DIRECT GEOLOCATION OF STATIONARY WIDEBAND RADIO SIGNAL BASED ON TIME DELAYS AND DOPPLER SHIFTS

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ABSTRACT

Contrary to the suboptimal (two-step) geolocation procedures, we propose a maximum likelihood estimation for the position of a stationary transmitter which its delayed and Doppler shifted signal is observed by moving receivers. The position is estimated based on the same data used in common methods. However, it is performed in a single step by maximizing a cost function that depends on the unknown position only.

Index Terms— Emitter location, Maximum likelihood estimation, Differential Doppler, Ambiguity function.

1. INTRODUCTION

Passive geolocation of a stationary transmitter based on the delayed and Doppler shifted signal observed by moving receivers is a well known technique as can be concluded from [1]-[4]. Since the receivers' location and velocity along their trajectory are known, the transmitter location can be estimated.

Common methods use two steps for localization. Each receiver first estimates the time delay and Doppler frequency along its trajectory. In the second step the system estimates the transmitter's position based on the results obtained in the first step. The two step methods are not guaranteed to yield optimal location results since in the first step the delay and Doppler estimates are obtained by ignoring the constraint that all measurements must be consistent with a single position. Thus, the lines of position obtained from the delay and Doppler estimates are not necessarily intersect in a single geographical location.

In a previous publication we proposed a single-step solution for narrowband signals by considering the Doppler shift only [9]. Herein we propose a maximum likelihood position estimation using a single step for wideband signals by also taking into account the time delay of the signal.

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2. PROBLEM FORMULATION

Consider a stationary radio transmitter located at position $\mathbf{p}$ and L moving receivers. The receivers are synchronized in frequency and time. Each receiver intercepts the transmitted signal at K short intervals along its trajectory. Let $\mathbf{p}_{\ell,k}$ and $\mathbf{v}_{\ell,k}$ where $k = 1, \ldots, K$ and $\ell = 1, \ldots, L$ denote the position and velocity vectors of the ℓ-th receiver at the k-th interception interval, respectively. The complex signal observed by the ℓ-th receiver at the k-th interception interval at time t is

$$r_{\ell,k}(t) = b_{\ell,k} s_k(t-\tau_{\ell,k}) e^{j2\pi f_{\ell,k} t} + w_{\ell,k}(t),\ 0 \le t \le T \quad (1)$$

where T is the observation time interval, $s_k(t)$ is the observed signal during the k-th interception interval, $b_{\ell,k}$ is an unknown complex path attenuation, and $\tau_{\ell,k} \triangleq \frac{1}{c}\|\mathbf{p}_{\ell,k} - \mathbf{p}\|$ is the signal's delay where c is the signal's propagation speed. Similarly to [5] we assume that $\tau_{\ell,k} \ll T$. Also, $w_{\ell,k}(t)$ is a white, zero mean, complex Gaussian noise with flat spectrum and,

$$f_{\ell,k} \triangleq f_c\,[1 + \mu_{\ell,k}(\mathbf{p})] \quad (2)$$

$$\mu_{\ell,k}(\mathbf{p}) \triangleq \frac{1}{c}\mathbf{v}_{\ell,k}^T(\mathbf{p} - \mathbf{p}_{\ell,k})/\|\mathbf{p} - \mathbf{p}_{\ell,k}\| \quad (3)$$

where f_c is the known nominal carrier frequency of the transmitted signal. Each receiver performs a down conversion of the intercepted signal by f_c and thus (2) can be replaced by $\bar{f}_{\ell,k} = f_c \mu_{\ell,k}(\mathbf{p})$. Assume that we collect N time samples, sampled with sampling interval T_s, of the down converted version of (1). Define

$$\begin{aligned} \mathbf{r}_{\ell,k} &\triangleq [r_{\ell,k}(t_1), \ldots, r_{\ell,k}(t_N)]^T \\ \mathbf{w}_{\ell,k} &\triangleq [w_{\ell,k}(t_1), \ldots, w_{\ell,k}(t_N)]^T \\ \mathbf{s}_k &\triangleq [s_k(t_1) \ldots, s_k(t_N)]^T \\ \mathbf{A}_{\ell,k} &\triangleq \text{diag}\{e^{j2\pi \bar{f}_{\ell,k} t_1}, \ldots, e^{j2\pi \bar{f}_{\ell,k} t_N}\} \end{aligned} \quad (4)$$

