2011 IEEE International Conference on Fuzzy Systems
June 27-30, 2011, Taipei, Taiwan

On the Type-1 and Type-2 Fuzziness Measures for Thresholding MRI Brain Images

R. Rajesh
Technology Development Centre
Network Systems and Technologies
Technopark, Thiruvananthapuram - 695581
Email: kollamrajeshr@ieee.org

N. Senthilkumaran, J. Satheeshkumar,
B. Shanmuga Priya, C. Thilagavathy, K. Priya
Department of Computer Applications
Bharathiar Universisty, Coimbatore - 641046, India
Email: jsathee@rediffmail.com

***Abstract*—The result of image thresholding is not always satisfactory due to the disturbing factors like vagueness, non-uniform illumination etc and to overcome these problems recently various researchers have proposed fuzzy image thresholding. The linear index of fuzziness for type-1 fuzzy sets by Zenzo et. al. and measure of ultrafuzziness for type-2 fuzzy sets by Tizhoosh has difficulties in handling MRI brain images with one level of gray value as background and other two levels of grayness as white matter and gray matter. Hence this paper proposes new modified thresholding measures for MRI brain images using type-1 and type-2 fuzzy sets. The results show the effectiveness of the proposed modified thresholding measures.**

***Index Terms*—type-1 fuzzy sets, type-2 fuzzy sets, thresholding, ultrafuzziness**

I. Introduction

Image thresholding, considered as the simplest form of segmentation, is an important task in most of the image processing applications. Lot of research work has already been appeared on robust thresholding techniques [1], [2], [3]. Recently fuzzy set theory have been used extensively in image thresholding [4], [7], [8], [9], [10], [11], [12], [13] due to the ability of fuzzy logic in handling ambiguity/vagueness in the presence of disturbing factors like non-uniform illumination, vagueness etc.

Fuzzy thresholding techniques can be classified into four categories [4] based on the way in which it is applied. They are namely (a) fuzzy clustering using fuzzy c-means, probabilistic c-means etc. (b) rule-based approaches (c) fuzzy geometrical approaches using spatial image information and geometrical measures(compactness, area coverage etc.) (d) information-theoretical approach by minimizing/maximizing measures of fuzziness (index of fuzziness, fuzzy entropy, fuzzy divergence etc.). Due to the simplicity and high speed of information-theoretical approach [7], [8], [9], [10], [11] it has been mostly used for image thresholding.

The linear index of fuzziness for type-1 fuzzy sets by Zenzo et. al. [11] and measure of ultrafuzziness for type-2 fuzzy sets by Tizhoosh [4] has difficulties in handling (we mean thresholding) MRI brain images. Here the difficulties in thresholding are due to the presence of 3 levels of grayness in MRI images, one level of gray value as background and other two levels of grayness as white matter and gray matter. Hence this paper proposes new modified thresholding measures for MRI brain images using type-1 and type-2 fuzzy sets. The results show the effectiveness of the proposed modified thresholding measures.

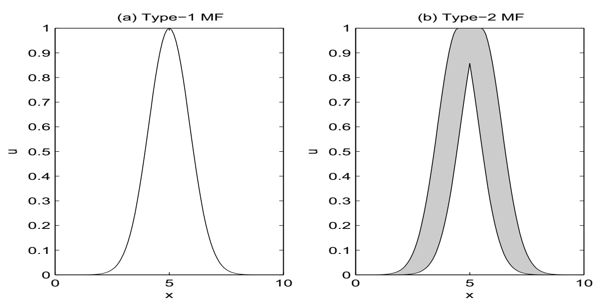

Fig. 1. Gaussian membership function with variance=0.9 of (a) type-1 with mean=5 (b) type-2 with uncertain means mean1=4.5, mean2=5.5

This paper is organized as follows. Section II deals with measure of fuzziness and section III introduces modified measures of fuzziness. Section IV presents the experimental results and section V concludes the paper.

