AoD-Adaptive Subspace Codebook for Channel Feedback in FDD Massive MIMO Systems

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***Abstract*—Channel feedback is essential for frequency division duplex (FDD) massive multiple-input multiple-output (MIMO) systems to realize precoding and power allocation. Traditional codebooks for channel feedback, where the required number of feedback bits is proportional to the number of base station (BS) antennas, can not scale up with massive MIMO due to the large number of BS antennas. To solve this problem, in this paper, we propose an angle-of-departure (AoD) adaptive subspace codebook to reduce the codebook size and feedback overhead. Specifically, by exploiting the channel property that path AoDs vary much slower than path gains, we propose an AoD-adaptive subspace codebook to quantize the channel vector. Within the angle coherence time, by utilizing the AoD information, the proposed codebook is able to track the channel vector better. We also provide the performance analysis of the proposed codebook, where we prove that the required number of feedback bits only scales linearly with the number of resolvable (path) AoDs, which is much smaller than the number of BS antennas. This quantitative result is verified by extensive simulations.**

I. Introduction

Massive multiple-input multiple-output (MIMO) can achieve orders of magnitude increase in spectral efficiency through simultaneously serving multiple users with a very large number of base station (BS) antennas [1]. Channel feedback is essential in frequency division duplex (FDD) massive MIMO, since the channel reciprocity can not be used to learn the channel state at the BS. Unfortunately, previous work on multiuser MIMO has shown that the codebook size for channel feedback should scale exponentially with the number of BS antennas to guarantee the capacity loss within an acceptable level [2], [3]. With a large number of BS antennas in massive MIMO systems, the codebook size and feedback overhead will be overwhelming.

Several channel feedback techniques have been proposed to reduce the codebook size and feedback overhead for FDD massive MIMO. In [4], a differential feedback scheme utilizing the temporal correlation of channels was proposed, which is only valid for slow-varying channels. An alternative is antenna-grouping based channel feedback [5], which consists of one codebook for antenna grouping pattern and the other dimension-reduced codebook. This approach is useful only when BS antennas are strongly correlated. In addition, we have proposed a joint channel training and channel feedback scheme based on compressive sensing [6], which can reduce the overhead for channel training and channel feedback when the channel impulse response is sparse.

In this paper, we propose an angle-of-departure (AoD) adaptive subspace codebook with significant reduction in codebook size and feedback overhead[1]. Our key insight is to leverage the observation that the path AoDs vary much slower than the path gains [7]. Within the angle coherence time, during which path AoDs remain unchanged, the channel vector is only distributed in the subspace of the full M-dimensional space (M is the number of BS antennas), which is called as "*channel subspace*" in this paper. We propose to design an AoD-adaptive codebook, where the quantization vectors are concentrated exactly on the normalized channel subspace which is much smaller than the full M-dimensional space. We also provide theoretical performance analysis of the proposed AoD-adaptive subspace codebook, where we prove that the required number of feedback bits to ensure a constant rate gap only scales linearly with the number of resolvable paths, which is much smaller than the number of BS antennas. This quantitative result is also verified by extensive simulations.

The most related work to this paper is the channel statistics-based codebooks [8], [9]. A rotated codebook based on channel statistics was proposed in [8] to track the spatially correlated channel vector. A compressive sensing based approach was proposed in [9], where a codebook is designed by considering both the channel statistics and the measurement matrix. Our work is different from the channel statistics-based codebooks due to the exploiting of AoD information to design an AoD-adaptive subspace codebook, which can track the channel vector better.

Notation: Boldface capital and lower-case letters stand for matrices and vectors, respectively. The conjugate transpose and inverse of a matrix are denoted by $(\cdot)^H$ and $(\cdot)^{-1}$, respectively. $\measuredangle(\mathbf{x},\mathbf{y})$ is the angle between $\mathbf{x}$ and $\mathbf{y}$, and $\sin^2(\measuredangle(\mathbf{x},\mathbf{y})) = 1 - \frac{|\mathbf{x}^H\mathbf{y}|^2}{\|\mathbf{x}\|^2\|\mathbf{y}\|^2}$. $E[\cdot]$ denotes the expectation operator. $\mathbf{I}_K$ denotes the identity matrix of size $K \times K$.

II. System Model

In this section, we first briefly introduce the massive MIMO downlink channel model, and then the channel feedback procedure. To quantify the performance of our strategy, we review the per-user rate calculated assuming zero-forcing (ZF) precoding based on the fed back CSI.

[1]Simulation codes are provided to reproduce the results presented in this paper: http://oa.ee.tsinghua.edu.cn/dailinglong/.