From the down converted version of (1) we get

$$\mathbf{r}_{\ell,k} = b_{\ell,k} \mathbf{A}_{\ell,k} \mathbf{F}_{\ell,k} \mathbf{s}_k + \mathbf{w}_{\ell,k} \quad (5)$$

where $\mathbf{F}_{\ell,k}$ is a down shift operator, i.e., $\mathbf{F}_{\ell,k}\mathbf{s}_k$ shifts $\mathbf{s}_k$ by $\lfloor \tau_{\ell,k}/T_s \rfloor$ indices. We assume that the covariance of $\mathbf{w}_{\ell,k}$ is $\sigma^2\mathbf{I}$.

The problem discussed herein can be stated as follows: Given $\{\mathbf{r}_{\ell,k}\}_{\ell,k=1}^{L,K}$, estimate the the transmitter's position $\mathbf{p}$.

3. THE DIRECT GEOLOCATION APPROACH

The log likelihood of $\{\mathbf{r}_{\ell,k}\}$ is equivalent, up to an additive constant and scaling to

$$C(\mathbf{p}) = \sum_{k=1}^{K}\sum_{\ell=1}^{L} \|\mathbf{r}_{\ell,k} - b_{\ell,k}\mathbf{A}_{\ell,k}\mathbf{F}_{\ell,k}\mathbf{s}_k\|^2 \tag{6}$$

The path attenuation scalars that minimize (6) are given by

$$\begin{aligned} \hat{b}_{\ell,k} &= [(\mathbf{A}_{\ell,k}\mathbf{F}_{\ell,k}\mathbf{s}_k)^H\mathbf{A}_{\ell,k}\mathbf{F}_{\ell,k}\mathbf{s}_k]^{-1}(\mathbf{A}_{\ell,k}\mathbf{F}_{\ell,k}\mathbf{s}_k)^H\mathbf{r}_{\ell,k} \\ &= (\mathbf{A}_{\ell,k}\mathbf{F}_{\ell,k}\mathbf{s}_k)^H\mathbf{r}_{\ell,k} \end{aligned} \tag{7}$$

where we assume, without loss of generality, that $\|\mathbf{s}_k\|^2 = 1$. Substituting (7) in (6) yields,

$$C_1(\mathbf{p}) = \sum_{k=1}^{K}\sum_{\ell=1}^{L} \|\mathbf{r}_{\ell,k}\|^2 - |(\mathbf{A}_{\ell,k}\mathbf{F}_{\ell,k}\mathbf{s}_k)^H\mathbf{r}_{\ell,k}|^2 \tag{8}$$

Since $\|\mathbf{r}_{\ell,k}\|^2$ is independent of the parameters, then instead of minimizing (8) we can now maximize

$$C_2(\mathbf{p}) = \sum_{k=1}^{K}\sum_{\ell=1}^{L} |(\mathbf{A}_{\ell,k}\mathbf{F}_{\ell,k}\mathbf{s}_k)^H\mathbf{r}_{\ell,k}|^2 = \sum_{k=1}^{K}\mathbf{s}_k^H\mathbf{Q}_k\mathbf{s}_k \tag{9}$$

where we defined the $N \times N$ hermitian matrix

$$\mathbf{Q}_k \triangleq \mathbf{V}_k\mathbf{V}_k^H \tag{10}$$

$$\mathbf{V}_k \triangleq [\mathbf{F}_{1,k}^H\mathbf{A}_{1,k}^H\mathbf{r}_{1,k}, \cdots, \mathbf{F}_{L,k}^H\mathbf{A}_{L,k}^H\mathbf{r}_{L,k}] \tag{11}$$

If the signal waveform is known, then $C_2(\mathbf{p})$ should be used.

