} + \frac{\delta_4}{2} + \frac{\delta_1^2}{2} + \frac{\delta_2^2}{2}\\ &+ \sum_{i=1}^{4}\sigma_i\left(|\tilde{a}_i|a_i - \tilde{a}_i^2\right)\end{aligned}$$

where δ_3 and δ_4 denote positive constants. If we choose $k_2 = 1+k_2^*, k_3 = 1+k_3^*, k_4 = (7/4)+k_4^*, k_5 = (1/2)+k_5^*, (1/\tau_2) = (3/4) + (p_1^2/2\delta_3) + \tau_2^*$, and $(1/\tau_3) = (1/2) + (p_2^2/2\delta_4) + \tau_3^*$, then we have

$$\begin{aligned}\dot{V} \leq & -k_2^* S_{11}^2 - \frac{1}{r}S_{12}v_r\sin\left(\arctan(k_1 S_{12} v_r)\right) - k_3^* S_{13}^2\\ &- k_4^*\|S_2\|^2 - k_5^*\|S_3\|^2 - \sum_{j=1}^{4}\frac{1}{2}\sigma_j\tilde{a}_j^2\\ &- \sum_{i=1}^{2}\left\{\tau_{i+1}^*\|y_{i+1}\|^2 + \left(1 - \frac{\|\Xi_i\|^2}{p_i^2}\right)\frac{p_i^2\|y_{i+1}\|^2}{2\delta_{i+2}}\right\} + \varepsilon\\ \leq & -2\zeta(V - V_p) + \varepsilon \quad (39)\end{aligned}$$

