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THE MODELLING OF ELECTRIC POWER SYSTEMS ON THE STATE SPACE AND CONTROLLING OF OPTIMAL LQR LOAD FREQUENCY

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ABSTRACT

The modelling of electric power systems on the state space and an optimal control system known as Linear Quadratic Regulator (LQR) for designing the load frequency control system are realized in this paper. In order to obtain high quality electric energy on electric power systems, the fluctuations on voltage and frequency must be decreased as much as possible. The frequency sensitivity of the system must be reduced to minimum level against to load changes. To obtain minimum frequency sensitivity against to load changes, an optimal control known as Linear Quadratic Regulator (LQR) is designed. In the study, the simplified and block diagram of the electric power system is employed. By using this diagram, the state equations for the power system are obtained on the state space. In the load frequency control of power system, control system via pole-placement and optimal LQR control method known as modern control with high reability rate are used. The proposed optimal LQR load frequency has been compared with control system via pole-placement. Load frequency control system of electric power system has been developed designed by using MATLAB/SIMULINK.

Keywords: *Controlling of optimal Lqr load frequency, modelling, Matlab/SIMULINK, Power systems*

1.INTRODUCTION

The purpose of operating the load frequency control is to keep uniform the frequency changes during the load sharing under definite limits. Controller must be sensitive against to changes in frequency and load. The main change parameters during operation of the power system are rotor angle (δ), fault angle (Δδ), change in frequency (Δf) and active power flowing between the connection lines (ΔPv) .

If the control system will only be realized at a computer medium , the mathematical model of the system is required. For this reason, in analysis and design of the control system, firstly the mathematical model must be established. There are two models widely used in the mathematical model of the power system. First of them is the transfer function model and the second one is the state variable approach [1] .

The state variable approach is used to define the non-linear systems as linear. To use the transfer function of the power system and the state equations, we must firstly make the system linear. The mathematical equations that define the power system to obtain the components of the power system are made linear with suitable and accepted hypothesis. Then the block diagram of the power system is established.

2. THE MODEL OF POWER SYSTEM

The block diagram of a power system containing the load frequency and automatic tension regulator is given in Fig. 1 above. To regulate the load

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frequency of the system in Fig. 1, by obtaining the linear equations of the power system, a block diagram is established for control. In the given power system, the linear expressions of the load, turbine, amplifier and feed-back element are obtained. The linear model is sufficient to express the dynamic behavior of the system around the working point [2].

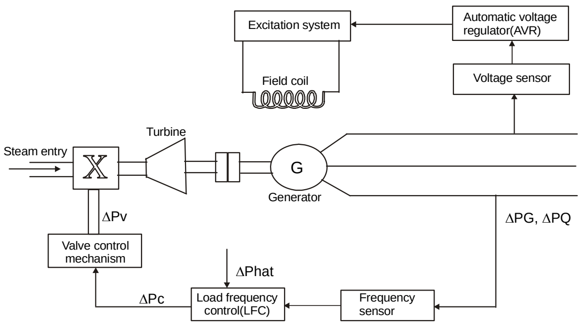

Fig. 1 *The block diagram containing the automatic tension and load frequency regulator of a power system*

To obtain the equation of the system on state space, it is necessary to determine the variable parameters in the power system. The block diagram of the load frequency control of a linear power system is given in Fig. 2 below.

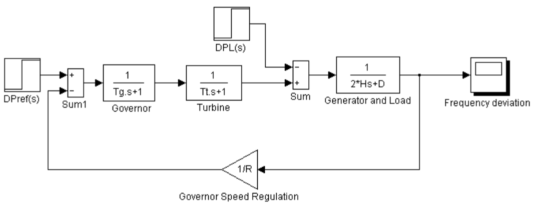

Fig. 2 *Simulation block diagram of the load frequency control of electric load system*

The control process is made with the feed-back state variables by means of the fix gaining regulators. The control system expression given in a manner of feed-back state variables.

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad (1)$$
$$y(t) = Cx(t)$$

is expressed like this. The block diagram of the feed-back power system is given in figure 3.

