Block Compressive Channel Estimation and Feedback for FDD Massive MIMO

Zhen Gao, Linglong Dai, Wei Dai, and Zhaocheng Wang

***Abstract*—Channel state information (CSI) at the base station (BS) is required to fully exploit the advantages of massive MIMO. However, due to massive BS antennas, the estimation and feedback of downlink CSI are very challenging in frequency division duplexing (FDD) massive MIMO. This paper proposes a block compressive channel estimation and feedback scheme for FDD massive MIMO, which can reduce the overhead for CSI acquisition substantially. Specifically, we first propose the non-orthogonal pilots, which is essentially different from conventional orthogonal pilots. Then, a block orthogonal matching pursuit (BOMP) algorithm is proposed to estimate CSI according to the feedback signal from users, where the analog channel feedback is adopted, and the spatial common sparsity of time-domain massive MIMO channels is exploited to reduce the overhead for CSI acquisition. Moreover, we exploit the temporal common sparsity of channels to estimate channels with reduced complexity. Simulation results demonstrate that the proposed scheme with significantly reduced CSI acquisition overhead can approach the performance bound.**

***Index Terms*—Massive MIMO, FDD, channel estimation and feedback, non-orthogonal pilot, block compressive sensing.**

I. INTRODUCTION

Massive MIMO has been widely recognized as a key enabling technology for future 5G communications due to its high system capacity and energy efficiency [1]. To fully exploit its advantages, accurate channel state information (CSI) at the base station (BS) is required for beamforming, user scheduling, etc. However, channel estimation and feedback are very challenging in frequency division duplexing (FDD) massive MIMO, since the user has to accurately acquire and feedback CSI associated with massive BS antennas, and thus the overhead for channel estimation and feedback could be prohibitively high [1]. On the other hand, FDD still dominates current wireless cellular systems, where the estimation and feedback for CSI in the downlink are requisite since the channel reciprocity does not exist. Therefore, it is urgent to explore an efficient channel estimation and feedback scheme to enable massive MIMO to be backward compatible with current FDD wireless networks.

For channel estimation, [2] exploited the temporal sparsity of wireless channels to estimate massive MIMO channels without considering the spatial correlation of MIMO channels. [3] estimated channels by assuming the perfect spatial correlation of the CSI known at the BS, which may be unrealistic in practice. Additinally, both [2] and [3] did not consider the channel feedback to the BS. For channel feedback, conventional quantized feedback can be unaffordable since the design, storage, and encoding of large codebooks can be challenging in massive MIMO. [4], [5] have proposed the analog channel feedback schemes by exploiting the spatial correlation of CSI to reduce the feedback overhead. However, the CSI is assumed to be perfectly known at the users, which is usually unrealistic.

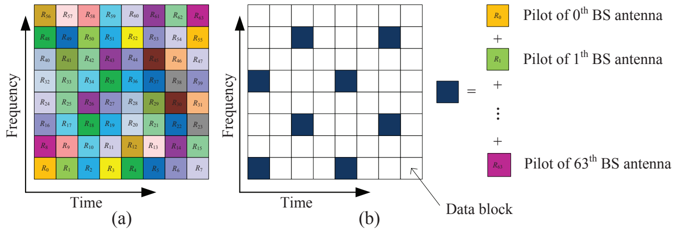

Fig. 1. Pilots for massive MIMO with $M = 64$. (a) Conventional orthogonal pilots; (b) Proposed non-orthogonal pilots.

Experiments and theoretical analysis have demonstrated that time-domain massive MIMO channel response impulses (CIRs) exhibit spatial common sparsity due to the finite significant scatterers and the compact BS antenna array [1]. Moreover, such spatial common sparsity remains almost unchanged within the coherence time due to the temporal correlation of channels, which is also referred as temporal common sparsity of wireless channels [2]. The spatial and temporal common sparsity of massive MIMO channels inspires us to utilize the emerging compressive sensing (CS) theory [6] to estimate and feedback channels with significantly reduced overhead. In this paper, we propose the block compressive channel estimation and feedback scheme for FDD massive MIMO. We first propose the non-orthogonal pilot design. Then we present the block compressive channel estimation and feedback scheme, where the block orthogonal matching pursuit (BOMP) algorithm is proposed to acquire CSI with reduced overhead. Furthermore, we exploit the temporal common sparsity of channels to estimate channels with reduced complexity.