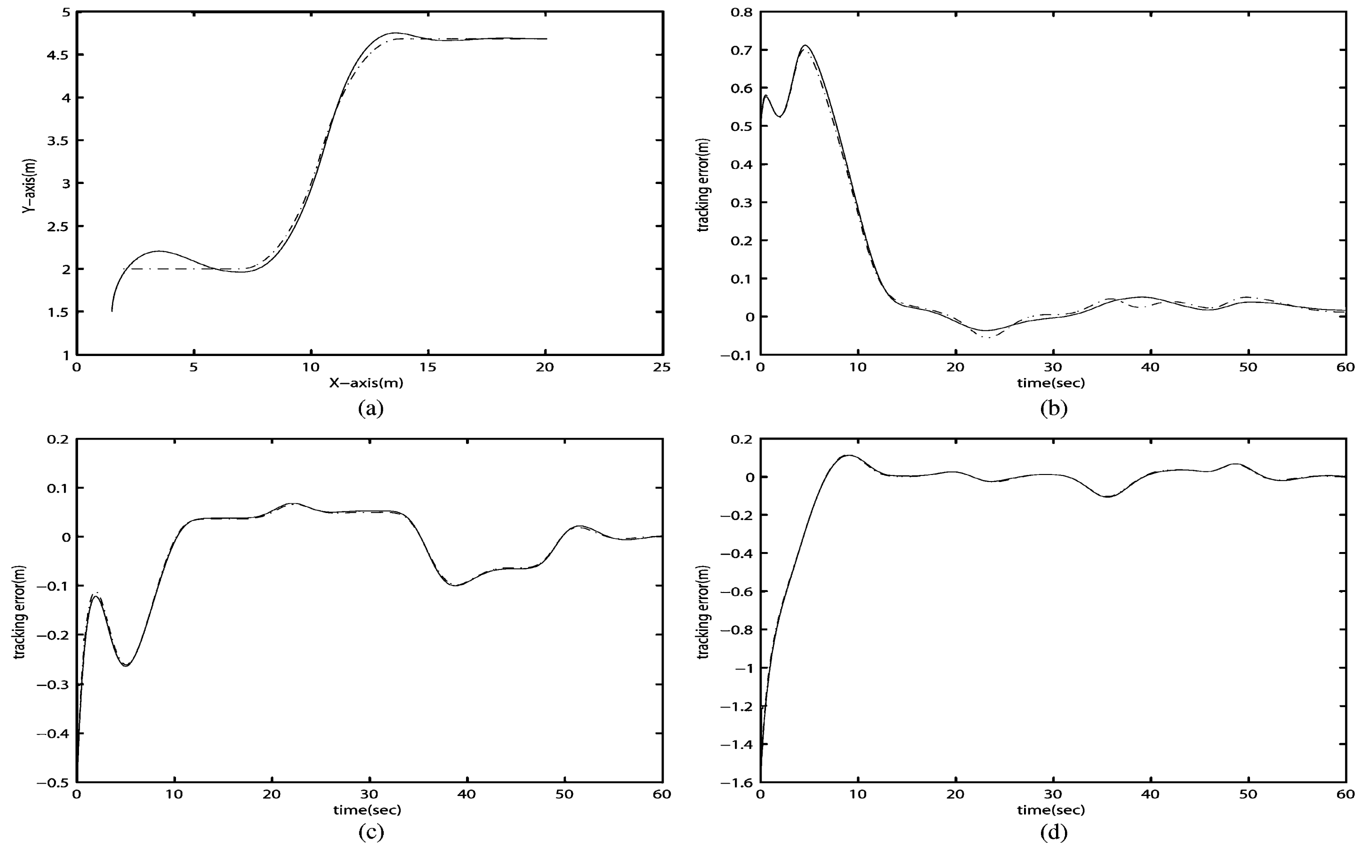

Fig. 2. Simulation results: (a) trajectory tracking result; (b) tracking error S_{11} (solid: DSC technique, dashed-dotted: backstepping technique); (c) tracking error S_{12} (solid: DSC technique, dashed-dotted: backstepping technique); (d) tracking error $\bar{S}_{13}$ (solid: DSC technique, dashed-dotted: backstepping technique).

where $k_2^*, k_3^*, k_4^*, k_5^* > 0$, $\tau_2^*, \tau_3^* > 0$, $\varepsilon = (\delta_1^2 + \delta_2^2 + \delta_3 + \delta_4 + \sum_{j=1}^{4} \sigma_j a_j^2)/2$, and $V_p = (1/2r)(S_{12}^2 - S_{12} v_r \sin(\arctan(k_1 S_{12} v_r)))$. The constant ζ is $0 < \zeta < \min[rk_2^*, 1, (r/R)k_3^*, M_m k_4^*, (1/L_{a,M}) k_5^*, \tau_2^*, \tau_3^*, (\sigma_1\gamma_1)/2, \ldots, (\sigma_4\gamma_4)/2]$ where M_m are the minimum eigenvalues of $M^{-1}NK_T$, and $L_{a,M}$ is the maximum eigenvalue of L_a. Since $S_{12} v_r \sin(\arctan(k_1 S_{12} v_r)) \geq 0$ for all S_{12} and all $t \geq 0$, (39) implies $\dot{V} \leq 0$ on $V = \mu$ when $\zeta > \varepsilon/2(\mu - V_p)$. Therefore, $V \leq \mu$ is an invariant set, i.e., if $V(0) \leq \mu$, then $V(t) \leq \mu$ for all $t \geq 0$. Therefore, we can prove that all error signals in the closed-loop system are semi-globally uniformly ultimately bounded. Besides, by increasing the design parameter ζ, i.e., adjusting k_{j+1}^*, τ_2^*, τ_3^*, γ_j, and σ_j $(j = 1, \ldots, 4)$, the tracking errors in the controlled closed-loop system can be made arbitrarily small. ■

Remark 4: In this remark, we comment that h_2 in (14) is well-defined for all $t \geq 0$ [27]. For any $k_1, k_2, k_3 \geq 0$, consider a set $\Psi(k_1, k_2, k_3) = \{(S_{11}, S_{12}, S_{13}, \hat{a}_1, \hat{a}_2) \in \mathbb{R}^5 : k_1 k_2 k_3 |S_{11}| < 1\}$. Then, let Ω_1 and Ω be sets given by $\Omega_1 = \{(S_{11}, S_{12}, S_{13}, \hat{a}_1, \hat{a}_2) \in \mathbb{R}^5 : V_1(t, S_{11}, S_{12}, S_{13}, \hat{a}_1, \hat{a}_2) < \varpi, \forall t \geq 0\}$ and $\Omega = \{V(t) \leq \mu, \forall t \geq 0\}$ where $0 < \varpi \leq \mu$ is a largest constant such that $\Omega_1 \subset \Psi(\|k_1\|_\infty, \|k_2\|_\infty, \|k_3\|_\infty) \subset \Omega$. From (39), since S_{11}, S_{12}, S_{13}, $\hat{a}_1$, and $\hat{a}_2$ remain in an invariant set Ω, $h_2(t)$ is well-defined for all $t \geq 0$.