A. Massive MIMO Downlink Channel Model

In this paper, we consider a millimeter wave (mmWave) massive MIMO system with M antennas at the BS and K single-antenna users ($M \gg K$). We consider the classical narrowband ray-based channel model for downlink channel vector $\mathbf{h}_k \in \mathcal{C}^{M\times 1}$ at the k-th user [10]

$$\mathbf{h}_k = \sum_{i=1}^{P_k} g_{k,i}\mathbf{a}(\theta_{k,i}), \tag{1}$$

where P_k is the number of resolvable paths from the BS to the k-th user, $g_{k,i}$ is the complex gain of the i-th propagation path of the k-th user, which is identically and independently distributed (i.i.d.) with zero mean and unit variance, and $\theta_{k,i}$ is the i-th path AoD of the k-th user. We assume an uniform linear arrays (ULA) of antennas at the BS [1], thus the steering vector $\mathbf{a}(\theta_{k,i}) \in \mathcal{C}^{M\times 1}$ denoting the antenna response of the i-th path of the k-th user is,

$$\mathbf{a}(\theta_{k,i}) = [1, e^{-j2\pi\frac{d}{\lambda}\sin(\theta_{k,i})}, \cdots, e^{-j2\pi\frac{d}{\lambda}(M-1)\sin(\theta_{k,i})}]^H, \tag{2}$$

where d is the antenna spacing at the BS, λ is the wavelength of the carrier frequency. In matrix form with $\mathbf{A}_k = [\mathbf{a}(\theta_{k,1}), \mathbf{a}(\theta_{k,2}), \cdots, \mathbf{a}(\theta_{k,P_k})] \in \mathcal{C}^{M\times P_k}$ and $\mathbf{g}_k = [g_{k,1}, g_{k,2}, \cdots, g_{k,P_k}]^H \in \mathcal{C}^{P_k\times 1}$, we have

$$\mathbf{h}_k = \mathbf{A}_k\mathbf{g}_k. \tag{3}$$

Further, we denote the concatenation of channel vectors for all K users as $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \cdots, \mathbf{h}_K] \in \mathcal{C}^{M\times K}$.

B. Channel Feedback

Although the training overhead to obtain the downlink channel vector at the user side is increased in massive MIMO systems, there are many effective downlink training methods [6], [11] proposed with reduced training overhead. Thus, in this paper, each user is assumed to know its channel vector.

Channel vector $\mathbf{h}_k$ is also required by the BS to perform power allocation and precoding, which is usually realized by channel feedback. The quantization of $\mathbf{h}_k$ is performed by the codebook $\mathbb{C}_k = \{\mathbf{c}_{k,1}, \mathbf{c}_{k,2}, \cdots, \mathbf{c}_{k,2^B}\}$, which consists of 2^B different M-dimensional unit-norm column vectors, where B is the number of feedback bits. The detailed codebook design will be discussed later in Section III. The k-th user quantizes its channel vector $\mathbf{h}_k$ to a quantization vector $\mathbf{c}_{k,F_k} \in \mathcal{C}^{M\times 1}$, where the quantization index F_k is computed according to

$$F_k = \underset{i\in[1,2^B]}{\arg\min}\sin^2(\measuredangle(\mathbf{h}_k, \mathbf{c}_{k,i})) = \underset{i\in[1,2^B]}{\arg\max}|\tilde{\mathbf{h}}_k^H\mathbf{c}_{k,i}|^2, \tag{4}$$

where $\tilde{\mathbf{h}}_k = \frac{\mathbf{h}_k}{\|\mathbf{h}_k\|}$ is the channel direction. F_k can be fed back from the k-th user to the BS by using B dedicated bits. After receiving these B channel feedback bits (thus the index F_k), the BS can generate the fed back channel vector $\hat{\mathbf{h}}_k = \|\mathbf{h}_k\|\mathbf{c}_{F_k}$. The concatenation of the fed back channel vectors can be denoted as $\hat{\mathbf{H}} = [\hat{\mathbf{h}}_1, \hat{\mathbf{h}}_2, \cdots, \hat{\mathbf{h}}_K] \in \mathcal{C}^{M\times K}$.