II. PROPOSED CHANNEL ESTIMATION AND FEEDBACK

The conventional orthogonal pilots are widely used in MIMO systems as shown in Fig. 1 (a). Clearly, conventional orthogonal pilots suffer from prohibitively high pilot overhead when the number of BS antennas M becomes large, e.g., $M = 64$ as shown in Fig. 1 (a). By contrast, the proposed non-orthogonal pilots, as shown in Fig. 1 (b), allows pilots of different BS antennas to occupy the completely same subcarriers. Hence, the pilot overhead could be substantially reduced. We use ξ to denote the placement set shared by pilot sequences $\{\mathbf{p}_m\}_{m=0}^{M-1} \in \mathbb{C}^{N_p}$, where elements of $\{\mathbf{p}_m\}_{m=0}^{M-1}$ following i.i.d. complex Gaussian distribution $\mathcal{CN}(0,1)$ is considered in this paper, and N_p is the channel estimation overhead (also the feedback overhead in the proposed scheme).

At the user, the received downlink pilot sequence $\mathbf{y} \in \mathbb{C}^{N_p}$ can be expressed as $\mathbf{y} = \sum_{m=0}^{M-1} \mathrm{diag}\{\mathbf{p}_m\}\mathbf{F}_L|_{\xi}\mathbf{h}_m + \mathbf{w} = \sum_{m=0}^{M-1} \boldsymbol{\Phi}_m\mathbf{h}_m + \mathbf{w} = \boldsymbol{\Phi}\tilde{\mathbf{h}} + \mathbf{w}$, where $\mathbf{F}_L \in \mathbb{C}^{N\times L}$ is a partial discrete Fourier transmission (DFT) matrix whose elements

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Algorithm 1 Proposed BOMP Algorithm.

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Input: Noisy measurement matrix $\mathbf{z}$ and sensing vector $\mathbf{\Phi}$.
Output: Estimated aggregate CIR $\tilde{\mathbf{h}}$.
$\Omega \leftarrow \emptyset$, $k \leftarrow 1$, $\hat{\tilde{\mathbf{h}}} \leftarrow \mathbf{0}$;
while $\|\mathbf{v}_k\|_2 < \|\mathbf{v}_{k-1}\|_2$, do
    $\mathbf{v}_k \leftarrow \mathbf{z} - \mathbf{\Phi}^H \hat{\tilde{\mathbf{h}}}$;   % $(\cdot)^H$ denotes the conjugate transpose.
    $\mathbf{c} \leftarrow \mathbf{\Phi}^H \mathbf{v}_k$;   % $\mathbf{c} = \left[c^{(0)}, c^{(1)}, \cdots, c^{(ML-1)}\right]^T$.
    $s(\tau) \leftarrow \sum_{m=0}^{M-1} \left|c^{(\tau+mL)}\right|^2$, $0 \le \tau \le L-1$;
    $\Omega \leftarrow \Omega \cup \max_{\tau} \{s(\tau), 0 \le \tau \le L-1\}$;
    $\Gamma \leftarrow \Omega \cup [\Omega + L] \cup \cdots \cup [\Omega + L(M-1)]$;   % $[\Omega + a]$ means to add $a$ to each element of the set $\Omega$.
    $\hat{\tilde{\mathbf{h}}}\Big|_{\Gamma} \leftarrow \mathbf{\Phi}_{\Gamma}^{\dagger} \mathbf{z}$;   % $(\cdot)^{\dagger}$ denotes the Moore-Penrose matrix inversion, and $\mathbf{\Phi}_{\Gamma}$ denotes the sub-matrix by selecting the columns of $\mathbf{\Phi}$ according to the set $\Gamma$.
    $k \leftarrow k + 1$;
end while
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are the first L columns of the N-dimension DFT matrix, $\mathbf{F}_L|_{\xi} \in \mathbb{C}^{N_p \times L}$ denotes the sub-matrix by selecting the rows of $\mathbf{F}_L$ according to the set ξ, N is the size of the OFDM symbol, $\mathbf{w} \in \mathbb{C}^{N_p}$ is the AWGN vector, $\mathbf{\Phi}_m = \mathrm{diag}\{\mathbf{p}_m\}\mathbf{F}_L|_{\xi}$, $\mathbf{\Phi} = [\mathbf{\Phi}_0, \mathbf{\Phi}_1, \cdots, \mathbf{\Phi}_{M-1}] \in \mathbb{C}^{N_p \times ML}$, $\mathbf{h}_m \in \mathbb{C}^L$ for $0 \le m \le M-1$ is the CIR associated with the mth BS antenna, $\tilde{\mathbf{h}} = [\mathbf{h}_0^T, \mathbf{h}_1^T, \cdots, \mathbf{h}_{M-1}^T]^T \in \mathbb{C}^{ML}$ is an aggregate CIR vector, and L is the channel length.