Remark 5: The design constants k_i, τ_2, τ_3, γ_j, and σ_j $(i = 1, \ldots, 5,\ j = 1, \ldots, 4)$ are only the sufficient condition and provide a guideline for the designers. The choice of design parameters for any given constants δ_3, δ_4, and a_i $(i = 1, \ldots, 4)$ has some suggestions as follows: (i) increasing k_i and γ_j $(i = 1, \ldots, 5,\ j = 1, \ldots, 4)$, and decreasing τ_2 and τ_3 help to increase ζ, subsequently reduces the bound ε/ζ of error, (ii) decreasing σ_j $(j = 1, \ldots, 4)$ helps to decrease ε, and reduces ε/ζ.

Remark 6: In the adaptation laws (15), (22), and (27), a σ-modification [28] is used for preventing parameter from drifting to infinity. We can also apply an e-modification [29] and a projection operator method [30] in place of a σ-modification.

IV. Simulations

In this section, we perform the simulation for the tracking control of the electrically driven nonholonomic mobile robot to demonstrate the validity of the proposed control method and compare it with the backstepping method. The physical parameters for the mobile robot are chosen as $R = 0.75$ m, $d = 0.3$ m, $r = 0.15$ m, $m_c = 30$ kg, $m_w = 1$ kg, $I_c = 15.625$ kg $\cdot$ m^2, $I_w = 0.005$ kg $\cdot$ m^2, $I_m = 0.0025$ kg $\cdot$ m^2, and $d_{11} = d_{22} = 5$ m. The parameters for the actuator dynamics are chosen as $R_a = \mathrm{diag}[2.5, 2.5]\ \Omega$, $L_a = \mathrm{diag}[0.048, 0.048]\ \Omega - \mathrm{s}$, $K_E = \mathrm{diag}[0.02, 0.02]$ V/rad/s, $K_T = \mathrm{diag}[0.2613, 0.2613]$ oz-in/A, and $N = \mathrm{diag}[62.55, 62.55]$. In this simulation, we assume that all of these parameters are unknown. The disturbances are