II. Measure of fuzziness

A. Type-1 fuzzy sets

A type-1 fuzzy set A in X is defined by a type-1 membership function $\mu_A(x)$ given by

$$A = \{(x, \mu_A(x)) | x \in X\} \quad (1)$$

Figure 1a shows the Gaussian membership function of type-1 fuzzy set with variance = 0.9 and mean = 5.

The linear index of fuzziness [11] for an $M \times N$ image subset $A \subseteq X$ with L gray levels $g \in [0, L-1]$ can be given by

$$\gamma_l(A) = \frac{2}{MN} \sum_{g=0}^{L-1} h(g) \times \min[\mu_A(g), 1 - \mu_A(g)] \quad (2)$$

where h(g) is the histogram. $\mu_X(g)$ is the membership function and can be defined by the standard S-function, Huang & Wang function, triangular membership function, LR type fuzzy number etc..

B. Type-2 fuzzy sets

Even though the application of T1 FS has seen success in all applications[5], [6], since it introduction by Zadeh in 1965, it has limited capabilities to directly model and minimize the effect of uncertainties [14], [15], [16], [17], [18]. This is due to the fact that T1 FS does not have measure of dispersion to capture more about linguistic uncertainties as equivalent to that of probability density function (pdf) [1] having variance as a measure of dispersion about the mean. In late 1975, Zadeh introduced the concept of type-2 fuzzy sets (T2 FS). T2 FS are described by membership functions (MFs) that are characterized by more parameters than MFs for T1 FSs and hence T2 FSs provide more degrees of freedom to handle uncertain environments [14]. Type-2 fuzzy sets are characterized by fuzzy membership functions that are three-dimensional with membership grade as a fuzzy set in [0, 1] rather than two dimensional membership functions with crisp number in [0, 1] as membership grade as in the case of T1 FS.

A type-2 fuzzy set ($\tilde{A}$) is given by

$$\tilde{A} = \{((x,u), \mu_{\tilde{A}}(x,u))|\ \forall x \in X,\ \forall u \in J_x \subseteq [0,1]\} \quad (3)$$

where $0 \le \mu_{\tilde{A}}(x,u) \le 1$ is a type-2 membership function. $\tilde{A}$ can also be expressed as

$$\tilde{A} = \int_{x\in X}\int_{u\in J_x} \mu_{\tilde{A}}(x,u)/(x,u), \qquad J_x \subseteq [0,1] \quad (4)$$

A more practical definition for a type-2 fuzzy set can be given as

$$\begin{aligned}\tilde{A} \;=\; & \{(x, \mu_U(x), \mu_L(x))|\ \forall x \in X, \\ & \mu_L(x) \le \mu(x) \le \mu_U(x), \mu \in [0,1]\}\end{aligned} \quad (5)$$

Figure 1b shows the Gaussian membership function of type-2 fuzzy set with variance = 0.9 and uncertain means 4.5 & 5.5.

The ultrafuzziness [4] $\tilde{\gamma}$ for an $M \times N$ image subset $\tilde{A} \subseteq X$ with L gray levels $g \in [0, L-1]$ can be given by

$$\gamma_l(A) = \frac{2}{MN}\sum_{g=0}^{L-1} h(g) \times [\mu_U(g) - \mu_L(g)] \quad (6)$$

where h(g) is the histogram, $\mu_{\tilde{A}}(g)$ is the membership function, $\mu_U(g) = [\mu_A(g)]^{1/\alpha}$ is the upper membership values, $\mu_L(g) = [\mu_A(g)]^{\alpha}$ is the lower membership value and $\alpha \in (1, 2]$.

III. Introducing Modified fuzziness measure

A. Modified index of fuzziness for type-1 fuzzy sets

The modified index of fuzziness for an $M \times N$ image subset $A \subseteq X$ with L gray levels $g \in [0, L-1]$ can be given by

$$\gamma_m(A) = \frac{2}{MN}\sum_{g=0}^{L-1} h(g) \times \mu_A(g) \times (1 - \mu_A(g)) \quad (7)$$

[1] pdfs usually require infinite number of moments. Since it is very difficult to determine all the moments, we make use of only first two moments namely mean and variance(measure of dispersion about mean)

where h(g) is the histogram. $\mu_A(g)$ is the membership function and can be defined by the standard S-function, Huang & Wang function, triangular membership function, LR type fuzzy number etc..