C. Per-User Rate

The BS can perform downlink precoding based on the fed back channel matrix $\hat{\mathbf{H}}$. In this paper, we consider the widely used simple linear ZF precoding, which is able to asymptotically achieve the near-optimal performance with low complexity when $M \to \infty$ [3]. The transmit signal $\mathbf{x} \in \mathcal{C}^{M\times 1}$ after ZF precoding is given by

$$\mathbf{x} = \sqrt{\frac{\gamma}{K}}\mathbf{V}\mathbf{s}, \tag{5}$$

where γ is the transmit power, $\mathbf{s} = [s_1, s_2, \cdots, s_K] \in \mathcal{C}^{K\times 1}$ is the signals intended for K users with the normalized power $E[|s_i|^2] = 1$, and $\mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_K] \in \mathcal{C}^{M\times K}$ is the ZF precoding matrix consisting of K different unit-norm precoding vectors $\mathbf{v}_i \in \mathcal{C}^{M\times 1}$. We denote $\mathbf{U} = \hat{\mathbf{H}}(\hat{\mathbf{H}}^H\hat{\mathbf{H}})^{-1}$, then the precoding vectors $\mathbf{v}_i$ can be described as the normalized i-th column of $\mathbf{U}$, i.e., $\mathbf{v}_i = \frac{\mathbf{U}(:,i)}{\|\mathbf{U}(:,i)\|}$.

After the channel, the received signal at the k-th user can be described as

$$\begin{aligned} y_k &= \mathbf{h}_k^H\mathbf{x} + n_k \\ &= \sqrt{\frac{\gamma}{K}}\mathbf{h}_k^H\mathbf{v}_k s_k + \sqrt{\frac{\gamma}{K}}\sum_{i=1,i\neq k}^{K}\mathbf{h}_k^H\mathbf{v}_i s_i + n_k, \end{aligned} \tag{6}$$

where n_k is the complex Gaussian noise at the k-th user with zero mean and unit variance. Thus, the signal-to-interference-plus-noise ratio (SINR) at the k-th user is

$$\text{SINR}_k = \frac{\frac{\gamma}{K}|\mathbf{h}_k^H\mathbf{v}_k|^2}{1 + \frac{\gamma}{K}\sum_{i=1,i\neq k}^{K}|\mathbf{h}_k^H\mathbf{v}_i|^2}. \tag{7}$$

Assuming Gaussian signaling, the per-user rate R is

$$\begin{aligned} R &= E\big[\log_2(1 + \text{SINR}_k)\big] \\ &= E\left[\log_2\left(1 + \frac{\frac{\gamma}{K}|\mathbf{h}_k^H\mathbf{v}_k|^2}{1 + \frac{\gamma}{K}\sum_{i=1,i\neq k}^{K}|\mathbf{h}_k^H\mathbf{v}_i|^2}\right)\right]. \end{aligned} \tag{8}$$

The per-user rate R depends on the precoding matrix $\mathbf{V}$, which is affected by the quality of fed back channel matrix $\hat{\mathbf{H}}$. In the following Section III, we present the proposed AoD-adaptive subspace codebook to provide the reliable channel feedback with low overhead.

III. Proposed AoD-Adaptive Subspace Codebook

In this section, we present the proposed AoD-adaptive subspace codebook at first. Then, we discuss how to obtain the AoDs during the angle coherence time.

A. AoD-Adaptive Subspace Codebook

The path AoD $\theta_{k,i}$ in (1) mainly depends on the surrounding obstacles around the BS, which may not physically change their position in much longer time than the channel coherence time. On the contrary, for the path gain $g_{k,i}$, one resolvable path is generated by a cluster of scatters surrounding the k-th user, which consists of a number of unresolvable paths. The resultant path gain $g_{k,i}$ seen by the k-th user depends on a number of unresolvable paths, Thus, path gains vary much faster than path AoDs [7]. Accordingly, the angle coherence time, during which the path AoDs can be regarded as static, is much longer than the channel coherence time.

During the angle coherence time, the channel vector $\mathbf{h}_k$ is distributed in the *channel subspace*. As shown in (1) and (3), the channel vector $\mathbf{h}_k$ is composed of P_k paths as $\mathbf{h}_k = \sum_i^{P_k} g_{k,i}\mathbf{a}(\theta_{k,i}) = \mathbf{A}_k\mathbf{g}_k$, where $\mathbf{A}_k$ is completely determined by path AoDs. Thus, $\mathbf{h}_k$ is actually distributed on the column space of $\mathbf{A}_k \in \mathcal{C}^{M\times P_k}$, which is formed by linear combination of $\mathbf{A}_k$'s column vectors. For example, due to the limited scattering of mmWave, the number of paths P_k is much smaller (e.g., $2 \sim 8$ for 6-60GHz [12]) than the number of BS antennas M (e.g., $M = 128, 256$). We also expect this to be true even with massive MIMIO since the number of resolvable paths P_k seen by the BS depends on the scatters around the BS whose number is usually limited. Thus, the column space of $\mathbf{A}_k$ is only a subspace of the full M-dimensional space. This subspace is referred to as *channel subspace* in this paper. Accordingly, the normalized channel vector $\hat{\mathbf{h}}_k$ is only distributed on a small part of the M-dimensional unit sphere, which is called as normalized channel subspace.