However, often the signal waveform is unknown and then one should look for $\{\mathbf{s}_k\}$ that maximize $C_2(\mathbf{p})$. This cost function is maximized by maximizing each of the K quadratic forms w.r.t. $\mathbf{s}_k$. Thus, the vector $\mathbf{s}_k$ should be selected as the eigenvector corresponding to the largest eigenvalue of $\mathbf{Q}_k$ denoted by $\lambda_{max}\{\mathbf{Q}_k\}$. The dimension of the matrix $\mathbf{Q}_k$ increases with the number of data samples. Determining the eigenvalues of $\mathbf{Q}_k$ can in turn result in high computation effort. However, the non-zero eigenvalues of $\mathbf{Q}_k$ are identical to the eigenvalues of the $L \times L$ matrix $\bar{\mathbf{Q}}_k \triangleq \mathbf{V}_k^H\mathbf{V}_k$. See [6, pp. 42-43]. This leads to a substantial reduction of the computation load whenever $L \ll N$. Therefore, (9) is equivalent to

$$C_3(\mathbf{p}) = \sum_{k=1}^{K}\lambda_{max}\{\bar{\mathbf{Q}}_k\} \tag{12}$$

The estimated transmitter's position is then given by

$$\hat{\mathbf{p}} = \underset{\mathbf{p}}{\text{argmax}}\{C_3(\mathbf{p})\} \tag{13}$$

A possible algorithm for unknown signals is displayed in Algorithm 1.

It is interesting to observe that the (i,j)-th element of $\bar{\mathbf{Q}}_k$ can be written as,

$$\begin{aligned} \bar{\mathbf{Q}}_k(i,j) &= \mathbf{r}_{i,k}^H\mathbf{A}_{i,k}\mathbf{F}_{i,k}\mathbf{F}_{j,k}^H\mathbf{A}_{j,k}^H\mathbf{r}_{j,k} \\ &\cong \frac{1}{T_s}e^{j2\pi(\bar{f}_{i,k}\tau_{i,k}-\bar{f}_{j,k}\tau_{j,k})}\rho_{i,j}^{(k)} \end{aligned} \tag{14}$$

where

$$\rho_{i,j}^{(k)} \triangleq \int_0^T r_{i,k}^*(t+\tau_{i,k})r_{j,k}(t+\tau_{j,k})e^{j2\pi(\bar{f}_{i,k}-\bar{f}_{j,k})t}\,dt \tag{15}$$

Note that $\bar{f}_{j,k}, \bar{f}_{i,k}, \tau_{j,k}, \tau_{i,k}$ are all functions of the assumed target position $\mathbf{p}$ and that $\rho_{i,j}^{(k)}$ is recognized as the complex ambiguity function evaluated at the geographical point $\mathbf{p}$ [5].

> Define the area of interest and determine a suitable grid of locations $\mathbf{p}_1, \mathbf{p}_2 \cdots \mathbf{p}_g$.
> **for** $j = 1$ **to** g **do**
>   Set $C_3(\mathbf{p}_j) = 0$
>   **for** $k = 1$ **to** K **do**
>   **for** $\ell = 1$ **to** L **do**
>   Evaluate $\tau_{\ell,k}, \bar{f}_{\ell,k}$
>   Evaluate $\mathbf{A}_{\ell,k}$, $\mathbf{F}_{\ell,k}$
>   **end**
>   Evaluate $\mathbf{V}_k$ according to (11)
>   Evaluate $\bar{\mathbf{Q}}_k = \mathbf{V}_k^H\mathbf{V}_k$
>   Let $C_3(\mathbf{p}_j) = C_3(\mathbf{p}_j) + \lambda_{\max}\{\bar{\mathbf{Q}}_k\}$
>   **end**
> **end**
> Find the grid point for which C_3 is the largest. This grid point is the estimated position.

Algorithm 1: A possible implementation of the DPD algorithm for unknown signals.