B. Modified measure of ultra fuzziness for type-2 fuzzy sets

The modified ultrafuzziness $\tilde{\gamma}$ for an $M \times N$ image subset $\tilde{A} \subseteq X$ with L gray levels $g \in [0, L-1]$ can be given by

$$\gamma_m(A) = \frac{2}{MN}\sum_{g=0}^{L-1} h(g) \times \mu_L(g) \times [\mu_U(g) - \mu_L(g)] \quad (8)$$

where h(g) is the histogram, $\mu_{\tilde{A}}(g)$ is the membership function, $\mu_U(g) = [\mu_A(g)]^{1/\alpha}$ is the upper membership values, $\mu_L(g) = [\mu_A(g)]^{\alpha}$ is the lower membership value and $\alpha \in (1, 2]$.

IV. Experiments and results for thresholding brain images

Image thresholding using type-1 fuzzy sets is done by thresholding with the minimum/maximum fuzziness obtained by shifting the membership function, defined over the image histogram, along the grey level range. Similarly, image thresholding using type-2 fuzzy sets can be done by thresholding with the minimum/maximum ultrafuzziness obtained by shifting the membership function, defined over the image histogram, along the grey level range.

Magnetic resonance images have three levels of grayness inherent in it. One level of gray value can be seen as background and other two levels of grayness can be seen as white matter and gray matter. Fig. 2 shows the white and gray matter in the brain slice. Due to the presence of 3 levels of greyness in MRI images, it is very difficult to threshold. Normal thresholding for two levels of greyness (namely separating background and foreground) will not work properly for MRI images.

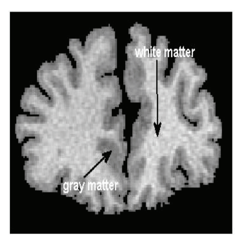

Fig. 2. White matter and gray matter corresponding to a brain slice

Several brain images have been experimented and here we show only the results of three slices of brain. Fig. 3 show the orginal brain slices and Fig. 4 show the corresponding preprocessed images.

Linear index of fuzziness (of type1) is calculated using eqn.2 and modified index of fuzziness (of type1) is calculated using eqn.7. Fig.5 show the plot of linear index of fuzziness and modified index of fuzziness corresponding to the preprocessed images (fig.4). It can be seen from the first image of Fig.5 that modified fuzziness index gives a threshold, $T_2 = 166$ for PImage1, which gives a much better thresolding result which can be seen from the first images

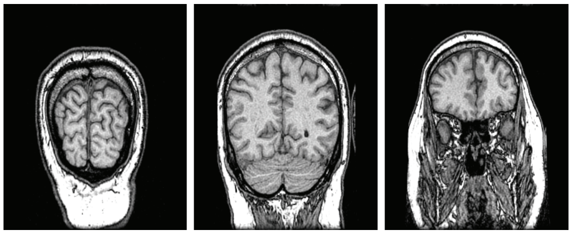

Fig. 3. Orginal images (from left) (a) Image1, (b) Image2, and (c) Image3

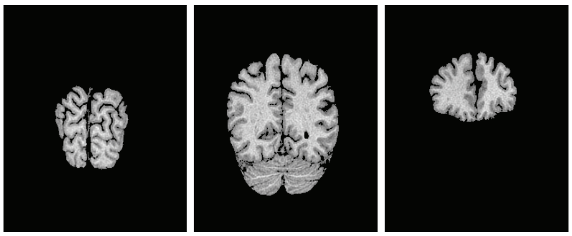

Fig. 4. Preprocessed images corresponding to Fig. 3 (from left) (a) PImage1, (b) PImage2, and (c) PImage3