We propose the AoD-adaptive subspace codebook where the quantization vectors are exactly distributed on the normalized channel subspace as shown in Fig. 1. In this section, we assume that the quantized AoDs can be obtained at both the BS and the k-th user as $\{\hat{\theta}_{k,1}, \hat{\theta}_{k,2}, \cdots, \hat{\theta}_{k,P_k}\}$; obtaining the AoDs will be discussed later in Section III-B. Thus, both the BS and users can generate the steering matrix $\hat{\mathbf{A}}_k = [\mathbf{a}(\hat{\theta}_{k,1}), \mathbf{a}(\hat{\theta}_{k,2}), \cdots, \mathbf{a}(\hat{\theta}_{k,P_k})] \in \mathcal{C}^{M\times P_k}$. Then, the quantization vector $\mathbf{c}_{k,i}$ of the proposed AoD-adaptive subspace codebook $\mathbb{C}_k = \{\mathbf{c}_{k,1}, \mathbf{c}_{k,2}, \cdots, \mathbf{c}_{k,2^B}\}$ is generated as

$$\mathbf{c}_{k,i} = \frac{1}{\sqrt{M}}\hat{\mathbf{A}}_k\mathbf{w}_i, \tag{9}$$

where unit-norm vector $\mathbf{w}_i \in \mathcal{C}^{P_k\times 1}$ can be the quantization vector of a traditional codebook such as Grassimannian codebook [2]. We have

$$\begin{aligned}\|\hat{\mathbf{A}}_k\mathbf{w}_i\|^2 &= \left\|\sum_{p=1}^{P_k} w_{i,p}\mathbf{a}(\hat{\theta}_{k,p})\right\|^2 \overset{(a)}{=} \sum_{p=1}^{P_k}\|w_{i,p}\mathbf{a}(\hat{\theta}_{k,p})\|^2 \\ &= M\Sigma_{p=1}^{P_k}|w_{i,p}|^2 = M,\end{aligned} \tag{10}$$

where (a) is true due to the orthogonality among column vectors $\mathbf{a}(\hat{\theta}_{k,p})$ of $\hat{\mathbf{A}}_k$ (see Appendix I). Thus, $\frac{1}{\sqrt{M}}$ in (9) is used to ensure the unit norm of $\mathbf{c}_{k,i}$.

The quantization vector $\mathbf{c}_{k,i} = \frac{1}{\sqrt{M}}\hat{\mathbf{A}}_k\mathbf{w}_i$ in the proposed subspace codebook $\mathbb{C}_k$ is distributed on the column space of $\hat{\mathbf{A}}_k$. If the BS and the k-th user obtain the exact AoDs, i.e., $\hat{\mathbf{A}}_k = \mathbf{A}_k$ (which is proved to be possible at expense of a small amount of additional overhead in next subsection III-B), the quantization vector $\mathbf{c}_{k,i}$ will be distributed exactly on the normalized channel subspace. Note that the proposed codebook is AoD adaptive, i.e., the quantization vector $\mathbf{c}_{k,i}$ in (9) can be updated according to the AoDs in current angel coherence time to track the current channel subspace better. Since the angle coherence time is much longer than channel coherence time, the updating frequency of the proposed AoD-adaptive subspace codebook is low.

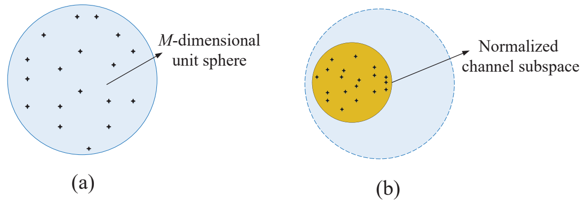

Fig. 1. Codebook comparison: (a) the classical Grassimannian codebook; (b) the proposed AoD-adaptive subspace codebook.