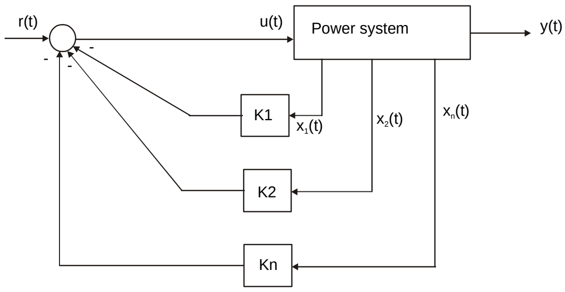

Fig. 3*.The control system design by means of pole placement*

We state the mathematical expression of the control system with pole placement as in Fig. 3 as;

$$u(t) = -Kx(t) \qquad (2)$$

Where K is defined in vector type as 1 xn of the feed-back gaining. The used control system aims to convert the state variables to zero when the state variables in the system are in undesired position. The control system expression expressed in equation (1) and (2) is given as follows when the fix gaining factor is considered.

$$\dot{X} = (A - BK)x(t) = A_f x(t) \qquad (3)$$

If in the characteristic equation of the obtained system

$$|sI - A + BK)| = 0 \qquad (4)$$

we put the variables in matrix form ,the state equation of the system is obtained as follows

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 & -a_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u(t) \qquad (5)$$

The state variables of the obtained control system is expressed as x(t), the entrance vector of the system as u(t) and the exit factor of the system as Cx(t). The state equations are obtained by using the block diagram in "s" domain of the load frequency control as given in Fig. 2.

$$(1 + \tau_g s)\Delta P_V(s) = \Delta Pef - \frac{1}{R}\Delta f(s)$$
$$(1 + \tau_T s)\Delta P_m(s) = \Delta P_V \qquad (6)$$
$$(2Hs + D)\Delta f(s) = \Delta P_m - \Delta P_L$$

Parameters that are expressed in (6).

τg: speed regulator time constant ,
ΔPv: change power in turbine valve position ,
ΔPref: Reference power change ,
R: Regulator gaining ,
τT: Turbine time constant,
ΔPm: Change at turbine mechanic exit power,
Δf: Change at frequency,
ΔPL: Change at load.

When the first derivatives of equation (6) is considered for the solution;

$$s\Delta P_V(s) = -\frac{1}{\tau_g}\Delta P_V - \frac{1}{R\tau_g}\Delta f(s) + \frac{1}{\tau}\Delta P_{ref}(s) \qquad (7)$$

$$s\Delta P_m(s) = \frac{1}{\tau_T}\Delta P_V - \frac{1}{\tau_T}\Delta P_m \qquad (8)$$

$$s\Delta f(s) = \frac{1}{2H}\Delta P_m - \frac{D}{2H}\Delta f(s) - \frac{1}{2H}\Delta P_L \qquad (9)$$

When the (7), (8) and (9) equations expressed in time domain are arranged in matrix form according to the equation (1) as state equations;

$$\begin{bmatrix} \dot{\Delta P_V} \\ \dot{\Delta P_m} \\ \dot{\Delta f} \end{bmatrix} = \begin{bmatrix} -\frac{1}{\tau_g} & 0 & -\frac{1}{R\tau_g} \\ \frac{1}{\tau_T} & -\frac{1}{\tau_T} & 0 \\ 0 & \frac{1}{2H} & -\frac{D}{2H} \end{bmatrix} \cdot \begin{bmatrix} \Delta P_V \\ \Delta P_m \\ \Delta f \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{-1}{2H} \end{bmatrix} \cdot \Delta P_L + \begin{bmatrix} \frac{1}{\tau_g} \\ 0 \\ 0 \end{bmatrix} \cdot \Delta P_{ref} \qquad (10)$$

$$\Delta f = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \Delta P_V \\ \Delta P_m \\ \Delta f \end{bmatrix} \qquad (11)$$

3. SOLUTION OF AN EXAMPLE APPLICATION BY USING Matlab/SIMULINK PROGRAM

Example application:
Parameters belong to a power station are given below,
Turbine time constant (τT)=0.5sn,
Speed regulator time constant (τg)=0.2sn,
Generator inertia constant (H)= 5sn,
Speed regulation rate= R pu,
The turbine exit power at nominal frequency is 250 MW. In case of the sudden 50 MW load increase, find the curve of change at load frequency by using the state equations.
Solution: the values given for the power system are used and "step " function is used at entry. If we analyze the solution at Matlab Command window ;

The change curve at frequency related to the step function applied to entry in case of uncontrolled state by using the data of the example application above is given in Fig 4. The change at frequency is very small and passing period to extinction takes about 8 seconds.

In Fig 5, when the control system with pole placement is applied, change at frequency becomes more stable than the previous uncontrolled state. Change at frequency in power systems must not take too long. This control system partly meets our requirement.