4. RELATIONS BETWEEN THE DIRECT AND INDIRECT APPROACH

To simplify the exhibition consider the case of only two moving receivers ($L = 2$), as discussed in [7],[8]. Then $\bar{\mathbf{Q}}_k$ is a 2×2 hermitian matrix, and its largest eigenvalue is given by

$$\begin{aligned} \lambda_{\max}\{\bar{\mathbf{Q}}_k\} &= \frac{1}{2}(\bar{\mathbf{Q}}_k(1,1) + \bar{\mathbf{Q}}_k(2,2) + \\ &\sqrt{(\bar{\mathbf{Q}}_k(1,1) - \bar{\mathbf{Q}}_k(2,2))^2 + 4|\bar{\mathbf{Q}}_k(1,2)|^2}) \end{aligned} \tag{16}$$

where

$$\bar{\mathbf{Q}}_k(i,i) \cong \frac{1}{T_s}\int_0^T |r_{i,k}(t)|^2\,dt \ , i=1,2 \quad (17)$$

$$\bar{\mathbf{Q}}_k(1,2) \cong \frac{1}{T_s} e^{j2\pi(\bar{f}_{1,k}\tau_{1,k}-\bar{f}_{2,k}\tau_{2,k})}\rho_{1,2}^{(k)} \quad (18)$$

Assuming that $\bar{\mathbf{Q}}_k(1,1) \cong \bar{\mathbf{Q}}_k(2,2)$, the cost function $C_3(\mathbf{p})$ can be replaced with

$$\tilde{C}_3(\mathbf{p}) = \sum_{k=1}^{K} |\bar{\mathbf{Q}}_k(1,2)| \cong \frac{1}{T_s}\sum_{k=1}^{K}\left|\rho_{1,2}^{(k)}\right| \quad (19)$$

where $|\rho_{1,2}^{(k)}|$ has been used in [5] for estimating the delay and Doppler at each of the interception intervals along the trajectory. Therefore, the proposed method selects the position that maximizes the sum of the distinct cost functions used in the two-step method.

5. CRAMÉR-RAO LOWER BOUND

The Cramér-Rao Lower Bound (CRLB) is a lower bound on the covariance of any unbiased estimator. The bound is given by the inverse of the Fisher Information Matrix (FIM). For complex Gaussian data vectors with parameters embedded in their mean, $\mathbf{m}$, and not in the covariance, the (i,j)-th element of the FIM is given by [10]

$$[\mathbf{J}]_{i,j} = \frac{2}{\sigma^2}\mathrm{Re}\left\{\frac{\partial \mathbf{m}^H}{\partial \psi_i}\frac{\partial \mathbf{m}}{\partial \psi_j}\right\} \quad (20)$$

where ψ_i is the i-th element of the unknown parameter vector. For simplicity, we assume that the parameter vector is the target coordinate vector only, i.e., $\psi_1 = x,\ \psi_2 = y$. Also, from (5), the data mean is given by

$$\begin{aligned}
\mathbf{m} &\triangleq [\mathbf{m}_1^T, \mathbf{m}_2^T, \cdots \mathbf{m}_K^T]^T \\
\mathbf{m}_k &\triangleq [\mathbf{m}_{1,k}^T, \mathbf{m}_{2,k}^T, \cdots \mathbf{m}_{L,k}^T]^T \\
\mathbf{m}_{\ell,k} &\triangleq b_{\ell,k}\mathbf{A}_{\ell,k}\mathbf{F}_{\ell,k}\mathbf{s}_k = b_{\ell,k}\mathbf{A}_{\ell,k}\tilde{\mathbf{s}}_{\ell,k} \quad (21)
\end{aligned}$$

where

$$\tilde{\mathbf{s}}_{\ell,k} \triangleq [s_k(t_1-\tau_{\ell,k}), \ldots s_k(t_N-\tau_{\ell,k})]^T \quad (22)$$

We are interested in the derivatives of the mean w.r.t. the target coordinates $\mathbf{p} = [x,y]^T$. Using the chain rule we get

$$\frac{\partial \mathbf{m}_{\ell,k}}{\partial x} = \frac{\partial \mathbf{m}_{\ell,k}}{\partial \bar{f}_{\ell,k}}\frac{\partial \bar{f}_{\ell,k}}{\partial x} + \frac{\partial \mathbf{m}_{\ell,k}}{\partial \tau_{\ell,k}}\frac{\partial \tau_{\ell,k}}{\partial x} \quad (23)$$