B. AoD Acquisition

In this section, we will discuss how the BS and users obtain AoDs within the angle coherence time. Firstly, users can easily estimate AoDs from channel vectors by the multiple signal classification (MUSIC) algorithm [13], where the channel vectors are regarded as measurements and AoDs can be estimated from the space spectral function of measurements. Since the BS also need to know AoDs to generate the AoD-adaptive subspace codebook $\mathbb{C}_k$ in (9), the estimated AoDs $\{\theta_{k,1}, \theta_{k,2}, \cdots, \theta_{k,P_k}\}$ can be quantized to $\{\hat{\theta}_{k,1}, \hat{\theta}_{k,2}, \cdots, \hat{\theta}_{k,P_k}\}$ using B_0P_k bits and then fed back to the BS. The average AoD feedback overhead is small due to the long angle coherence time. Next, we will focus on the quantitative performance analysis of the proposed AoD-adaptive subspace codebook.

IV. Performance Analysis

In this section, we calculate the rate gap between the ideal case of perfect CSIT and practical case of limited channel feedback using a random vector quantization (RVQ) framework. We then analyze the quantization error of the proposed structure. Finally, we derive an lower bound of the required number of feedback bits to ensure a constant rate gap.

A. Rate Gap

In the ideal case of perfect CSIT at the BS, i.e., $\hat{\mathbf{H}} = \mathbf{H}$, the ZF precoding vector $\mathbf{v}_{\text{ideal},i} \in \mathcal{C}^{M\times 1}$ is obtained as the normalized i-th column of $\mathbf{H}(\mathbf{H}^H\mathbf{H})^{-1}$. Thus, the corresponding per-user rate is $R_{\text{ideal}} = E\left[\log_2(1 + \frac{\gamma}{K}|\mathbf{h}_k^H\mathbf{v}_{\text{ideal},k}|^2)\right]$. However, in the practical case of limited channel feedback, ZF precoding is performed based on the fed back channel matrix $\hat{\mathbf{H}}$, and the per-user rate R is shown in (8). Following Theorem 1 of [3], the rate gap $\Delta R(\gamma) = R_{\text{ideal}} - R$ can be upper bounded as

$$\Delta R(\gamma) \le \log_2\left(1 + \frac{(K-1)\gamma}{K}E[\|\mathbf{h}_k\|^2]E[\sin^2(\measuredangle(\tilde{\mathbf{h}}_k, \hat{\mathbf{h}}_k)]\right), \tag{11}$$

where $\tilde{\mathbf{h}}_k$ is the normalized channel vector in (4). We observe that the rate gap mainly depends on the quantization error $E[\sin^2(\measuredangle(\tilde{\mathbf{h}}_k, \hat{\mathbf{h}}_k))]$.

B. Quantization Error

In this section, we compute the quantization error $E[\sin^2(\measuredangle(\tilde{\mathbf{h}}_k, \hat{\mathbf{h}}_k))]$ in (11) when the proposed AoD-adaptive subspace codebook is considered. In the rest of this paper, we omit the subscript k for simplicity. Since $\|\tilde{\mathbf{h}}\| = 1$ and $\frac{\hat{\mathbf{h}}}{\|\hat{\mathbf{h}}\|} = \mathbf{c}_F$, $E[\sin^2(\measuredangle(\tilde{\mathbf{h}}, \hat{\mathbf{h}}))]$ can be expressed as

$$E[\sin^2(\measuredangle(\tilde{\mathbf{h}}, \hat{\mathbf{h}}))] = 1 - E[|\tilde{\mathbf{h}}^H \mathbf{c}_F|^2], \tag{12}$$

where $\tilde{\mathbf{h}} = \frac{\mathbf{Ag}}{\|\mathbf{h}\|}$ according to (3). Similar to (10), it can be shown that $\|\mathbf{h}\| = \|\mathbf{Ag}\| = \|\mathbf{g}\|\sqrt{M}$. By denoting $\tilde{\mathbf{g}} = \frac{\mathbf{g}}{\|\mathbf{g}\|}$, we have $\tilde{\mathbf{h}} = \frac{\mathbf{A}\tilde{\mathbf{g}}}{\sqrt{M}}$. Combining $\tilde{\mathbf{h}} = \frac{\mathbf{A}\tilde{\mathbf{g}}}{\sqrt{M}}$ and (9), we have

$$E[|\tilde{\mathbf{h}}^H \mathbf{c}_F|^2] = E\left[\left|\frac{1}{M}\tilde{\mathbf{g}}^H \mathbf{A}^H \hat{\mathbf{A}} \mathbf{w}_F\right|^2\right] \overset{(a)}{\approx} E\left[\left|\frac{\mathcal{K}}{M}\tilde{\mathbf{g}}^H \mathbf{w}_F\right|^2\right] = \left|\frac{\mathcal{K}}{M}\right|^2 E[|\tilde{\mathbf{g}}^H \mathbf{w}_F|^2], \tag{13}$$