For channel feedback, the user directly feedbacks the received pilot sequence $\mathbf{y}$ to the BS, where the feedback channel can be considered as the AWGN channel [5]. Hence, the acquired feedback signal at the BS can be expressed as $\mathbf{z} = \mathbf{\Phi}\tilde{\mathbf{h}} + \mathbf{n}$, where $\mathbf{n} \in \mathbb{C}^{N_p}$ is the equivalent noise vector [5].

Conventionally, $N_p \ge ML$ can guarantee the reliable CSI acquisition, which will lead to the unaffordable overhead due to large M in massive MIMO. The inherent sparsity of time-domain channels inspires us to exploit the emerging CS theory to acquire channels with reduced overhead [6]. Moreover, the spatial common sparsity of time-domain massive MIMO channels can be also integrated for further performance enhancement, i.e., $\mathrm{supp}\{\mathbf{h}_0\} = \mathrm{supp}\{\mathbf{h}_1\} = \cdots = \mathrm{supp}\{\mathbf{h}_{M-1}\}$, where $\mathrm{supp}\{\mathbf{h}_m\} = \left\{\tau : \left|{h_m}^{(\tau)}\right| > 0, 0 \le \tau \le L-1\right\}$ and ${h_m}^{(\tau)}$ is the τth element of $\mathbf{h}_m$. Based on the classical orthogonal matching pursuit (OMP) algorithm, we propose the BOMP algorithm as shown in **Algorithm 1**, which can estimate $\tilde{\mathbf{h}}$ with enhanced accuracy by exploiting the spatial common sparsity.

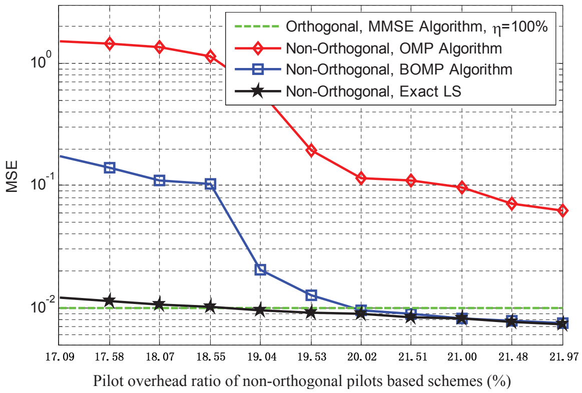

Fig. 2. MSE performance comparison of different CSI acquisition schemes. Note that the orthogonal pilot based MMSE algorithm adopts the fixed pilot overhead ratio with $\eta = 100\%$.

Furthermore, we can exploit the temporal common sparsity of channels within the coherence time, i.e., $\mathrm{supp}\{\mathbf{h}_{m,1}\} = \mathrm{supp}\{\mathbf{h}_{m,2}\} = \cdots = \mathrm{supp}\{\mathbf{h}_{m,R}\}$, where R ensures the common sparsity of CIRs in R successive time slots. However, the channel gains change along with the time and need to be estimated frequently, since the variance rate of channel gains is much faster than that of channel delays [2]. Hence, by exploiting such temporal common sparsity, we can use the simple least squires (LS) estimator to estimate channels in the following $R-1$ time slots when the support of channels is acquired by using the BOMP algorithm in the first time slot, which can reduce the complexity of CSI acquisition.

III. SIMULATION RESULTS

A simulation study was carried out to investigate the performance of the proposed block compressive channel estimation and feedback scheme for FDD massive MIMO systems. Carrier frequency was $f_c = 2$ GHz, system bandwidth was $f_s = 10$ MHz, $N = 2048$, $M = 32$, the length of estimated channels was $L = 64$ [2], and the signal to noise ratio (SNR) was 20 dB. Orale LS algorithm is also plotted as the performance benchmark. The ITU-VA channel model with six-paths was adopted [2]. As shown in Fig. 2, the proposed scheme can approach the performance bound with the pilot overhead ratio $\eta = N_p/N = 20.02\%$. In other words, the average overhead for CSI acquisition per antenna is $N_p/M = 12.81$, which approaches $2 \times 6 = 12$, the theoretical bound to recovery the six-path channels according to the CS theory [6]. However, to achieve such performance, the conventional orthogonal pilot based minimum mean square error (MMSE) algorithm needs the pilot overhead ratio $\eta = 100\%$, since the conventional scheme requires $N_p = ML$.

IV. CONCLUSIONS

We have proposed a block compressive channel estimation and feedback scheme for FDD massive MIMO, where non-orthogonal pilots was introduced to reduce the overhead for CSI acquisition. By exploiting the spatial common sparsity of massive MIMO channels, the proposed BOMP algorithm can approach the theoretical bound for sparse signal recovery in C-S. Moreover, we can reduce the complexity of CSI acquisition by utilizing the temporal common sparsity of channels.

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