where

$$\begin{aligned}
\frac{\partial \mathbf{m}_{\ell,k}}{\partial \bar{f}_{\ell,k}} &= b_{\ell,k}\dot{\mathbf{A}}_{\ell,k}\tilde{\mathbf{s}}_{\ell,k}, \quad \frac{\partial \mathbf{m}_{\ell,k}}{\partial \tau_{\ell,k}} = b_{\ell,k}\mathbf{A}_{\ell,k}\dot{\tilde{\mathbf{s}}}_{\ell,k} \\
\dot{\mathbf{A}}_{\ell,k} &= j2\pi \mathrm{diag}\{t_1 e^{j2\pi\bar{f}_{\ell,k}t_1}, \ldots, t_N e^{j2\pi\bar{f}_{\ell,k}t_N}\} \\
\dot{\tilde{\mathbf{s}}}_{\ell,k} &= -[\dot{s}_k(t_1-\tau_{\ell,k}), \ldots \dot{s}_k(t_N-\tau_{\ell,k})]^T \quad (24)
\end{aligned}$$

Note that

$$\begin{aligned}
\frac{c}{f_c}\bar{f}_{\ell,k} &= \frac{\mathbf{v}_{\ell,k}^T(\mathbf{p}-\mathbf{p}_{\ell,k})}{d_{\ell,k}} \quad (25)\\
\frac{c}{f_c}\dot{\bar{f}}_{\ell,k}^{(x)} &= \frac{c}{f_c}\frac{\partial \bar{f}_{\ell,k}}{\partial x} = \frac{v_{\ell,k}^x}{d_{\ell,k}} - \frac{\|\mathbf{v}_{\ell,k}\|}{d_{\ell,k}}\cos\phi_{\ell,k}\cos\theta_{\ell,k} \\
\frac{c}{f_c}\dot{\bar{f}}_{\ell,k}^{(y)} &= \frac{c}{f_c}\frac{\partial \bar{f}_{\ell,k}}{\partial y} = \frac{v_{\ell,k}^y}{d_{\ell,k}} - \frac{\|\mathbf{v}_{\ell,k}\|}{d_{\ell,k}}\cos\phi_{\ell,k}\sin\theta_{\ell,k}
\end{aligned}$$

where, ϕ is the angle between the receiver velocity vector and the line connecting the receiver and the transmitter, θ is the angle between the line connecting the receiver and the transmitter and the x axis, and $c\tau_{\ell,k} = d_{\ell,k} \triangleq \|\mathbf{p}-\mathbf{p}_{\ell,k}\|$. Also note that

$$c\dot{\tau}_{\ell,k}^{(x)} \triangleq c\frac{\partial \tau_{\ell,k}}{\partial x} = \cos\theta_{\ell,k}; \quad c\dot{\tau}_{\ell,k}^{(y)} \triangleq c\frac{\partial \tau_{\ell,k}}{\partial y} = \sin\theta_{\ell,k}$$

The elements of the FIM are given by

$$\begin{aligned}
[\mathbf{J}]_{1,1} &= \frac{2}{\sigma^2}\sum_{k,\ell}|b_{\ell,k}|^2\|\dot{\mathbf{A}}_{\ell,k}\tilde{\mathbf{s}}_{\ell,k}\dot{\bar{f}}_{\ell,k}^{(x)} + \mathbf{A}_{\ell,k}\dot{\tilde{\mathbf{s}}}_{\ell,k}\dot{\tau}_{\ell,k}^{(x)}\|^2 \\
[\mathbf{J}]_{2,2} &= \frac{2}{\sigma^2}\sum_{k,\ell}|b_{\ell,k}|^2\|\dot{\mathbf{A}}_{\ell,k}\tilde{\mathbf{s}}_{\ell,k}\dot{\bar{f}}_{\ell,k}^{(y)} + \mathbf{A}_{\ell,k}\dot{\tilde{\mathbf{s}}}_{\ell,k}\dot{\tau}_{\ell,k}^{(y)}\|^2 \\
[\mathbf{J}]_{1,2} &= \frac{2}{\sigma^2}\sum_{k,\ell}|b_{\ell,k}|^2\,\mathrm{Re}\{[\dot{\mathbf{A}}_{\ell,k}\tilde{\mathbf{s}}_{\ell,k}\dot{\bar{f}}_{\ell,k}^{(x)} + \mathbf{A}_{\ell,k}\dot{\tilde{\mathbf{s}}}_{\ell,k}\dot{\tau}_{\ell,k}^{(x)}]^H \\
&\qquad [\dot{\mathbf{A}}_{\ell,k}\tilde{\mathbf{s}}_{\ell,k}\dot{\bar{f}}_{\ell,k}^{(y)} + \mathbf{A}_{\ell,k}\dot{\tilde{\mathbf{s}}}_{\ell,k}\dot{\tau}_{\ell,k}^{(y)}]\} \quad (26)
\end{aligned}$$