where (a) is true due to $\mathbf{A}^H \hat{\mathbf{A}} \approx \mathcal{K}\mathbf{I}_P$ as proved in **Lemma 1** (see Appendix I). To enable the performance analysis of the proposed approach, in this section, we employ the RVQ framework where $\mathbf{w}_i$ in (9) is isotropically distributed in unit sphere, which is usually adopted for analysis. Thus, both $\tilde{\mathbf{g}}$ and $\mathbf{w}_F$ are isotropically distributed vectors on the P-dimensional unit sphere, and we have [3]

$$E[|\tilde{\mathbf{g}}^H \mathbf{w}_F|^2] > 1 - 2^{-\frac{B}{P-1}}. \tag{14}$$

Combining (12), (13), and (14), we have

$$E[\sin^2(\measuredangle(\tilde{\mathbf{h}}, \hat{\mathbf{h}}))] < 1 - \left|\frac{\mathcal{K}}{M}\right|^2 (1 - 2^{-\frac{B}{P-1}}). \tag{15}$$

By substituting (21) in Appendix I into (15) and denoting $\beta = \frac{(M^2-1)}{3}(\pi\frac{d}{\lambda})^2 r^2 2^{-2B_0} \ll 1$, we can obtain the upper bound of the quantization error $E[\sin^2(\measuredangle(\tilde{\mathbf{h}}, \hat{\mathbf{h}}))]$ as

$$E[\sin^2(\measuredangle(\tilde{\mathbf{h}}, \hat{\mathbf{h}}))] < \beta(1 - 2^{-\frac{B}{P-1}}) + 2^{-\frac{B}{P-1}}. \tag{16}$$

We can observe from (16) that a small β leads to a small quantization error due to $2^{-\frac{B}{P-1}} \ll 1$, where a small β can be achieved with a large number of AoD feedback bits B_0. In addition, since $\beta \ll 1$, a small quantization error can be achieved with a large number of feedback bits B.

C. Feedback Bits

Finally, we will discuss the required number of feedback bits B to guarantee a constant rate gap $\Delta R(\gamma)$. By substituting (16) into (11), we can obtain the rate gap as

$$\Delta R(\gamma) \leq \log_2\left(1 + \frac{(K-1)\gamma}{K}E[\|\mathbf{h}_k\|^2]2^{-\frac{B}{P-1}} + \frac{(K-1)\gamma}{K}E[\|\mathbf{h}_k\|^2]\beta(1 - 2^{-\frac{B}{P-1}})\right). \tag{17}$$

If we consider the perfect case of AoD acquisition, i.e., B_0 is large enough, we have $\beta = 0$, and (17) can be simplified as

$$\Delta R(\gamma) \leq \log_2\left(1 + \frac{(K-1)\gamma}{K}E[\|\mathbf{h}_k\|^2]2^{-\frac{B}{P-1}}\right). \tag{18}$$

Let the rate gap $\Delta R(\gamma) \leq \log_2(b)$ bps/Hz, then the number of feedback bits B should scale according to

$$B \geq \frac{P-1}{3}\text{SNR} + (P-1)\log_2\frac{K-1}{b-1}, \tag{19}$$

where $\text{SNR} = 10\log_{10}\frac{\gamma}{K}E[\|\mathbf{h}_k\|^2]$ is the signal-to-noise-ratio (SNR) at the receiver. We can observe that the slope of the required number of feedback bits B is $P-1$ when SNR increases. In other words, the required number of feedback bits only scales linearly with $P-1$ to maintain a constant rate gap. Since $P \ll M$, the proposed AoD-adaptive subspace codebook can reduce the codebook size and feedback overhead significantly.

V. Simulation Results

A simulation study was carried out to verify the performance of the proposed AoD-adaptive subspace codebook. The main system parameters are set as: 1) The number of BS antennas M, the number of users K and the number of resolvable paths P are $(M, K, P) = (128, 8, 3)$; 2) The AoDs follow the uniform distribution $\mathcal{U}[-\frac{1}{2}\pi, \frac{1}{2}\pi]$; 3) The number of feedback bits $B = \frac{P-1}{3}\text{SNR} - 2.44$ (let $b = 4$ in (19)).