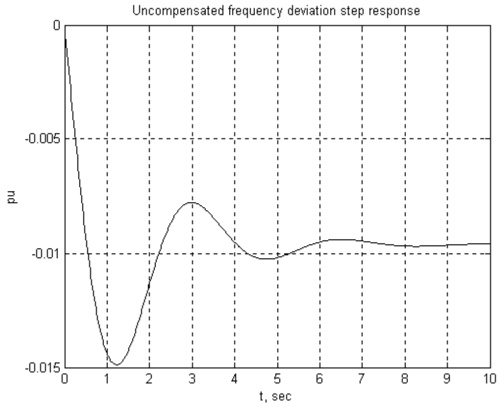

Fig. 4 *Change curve at uncontrolled load frequency*

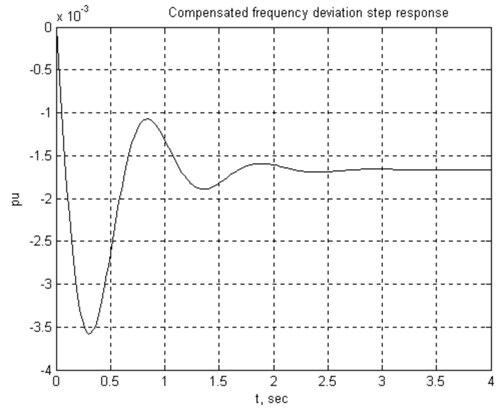

Fig. 5 *Change curve at load frequency in case of using of the control system with pole placement*

We obtain the simulation block diagram established by using the transfer function belongs to load frequency change of the power system given in s domain in Fig. 2 using the state variables equation. The simulation block diagram of the power system we obtained by using the state variables is given in Fig.6.

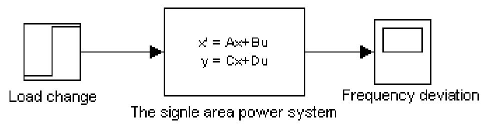

Fig. 6 *The block diagram established to observe the change at load by using the state variables*

4. OPTIMAL CONTROL DESIGN FOR THE LOAD FREQUENCY CONTROL AGAINTST TO LOAD CHANGES

Optimal control is the branch of the modern control theory. The control design process is realized depending on system variables and high performance operation of the system in ensured [3]. In the study, optimal control design for the linear systems with quadratic performance (Linear Quadratic Regulator (LQR) is established. The purpose of optimal regulator design is to determine the optimal control rule u*(x,t). For the studied power system, the performance index minimized from the first entry values to the final state values is transferred.

The performance index of the system is selected to obtain the best process between the cost and performance of the control system. Performance index is widely used in optimal control design process. It is expressed as quadratic performance depended on minimum fault and minimum energy criteria. The system is identified by the state equation given below.

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{12}$$

The problem faced during the system controls is to find the control rule K(t) vector. The control rule vector below is expressed with the given equation.

$$u(t) = -K(t)x(t) \tag{13}$$

The control rule vector minimizes the quadratic performance index values in the dynamic system expression defined in equation (13).[4] Linear Quadratic Performance index value (J) is expressed as follows.

$$J = \int_{t_0}^{t_f} (x'Qx + u'Ru)dt \tag{14}$$

Where, Q is the positive semi-defined matrix and R is the real symmetric matrix. If the marked minors of all the elements of Q matrix are not negative, Q matrix is positive semi-defined matrix. Selection of Q and R elements are determined according to the weighted relations of state variables and control entries [5].

To obtain the above explained solution, we may use the Lagrange multiplies method. With using of Lagrange multiplies method, equation (13) is added to equation (14) and limited problem (λ)solution within definite limitations is made. Solution of this function aims to minimize the below given limitless function.

$$\mathrm{L}(\mathrm{x}, \lambda, \mathrm{u}, \mathrm{t}) = \left[\mathrm{x}'\mathrm{Q}\mathrm{x} + u'Ru\right] + \lambda'\left[Ax + Bu - \dot{x}\right] \tag{15}$$

To calculate the optimal values, the above given equation is equalized to zero and its partial derivatives are taken. (*) symbol in the following expressions expresses the optimal values.

$$\frac{\partial L}{\partial \lambda} = AX^* + Bu^* - \dot{x}^* = 0 \Rightarrow \dot{x}^* = AX^* + Bu^* \tag{16}$$

$$\frac{\partial L}{\partial u} = 2Ru^* + \lambda'B = 0 \Rightarrow u^* = -\frac{1}{2}R^{-1}\lambda'B \tag{17}$$

$$\frac{\partial L}{\partial x} = 2x'^*Q + \lambda' + \lambda'A = 0 \Rightarrow \lambda = -2Qx^* - A'\lambda \tag{18}$$