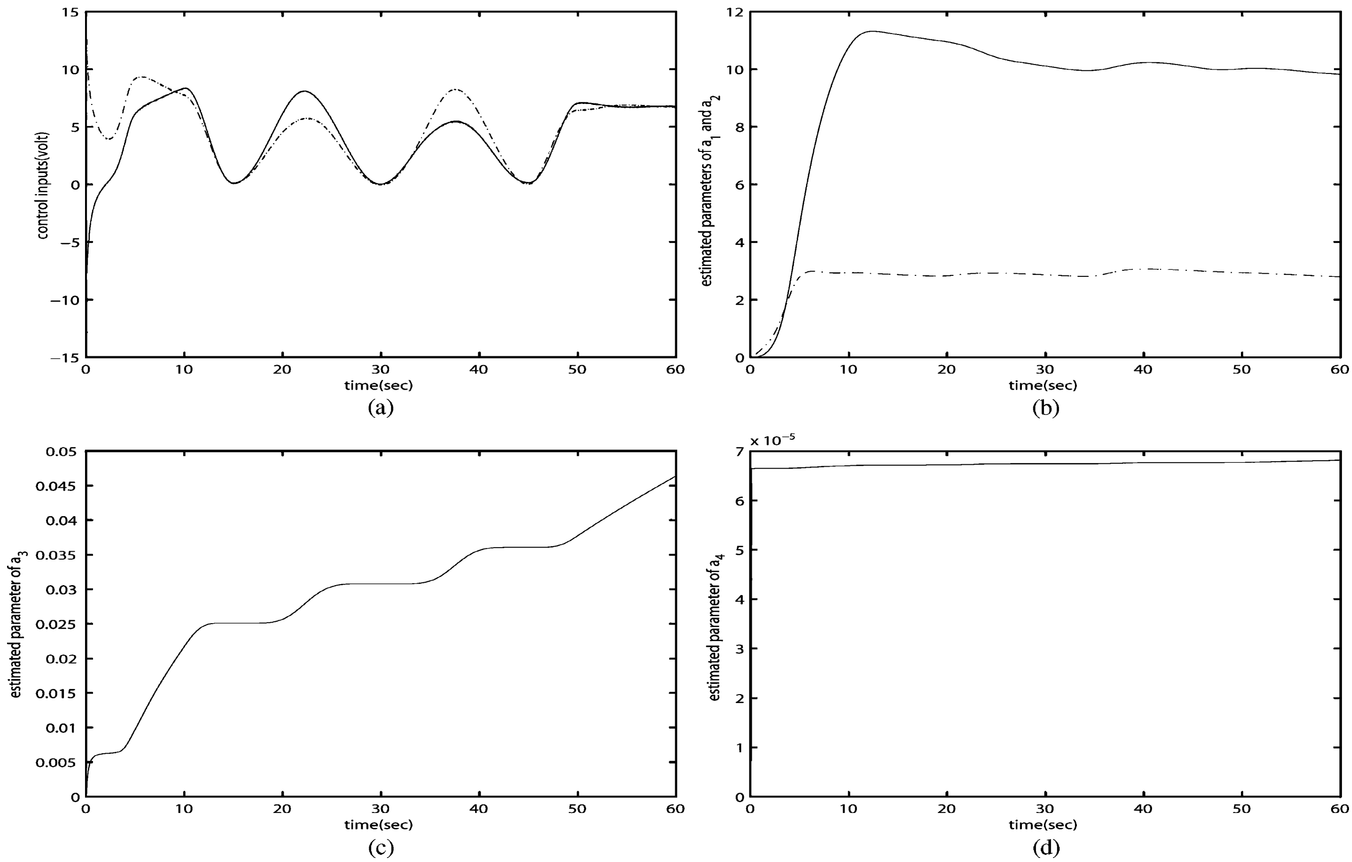

Fig. 3. Simulation results: (a) control inputs (solid: u_1, dashed-dotted: u_2); (b) estimated parameters (solid: $\hat{a}_1$, dashed-dotted: $\hat{a}_2$); (c) estimated parameter $\hat{a}_3$; (d) estimated parameter $\hat{a}_4$.

chosen to be Gaussian random noises with mean 0 and variance 0.5, and the upper bounds of disturbances are assumed as $d_{m1} = d_{m2} = 0.5$ N.

The controller parameters and adaptation gains for the proposed control systems are chosen as $k_1 = 3$, $k_2 = 3$, $k_3 = 2$, $k_4 = 2$, $k_5 = 2$, $\sigma_i = 0.001$, $i = 1, \ldots, 4$, $\gamma_1 = \gamma_2 = 5$, $\gamma_3 = 0.00001$, $\gamma_4 = 0.0000000001$, and $\tau_2 = \tau_3 = 0.01$. The reference velocities v_r, ω_r for generating the reference trajectory are chosen as follows:

$$
\begin{aligned}
0 \le t < 5: &\quad v_r = 0.25\left(1 - \cos\frac{\pi t}{5}\right), \omega_r = 0\\
5 \le t < 10: &\quad v_r = 0.5, \omega_r = 0\\
10 \le t < 15: &\quad v_r = 0.25\left(1 + \cos\frac{\pi t}{5}\right), \omega_r = 0\\
15 \le t < 30: &\quad v_r = 0.25\left(1 - \cos\frac{2\pi t}{15}\right), \omega_r = v_r/5\\
30 \le t < 45: &\quad v_r = 0.25\left(1 - \cos\frac{2\pi t}{15}\right), \omega_r = -v_r/5\\
45 \le t < 50: &\quad v_r = 0.25\left(1 + \cos\frac{\pi t}{5}\right), \omega_r = 0\\
50 \le t < 60: &\quad v_r = 0.5, \omega_r = 0.
\end{aligned}
$$

The initial postures for the reference robot and the actual robot are $(x_r, y_r, \theta_r) = (2, 2, 0)$ and $(x, y, \theta) = (1.5, 1.5, \pi/2)$, respectively. Fig. 2(a) shows the tracking result for the proposed control method. The tracking errors for the DSC and backstepping techniques are compared in Fig. 2(b)–(d). As [22] have already reported, it can be seen that the control performance is not significantly different between the DSC technique and the backstepping technique. In spite of the similar performance, the proposed controller based on the DSC technique can be designed more simply than the backtepping controller as stated in Remark 3. This paper focuses on this simplicity of the controller design for electrically driven nonholonomic mobile robots. Fig. 3(a) shows the boundedness of the control input for the proposed control method. The estimates of unknown parameters for the proposed control method are shown in Fig. 3(b)–(d).

V. Conclusion

In this paper, a simple adaptive controller for electrically driven nonholonomic mobile robots with parametric uncertainties and disturbances has been proposed. The dynamics, the kinematics, and the actuator dynamics of mobile robots with parametric uncertainties and disturbances have been considered. The DSC technique has been extended to design the controller for path tracking of mobile robots including the actuator dynamics, and the adaptive control technique has been applied to deal with parametric uncertainties and disturbances. In addition, the simplified parameter estimation technique has been presented to reduce the number of tuning parameters. From the Lyapunov stability theory, we have proved that all signals in the closed-loop system are semi-globally uniformly ultimately bounded. Finally, from the simulation results, it has been shown that the proposed controller has good tracking performance and the robustness against the parametric uncertainties and disturbances.

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