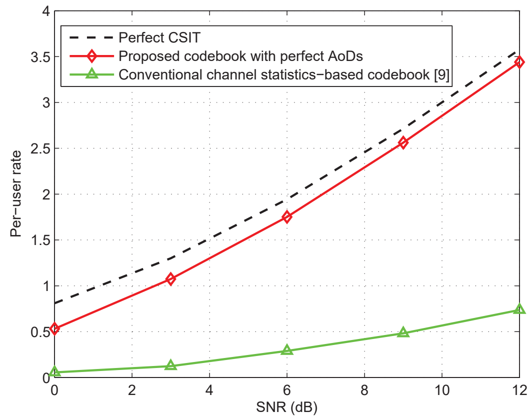

Fig. 2. Comparison of the per-user rate between the ideal case of perfect CSIT and the practical cases of limited channel feedback.

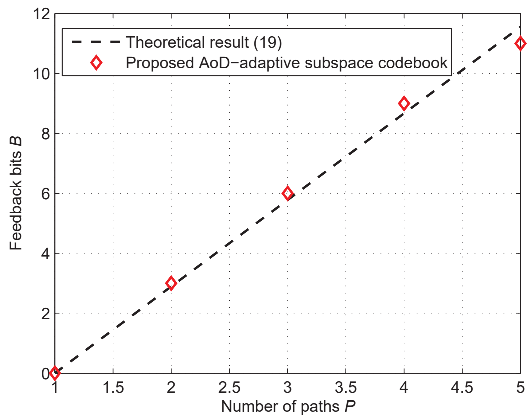

Fig. 3. The number of feedback bits B against number of paths P.

Fig. 2 compares the per-user rate between the ideal case of perfect CSIT and the practical cases of limited channel feedback, where the proposed AoD-adaptive subspace codebook and the channel statistics-based codebooks [9] are considered. We observe that the rate gap between the ideal case of perfect CSIT and the practical case of using the proposed AoD-adaptive subspace codebook remains constant when SNR at the receiver in (19) increases, which is consistent with our theoretical analysis in Section IV-C. On the contrary, for the conventional channel statistics-based codebook, the rate gap increases with the SNR. In addition, the proposed AoD-adaptive subspace codebook outperforms the classical channel statistics-based codebook in terms of the per-user rate.

Fig. 3 shows the required number of feedback bits B to limit the rate gap between the ideal case of perfect CSIT and the practical case of using the proposed AoD-adaptive subspace codebook within 0.18 bps/Hz. We observe that the required number of feedback bits B scales linearly with the number of resolvable paths P. It is consistent with the theoretical result (19) which is also shown in Fig. 3 for comparison.

VI. Conclusions

In this paper, we have proposed the AoD-adaptive subspace codebook for channel feedback in FDD massive MIMO systems. By exploiting the channels property that path AoDs vary much slower than path gains, the proposed codebook can achieve significant reduction of codebook size and feedback overhead. We have also provided performance analysis of the proposed codebook, where we have proved that the required number of feedback bits only scales linearly with the number of paths, which is much smaller than the number of BS antennas. This quantitative result is also verified by extensive simulations. In the future, the impact of AoD quantization error on the per-user rate can be further analyzed.

Appendix I

Lemma 1: The steering vectors of paths with different AoDs (column vectors of $\mathbf{A}$) are asymptotically orthogonal to each other, i.e., $\mathbf{A}^H\mathbf{A} \approx M\mathbf{I}_P$. When the quantization error of AoD is small, $\mathbf{A}^H\hat{\mathbf{A}} \approx \mathcal{K}\mathbf{I}_P$ where $\left|\frac{\mathcal{K}}{M}\right|^2 \geq 1 - \frac{(M^2-1)}{12}(\pi\frac{d}{\lambda})^2 r^2 2^{-2B_0}$.

Proof: The (p,q)-th element $\mathbf{a}(\theta_p)^H\mathbf{a}(\hat{\theta_q})$ of $\mathbf{A}^H\hat{\mathbf{A}}$ is $\mathbf{a}(\theta_p)^H\mathbf{a}(\hat{\theta_q}) = \sum_{m=0}^{M-1} e^{-j2\pi\frac{d}{\lambda}m[\sin(\theta_p)-\sin(\hat{\theta_q})]}$. Denoting $\delta_{p,q} = \sin(\theta_p) - \sin(\hat{\theta_q})$, we have

$$\left|\mathbf{a}(\theta_p)^H\mathbf{a}(\hat{\theta_q})\right| = M\left|\Upsilon(\frac{d}{\lambda}\delta_{p,q})\right|, \tag{20}$$

where $\Upsilon(x) \triangleq \frac{\sin(M\pi x)}{M\sin(\pi x)}$. According to the characteristics of $\Upsilon(x)$, when $|x| \gg \frac{1}{M}$, i.e., $|\delta_{p,q}| \gg \frac{\lambda}{Md}$, we have $|\Upsilon(x)| \approx 0$ [14]. Now we consider the following two cases:

i) If $\hat{\mathbf{A}} = \mathbf{A}$, i.e., $\delta_{p,q} = \sin(\theta_p) - \sin(\theta_q)$, the absolute diagonal element $|\mathbf{a}(\theta_p)\mathbf{a}(\theta_p)^H| = M|\Upsilon(\frac{d}{\lambda}\delta_{p,p})| = M|\Upsilon(0)| = M$. For the non-diagonal element $\mathbf{a}(\theta_p)\mathbf{a}(\theta_q)^H$, since AoDs θ_p and θ_q are distinguished enough, i.e., $|\delta_{p,q}| = |\sin(\theta_p) - \sin(\theta_q)| \gg \frac{\lambda}{Md}$, the absolute non-diagonal element $|\mathbf{a}(\theta_p)\mathbf{a}(\theta_q)^H| = M|\Upsilon(\frac{d}{\lambda}\delta_{p,q})| \approx 0$. Therefore, we have $\mathbf{A}\mathbf{A}^H \approx M\mathbf{I}_P$.

ii) Otherwise $\hat{\mathbf{A}} \neq \mathbf{A}$, i.e., $\delta_{p,q} = \sin(\theta_p) - \sin(\hat{\theta}_q)$, the absolute diagonal element $|\mathbf{a}(\theta_p)\mathbf{a}(\hat{\theta}_p)^H| = M|\Upsilon(\frac{d}{\lambda}\delta_{p,p})|$, where $|\delta_{p,p}| = |\sin(\theta_p) - \sin(\hat{\theta}_p)|$ is the AoD quantization error. With uniform quantization, $|\delta_{p,p}| \leq r2^{-B_0}$, where r is the difference between the maximum and minimum values over which $\sin(\theta_p)$ is quantized and B_0 is the number of quantization bits. By denoting $\mathcal{K} = \mathbf{a}(\theta_p)\mathbf{a}(\hat{\theta}_p)^H$ and using (20), we have

$$\begin{aligned}\left|\frac{\mathcal{K}}{M}\right|^2 &= \frac{\sin^2(\pi\frac{d}{\lambda}\delta_{p,p}M)}{M^2\sin^2(\pi\frac{d}{\lambda}\delta_{p,p})} \overset{(a)}{\approx} 1 - \frac{M^2-1}{3}\left(\pi\frac{d}{\lambda}\right)^2\delta_{p,p}^2 \\ &\overset{(b)}{\geq} 1 - \frac{M^2-1}{3}\left(\pi\frac{d}{\lambda}\right)^2 r^2 2^{-2B_0},\end{aligned} \tag{21}$$

where (a) is obtained by the second order Taylor's expansion of $\sin(\cdot)$, and (b) holds true as $|\delta_{p,p}| \leq r2^{-B_0}$. For the non-diagonal element $\mathbf{a}(\theta_p)\mathbf{a}(\hat{\theta}_q)^H$,

$$\begin{aligned}|\delta_{p,q}| &= |\sin(\theta_p) - \sin(\hat{\theta}_q)| \\ &= |\sin(\theta_p) - \sin(\theta_q) + \sin(\theta_q) - \sin(\hat{\theta}_q)| \\ &\geq |\sin(\theta_p) - \sin(\theta_q)| - |\sin(\theta_q) - \sin(\hat{\theta}_q)| \\ &\overset{(a)}{\geq} |\sin(\theta_p) - \sin(\theta_q)| - r2^{-B_0},\end{aligned} \tag{22}$$

where (a) is true since $|\sin(\theta_q) - \sin(\hat{\theta}_q)| \leq r2^{-B_0}$. We assume that B_0 is properly chosen (large enough). Then, we can obtain $|\delta_{p,q}| \gg \frac{\lambda}{dM}$. Thus, it holds that the absolute non-diagonal element $|\mathbf{a}(\theta_p)\mathbf{a}(\hat{\theta}_q)^H| = M|\Upsilon(\frac{d}{\lambda}\delta_{p,q})| \approx 0$. Therefore, we have $\mathbf{A}\hat{\mathbf{A}}^H \approx \mathcal{K}\mathbf{I}_P$.

In summary, $\mathbf{A}\mathbf{A}^H \approx M\mathbf{I}_P$, and $\mathbf{A}\hat{\mathbf{A}}^H \approx \mathcal{K}\mathbf{I}_P$ where $\left|\frac{\mathcal{K}}{M}\right|^2 \geq 1 - \frac{(M^2-1)}{3}(\pi\frac{d}{\lambda})^2 r^2 2^{-2B_0}$. ■

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