of Fig.6 and Fig.7. Modified fuzziness index of PImage2 and PImage3, as seen in the second and third images of Fig.5, shows two twists corresponding to gray and white matter in the images. But such twists cannot be seen in the linear index of fuzziness of PImage2 and PImage3, as seen in the second and third images of Fig.5. The threshold values corresponding to the secondary twist helps to separate (and hence threshold) white matter. The thresholding results corresponding to the thresholds thus obtained (Fig.7) performs better than the linear thresholds (Fig.6) (Please see the second and third images of corresponding figures).

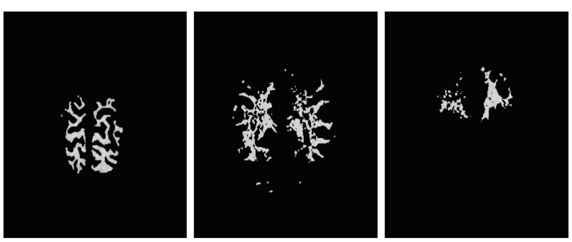

Fig. 6. Thresholded images with threshold obtained from fuzziness index of type1. Thresholded (a) PImage1 with T=176, (b) PImage2 with T=199, and (c) PImage3 with T=199

Ultrafuzziness (of type2) is calculated using eqn.6 and modified index of fuzziness (of type2) is calculated using eqn.8. Fig.8 show the plot of ultrafuzziness and modified ultrafuzziness corresponding to the preprocessed images (fig.4).

Modified ultrafuzziness shows two twists corresponding to

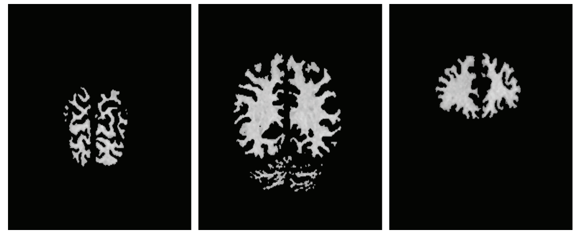

Fig. 7. Thresholded images with threshold obtained from modified fuzziness index of type1. Thresholded (a) PImage1 with T=166, (b) PImage2 with T=165, and (c) PImage3 with T=147

TABLE I
THRESHOLDS FOR THE IMAGES

	PImage1	PImage2	PImage3
Type-1	176	199	199
mType-1	166	165	147
Type-2	176	199	150
mType-2	156	150	133

gray and white matter in the images. The threshold values corresponding to the secondary twist helps to separate (and hence threshold) white matter. It can be seen from the figures that modified ultrafuzziness gives a thresholding result which is much better than the threshold of ultrafuzziness and it clear from the images of Fig.9 and Fig.10.

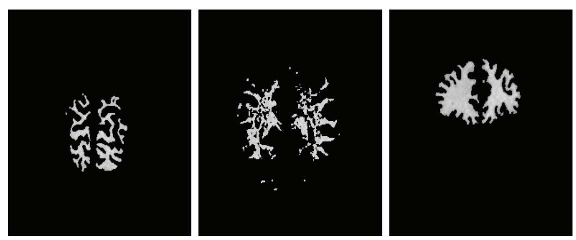

Fig. 9. Thresholded images with threshold obtained from ultrafuzziness of type2. Thresholded (a) PImage1 with T=176, (b) PImage2 with T=199, and (c) PImage3 with T=150

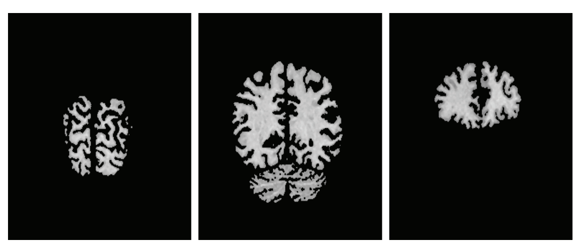

Fig. 10. Thresholded images with threshold obtained from modified ultrafuzziness of type2. Thresholded (a) PImage1 with T=156, (b) PImage2 with T=150, and (c) PImage3 with T=133