In optimal solution, when symmetric and positive semi-defined p(t) matrix that is variable depending on time is considered, the following expression is given.

$$\lambda = 2p(t)x^* \tag{19}$$

If the equation (2) expression is expressed as equation (17) manner, it gives the optimal close cycle rule.

$$u^*(t) = -R^{-1}B'p(t)x^* \tag{20}$$

Of the derivative of equation (18) is taken, it is expressed as follow,

$$\dot{\lambda} = 2(\dot{p}x^* + p\dot{x}^*) \tag{21}$$

Consequently, when symmetric and positive semi-defined matrix's equation (18) and (19) is solved, the following is obtained [2].

$$\dot{p}(t) = -p(t)A - A'p(t) - Qp(t)BR^{-1}B'p(t) \tag{22}$$

In solution of the above equation, Riccati equation solution is used. The limit condition for equation (21) is $p(t_f)=0$. For the time domain solution of the Diccati equation, **[τ,p,K,t,x]=riccati** function is developed. Riccati equation matrix p(τ), optimal feed-back gaining K(τ), and state response of the system at the beginning x(t) are solved.

Optimal control gaining is the feed-back and time adjustable state variable. In many practical applications, it is sufficient to use the stable state feed-back gaining.

For the systems that do not change with linear time, $\dot{p}=0$. For the solution of Riccati equation, **[k,p]=lgr2(A,B,Q,R]** function in Matlab Control System Toolbox is used [4] If we use the LQG control as optimal gaining, by using the Linear Quadratic Regulator(**lqry**) function, we obtain K feed-back gaining

LQR design procedure is simpler and clearer than the traditional control design. In traditional controller, the gaining matrix (K) may be directly selected.

The control experts prefer Q and R parameters to design the optimal LQR. Then by using the Riccati equations, K feed-back gaining matrix is selected [3]. If the system response is not stable, the new Q and R weight matrixes are determined. This is an important advantage for all the control cycles.

4.1 Load frequency control with optimal LQR controller

In the above given example application, lets use LQR controller for the load frequency control. For solution, Matlab/Simulink software program is used. The state matrixes of the system and changes at load are entered in Matlab m.file file as given below.

```
PL= 0.2;
A = [-5  0  -100; 2  -2  0; 0  0.1  -0.08];
B = [0;  0;  -0.1]; BPL=PL*B;
C = [0  0  1];
D = 0;
Q = [20 0  0; 0  10 0; 0  0  5];
R = 0.15;
[K, P] = lqr2(A, B, Q, R)
Af = A - B*K
t = 0:0.02:1;
[y, x] = step(Af, BPL, C, D, 1, t);
plot(t, y), grid
xlabel('t, sec'), ylabel('pu')
title('Yük Frekansı Adım Cevabı’)
```

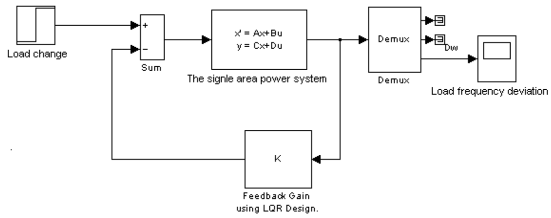

Fig.7 The simulation block diagram of optimal LQR load frequency of the signle area power system

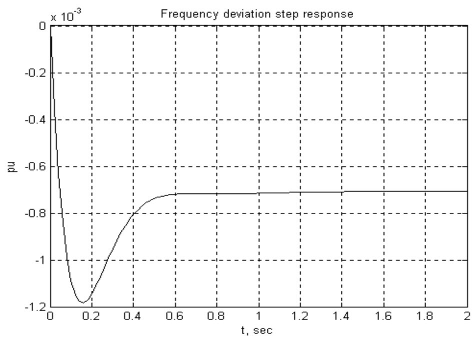

Fig. 8 *Change at load frequency in the power system with optimal controller*

As seen in figure 8, the load frequency of the system according to the controller with load placement becomes stable in a short time. The load frequency is not affected much with the changes in the system parameter and load thanks to LQR optimal control.

5.CONCLUSION

In the study, the optimal LQR controller is used to develop and secure the system performance under changes occur in the power system parameters. With this high quality and performance controller, no modification is made in the controller structure against to changing parameters and loads. This demonstrates us that optimal LQR controller is more robust against to changes occur in the system than the other traditional controllers.

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