with $[\mathbf{J}]_{2,1} = [\mathbf{J}]_{1,2}$.

6. NUMERICAL EXAMPLES

In this section we describe numerical examples which compare the proposed method with the two-steps approach [5]. Figure 1 shows a stationary emitter and two receivers flying at the same speed of 300 [meters/sec] from left to right. The receivers estimate the emitter position and the resulting mean square location error (RMSE) is displayed in Figure 2.

The transmitted signal is a modulated carrier at 1 [GHz]. The modulating signal is a pulse train with a bandwidth of 3.3 [MHz] and a duty cycle of 15%. The sampling rate used by the receivers is 8.4 [MHz]. The observation time interval is 61 [μsec].

Each of the receivers intercepts the signal 5 times, once every 1000 [meters].

As observed in Figure 2, the proposed approach outperforms the two-step approach. Also, as expected, better results are obtained when the signal waveform is known.

7. CONCLUSIONS

We presented a maximum likelihood estimator for position determination of a stationary radio transmitter based on the

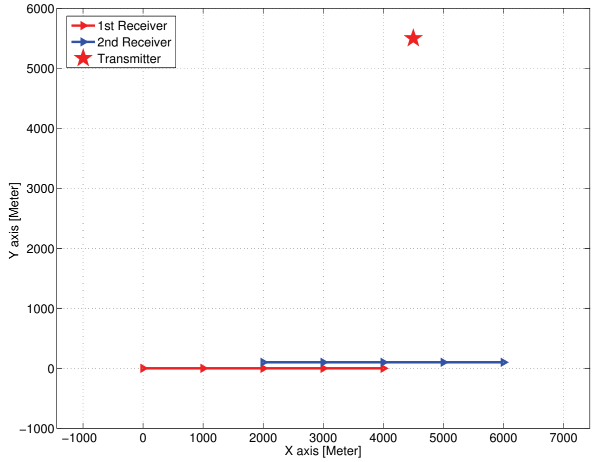

Fig. 1. Receivers and transmitter geometry.

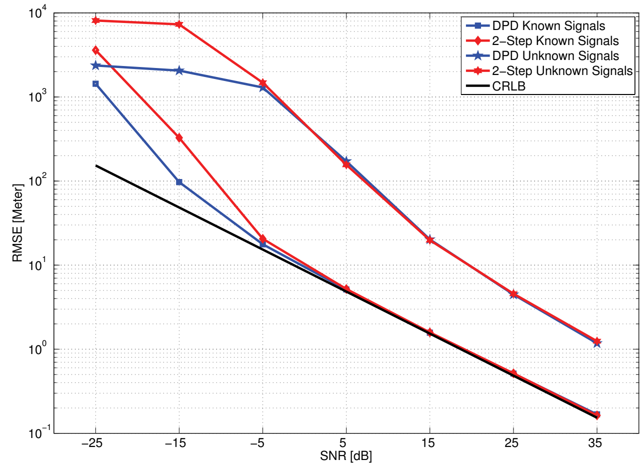

Fig. 2. RMSEs of the DPD method, the 2-step method, and the CRLB versus SNR for known and unknown transmitted signals.

delayed and Doppler shifted signals observed by moving receivers. Contrary to the conventional methods the position is determined by a single step without first explicitly estimating the Doppler shift and the differential delays. As expected, the performance of the single step approach is better. However, in order to carry out the proposed method all the observed signals must be transmitted to a common processor. The signal transmission requires more bandwidth than the transmission of just Doppler and delay measurements.

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