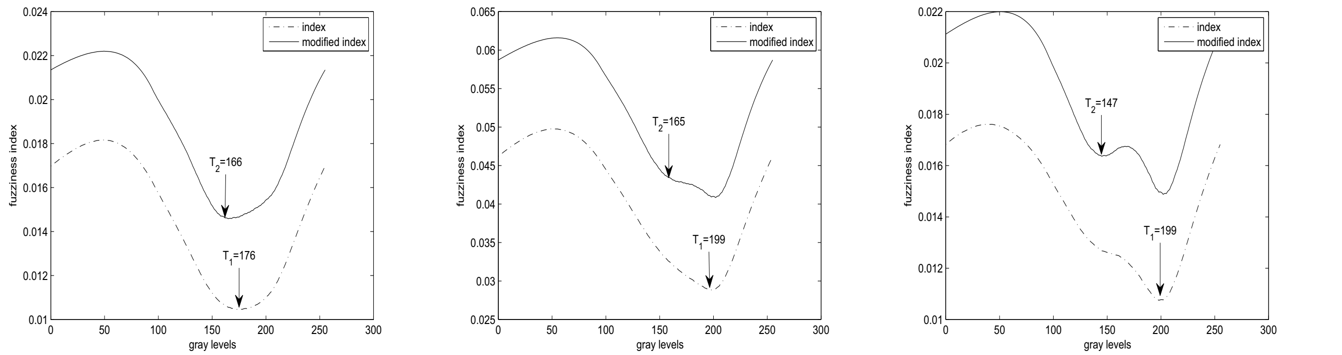

Fig. 5. Fuzziness index (of type1) and modified fuzziness index (of type1) for the images (from left) (a) PImage1, (b) PImage2, and (c) PImage3

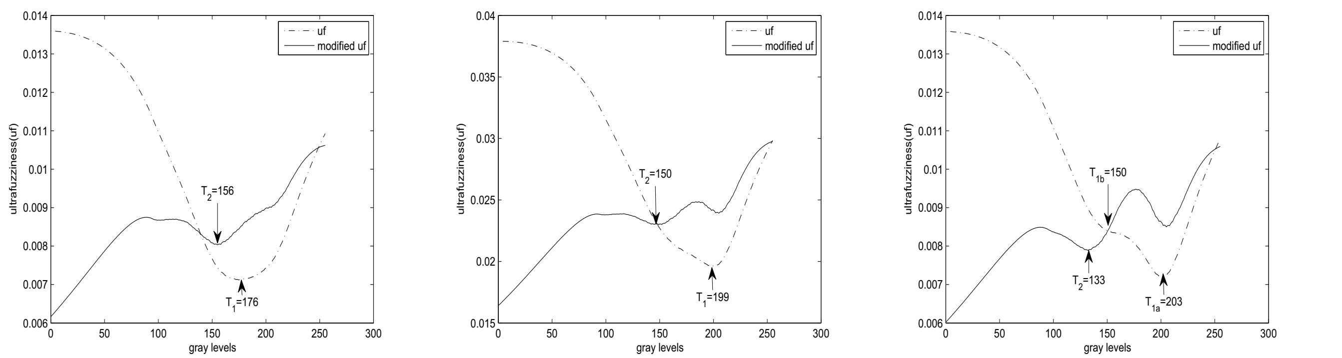

Fig. 8. Ultrafuzziness and modified ultrafuzziness for the images (from left) (a) PImage1, (b) PImage2, and (c) PImage3

V. Conclusion

Due to the difficulties in handling MRI brain images with one level of gray value as background and other two levels of grayness as white matter and gray matter, the paper has presented modified index of fuzziness for type1 fuzzy sets and modified ultrafuzziness for type2 fuzzy sets for thresolding brain images. The results have shown the effectiveness of the proposed modified thresholding measures.

