On PM Tubular Linear Synchronous Motor Modelling

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Abstract: In this paper a new approach is made on PM tubular linear synchronous motor (PM-TLSM) modelling, considering a non linear model. This model allows the incorporation of PM elements characteristics. The straight forward approach induces robust simulation results for tubular linear machines, without resorting to complex variable transformations.

Key-Words: Linear machines, Tubular motor, Permanent-magnets, Modelling and Simulation.

1 Introduction

A linear motor applies force directly to a load, and does not require any intermediate mechanism to convert rotary motion into linear motion. Linear motors are capable of high speeds, fast acceleration, and accurate positioning.

The linear motors are employed in a wide range of electromechanical systems, such as robotics and factory automation, processing and packaging equipment, machine tools, transfer equipment, aerospace industry and transportation systems. An industrial applications issue of the linear actuators is present in [1].

When providing motion for people-moving devices or virtual reality capsules, it's important to provide a clean environment. The linear motors can offer significant advantages over conventional linear motion systems, such as hydraulic linear actuators, motors drives by cams, ballscrews and belts, in terms of efficiency, low cost maintenance, no power loss in rotary-linear conversion, speed and force control and positional accuracy [2].

The linear synchronous machines (LSM) are selected for some applications, because they have significantly higher force production then linear induction and switched reluctance motors. This preference is justified by the improvements of permanent magnet materials and power electronics devices [3].

There are several topologies of LSM, these topologies vary with some parameters i.e. excitation mode, type and permanent magnets placement, slotted or slotless armature design.

A permanent magnet tubular linear synchronous motor (PM-TLSM) topology is considered in this paper. The different PM-TLSM topologies and their comparative studies are present in [4]-[6].

2 PM Tubular Linear Synchronous Motor

2.1 Development of linear motor geometries

To understand the linear machines modelling based on rotary machines knowledge, geometrically it can be “cut” along radial plane and “unrolled”. This is exemplified in Figure 1(a) and Figure 1(b). Applying this process to a PM rotary synchronous motor a PM flat LSM is obtained. The PM-TLSM can be developed from flat linear motors by “rolling” them around their longitudinal axis. The permanent magnets, initially oriented along the length of the linear shaft, now form a stack of alternating magnetic polarity rings, Figure 1(c).

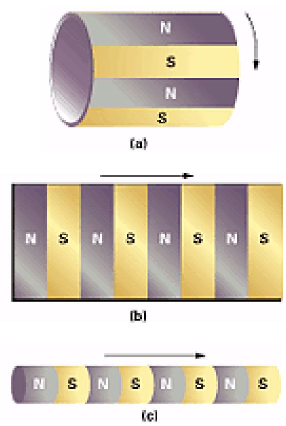

Fig.1: (a) Rotary motor; (b) Flat linear motor; (c) Tubular linear motor.

2.2 Motor Description

The PM-TLSM used in this work is the CLD4206D model from California Linear Devices, Inc. The Figure 2 illustrates a cutaway view of the CLD4206D tubular linear motor, showing the shaft containing rings rare-earth neodymium-iron-boron magnets. The magnetic field provides the interaction between stator coils and the moving part. This characteristic gives a very accurate method for achieving force and motion control.

Fig.2: A cutaway view of the PM-TLSM [7].

The Table 1 lists the most important features of the CLD4206D tubular linear motor, [7].

Table 1: The CLD4206D performance parameters.

Parameter	Units	Value
Peak Force (1 sec)	N	5249
Peak Force (3 sec)	N	4537
Peak Current (1 sec)	A	90
Continuous Static Force (nat. convection)	N	1156
Force Sensitivity (at 50% of 3 sec Peak Current)	N/A	97,9
Back EMF Constant (ph-ph)	Vpk/(in/sec)	126,8
DC Winding Resistance (ph-ph at 25 deg C)	Ω	2,2
Winding Inductance (ph-ph)	mH	15,1
Motor Constant	N/sqrt(watt)	76,1
Detent Force (peak)	N	44
Thermal Resistance (nat convection)	deg. C/W	0,13
Maximum Stroke	m	0,305
Weight	kg	26,3

2.3 PM-TLSM Parameter Identification

To properly design and optimize a control system of an electrical machine in which both dynamic performance and energy efficiency are expected, the machine model and its parameters must be precisely known.

The parameters for LSM models are evaluated by direct measurement, derivation from tests results, analysis, estimation, and a combination of them. The circuit parameters developed for rotary synchronous modelling are well known. Similar methods for LSM parameters determination are still in development [8].