Acknowledgment

This research is supported by University Grants Commission, India through a major research project grant (UGC F. No. 33-62/2007 (SR) dated 28th Feb 2008). N. Senthilkumaran, Thilagavathy, K. Priya and Shanmuga Priya are currently doing Ph.D in Bharathiar University.

References

[1] K.C. Lin, Fast image thresholding by finding the zeros of the first derivative of between class variance, Mach. Vis. Appl. 13 (5-6) (2003) 254-262.
[2] P.L. Rosin, Unimodel thresholding, Pattern Recognition 34 (11) (2001) 2083-2096.
[3] L. Snidaro, G.L. Foresti, Real-time thresholding with Euler numbers, Pattern Recognition Lett. 24 (9-10) (2003) 1543-1554
[4] H.R. Tizhoosh, Image thresholding using type II fuzzy sets, Pattern Recognition Lett. 38 (2005) 2363-2372.
[5] P. Balamurugan, R. Rajesh, Fuzzy Logic Approach using Guided Gray Level Coherence Features for Greenery and Non-Greenery Image Classification, Far East Journal of Experimental and Theoretical Artificial Intelligence, ISSN: 0974-3261, vol. 2(1), pp. 47-58, 2008
[6] R. Rajesh, M.R. Kaimal, T-S Fuzzy Model with Nonlinear Consequence and PDC Controller for a Class of Nonlinear Control Systems, Applied Soft Computing Journal, Elsevier (ISSN: 1568-4946), vol. 7(3), pp. 772-782, June 2007.
[7] L.K. Huang, M.J. Wang, Image thresholding by minimizing the measure of fuzziness, Pattern Recognition 28 (1995) 41-51.
[8] N.R. Pal, D. Bhandarai, D.D. Majumder, Fuzzy divergence, probability measure of fuzzy events and image thresholding, Pattern Recognition Lett. 13 (1992) 857-867.
[9] H.R. Tizhoosh, H. Haubecker, Fuzzy image processing: an overview, in: B. Jahne, H. Haubecker, P. Geibier (Eds.), Handbook on computer vision and applications, Academic Press, Boston, 1998.
[10] Q. Wang, Z. Chi, R. Zhao, Image thresholding by maximizing the index of nonfuzziness of the 2-D grayscale histogram, Computer Vision Image understanding 85 (2) (2002) 100-116.
[11] S.D. Zenzo, L. Cinque, S. Levialdi, Image thresholding using fuzzy entropies, SMC 28(1) (1998) 15-23.
[12] S.K. Pal, C.A. Murthy, Fuzzy thresholding: mathematical framework bound functions and weighted moving average technique, Pattern Recognition Lett. 11 (1990) 197-206.
[13] N. Senthilkumaran, R. Rajesh, ”Edge Detection Techniques for Image Segmentation - A Survey of Soft Computing Approaches”, International Journal of Recent Trends in Engineering, Academy Publisher (Finland), Issue. 1, Vol. 1, No. 2, June 2009
[14] J.M. Mendel, Uncertain Rule-based fuzzy logic systems, Prentice-Hall, Englewood Cliffs, NJ, 2001.
[15] J. M. Mendel and R. I. Bob John, Type-2 fuzzy sets made simple, IEEE Trans. on Fuzzy Systems, vol. 10, pp. 117-127, April 2002.
[16] Q. Liang and J. M. Mendel, Interval type-2 fuzzy logic systems: theory and design, IEEE Trans. on Fuzzy Systems, Vol. 8, pp. 535-550, Oct. 2000.
[17] N. N. Karnik, J. M. Mendel and Q. Liang, Type-2 fuzzy logic systems, IEEE Trans on Fuzzy Systems, vol. 7, pp. 643-658, Dec. 1999.
[18] N. N. Karnik and J. M. Mendel, Centroid of a type-2 fuzzy set, Information Sciences, vol. 132, pp. 195-220, 2001.