Initial tests were conducer with the PM-TLSM, to determine the pole pitch and to verify if the winding inductance was constant to any shaft position. These tests allow a more accurate PM-TLSM model.

In this machine the relationship between synchronous linear speed, v_s, and pole pitch, τ_p, of the PM-TLSM, [9] is given by (1).

$$v_s = 2 \cdot f \cdot \tau_p \qquad (1)$$

In (1) f is the supply frequency in Hertz.

Employing a power drive to command the speed of the CLD4206D motor, it is possible to determine the pole pitch parameter.

In Figure 3 to 5, the images of the single phase current, for three different shaft speeds, are presented.

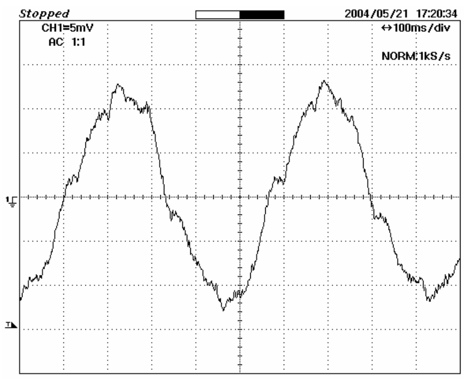

Fig.3: Test 1 – Current for a constant shaft speed, $v_s = 0{,}05\,\mathrm{ms}^{-1}$.

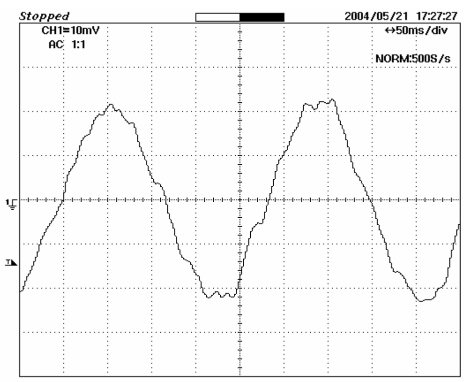

Fig.4: Test 2 – Current for a constant shaft speed, $v_s = 0{,}10\,\mathrm{ms}^{-1}$.

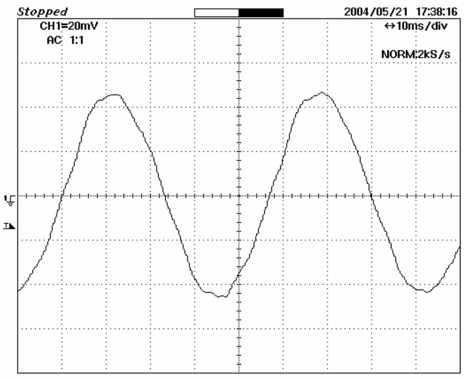

Fig.5: Test 3 – Current for a constant shaft speed, $v_s = 0{,}50\,\mathrm{ms}^{-1}$.

After determining the current frequencies, using the equation (1) the pole pitch value was find, as shows in Table 2.

Table 2: Pole pitch calculations results.

Tests	Speed, v_s ($\mathrm{m.s^{-1}}$)	Frequency, f (Hz)	Pole pitch, τ_p (m)
Test 1	0,05	2,14	$11{,}67\times10^{-3}$
Test 2	0,10	4,29	$11{,}67\times10^{-3}$
Test 3	0,50	21,43	$11{,}67\times10^{-3}$

The windings inductance was measure with a RLC bridge for several shaft positions. These measures are shown in Figure 6.

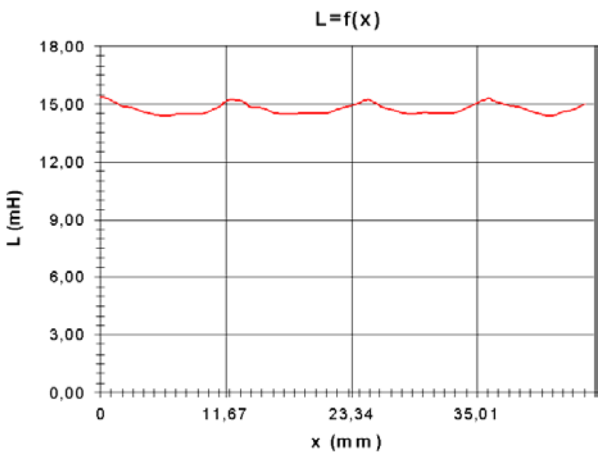

Fig.6: A phase-phase windings inductance measure as a function of the shaft position.

Figure 6 depicts the windings inductance has a small variation, about 1mH, associate to the pole pitch.

3 Motor Model

In order to simplify the PM-TLSM model, the following assumptions are considered:

- windings stator inductance is constant to any shaft position;
- shaft length is considered infinite, so the end effects are neglected;
- magnetic circuit model do not includes magnetic saturation;
- permeability of the iron is infinite, so the iron losses are neglected;
- flux density reaction produced by stator current is neglected when compared with the PM flux density.

Basing on Faraday's law, the equations that describes the electric part of the PM-TLSM are given in (2).

$$\begin{cases} u_a = R_a \cdot i_a + \dfrac{d\Psi_a}{dt} \\ u_b = R_b \cdot i_b + \dfrac{d\Psi_b}{dt} \\ u_c = R_c \cdot i_c + \dfrac{d\Psi_c}{dt} \end{cases} \quad (2)$$

In (2) Ψ_a, Ψ_b and Ψ_c represent the linkage fluxes on *a*, *b*, *c* phases, respectively, (3).

$$\begin{cases} \Psi_a = L_a \cdot i_a + M_{ab} \cdot i_b + M_{ac} \cdot i_c + \Psi_{ma} \\ \Psi_b = M_{ba} \cdot i_a + L_b \cdot i_b + M_{bc} \cdot i_c + \Psi_{mb} \\ \Psi_c = M_{ca} \cdot i_a + M_{cb} \cdot i_b + L_c \cdot i_c + \Psi_{mc} \end{cases} \quad (3)$$

The linkage fluxes Ψ_{ma}, Ψ_{mb}, and Ψ_{mc}, established by permanent magnets on three phases windings have a co-sinusoidal variation, (4) whose maximum value is represented by Ψ_m.

$$\begin{cases} \Psi_{ma} = \Psi_m \cdot \cos\left(\dfrac{\pi}{\tau_p} x\right) \\ \Psi_{mb} = \Psi_m \cdot \cos\left(\dfrac{\pi}{\tau_p} x - \dfrac{2\pi}{3}\right) \\ \Psi_{mc} = \Psi_m \cdot \cos\left(\dfrac{\pi}{\tau_p} x + \dfrac{2\pi}{3}\right) \end{cases} \quad (4)$$

Considering that the PM-TLSM stator have the same resistance value for all phases windings, as well as the same values for the self-induction and mutual induction coefficients, then (5).

$$\begin{aligned}&R_a = R_b = R_c = R_s\\&L_a = L_b = L_c = L_s\\&M_{ab} = M_{ba} = M_{ac} = M_{ca} = M_{bc} = M_{cb} = M_s\end{aligned} \tag{5}$$

The linkage fluxes Ψ_a, Ψ_b, and Ψ_c derivatives become (6).

$$\begin{cases}\dfrac{d\Psi_a}{dt} = L_s\dfrac{di_a}{dt} + M_s\dfrac{di_b}{dt} + M_s\dfrac{di_c}{dt} - \\ \dfrac{d\Psi_b}{dt} = M_s\dfrac{di_a}{dt} + L_s\dfrac{di_b}{dt} + M_s\dfrac{di_c}{dt} - \\ \dfrac{d\Psi_c}{dt} = M_s\dfrac{di_a}{dt} + M_s\dfrac{di_b}{dt} + L_s\dfrac{di_c}{dt} - \end{cases}$$

$$\begin{aligned}&-\frac{\pi}{\tau_p}\cdot\frac{dx}{dt}\cdot\Psi_m\cdot\sin\left(\frac{\pi}{\tau_p}x\right)\\&-\frac{\pi}{\tau_p}\cdot\frac{dx}{dt}\cdot\Psi_m\cdot\sin\left(\frac{\pi}{\tau_p}x-\frac{2\pi}{3}\right)\\&-\frac{\pi}{\tau_p}\cdot\frac{dx}{dt}\cdot\Psi_m\cdot\sin\left(\frac{\pi}{\tau_p}x+\frac{2\pi}{3}\right)\end{aligned} \tag{6}$$

Accordingly with the equations (6), the equations (2) can be rewritten in the matrix form, i.e. (7).

$$\begin{bmatrix}u_a\\u_b\\u_c\end{bmatrix} = \begin{bmatrix}R_s & 0 & 0\\0 & R_s & 0\\0 & 0 & R_s\end{bmatrix}\times\begin{bmatrix}i_a\\i_b\\i_c\end{bmatrix} + \begin{bmatrix}L_s & M_s & M_s\\M_s & L_s & M_s\\M_s & M_s & L_s\end{bmatrix}\times$$

$$\times\frac{d}{dt}\begin{bmatrix}i_a\\i_b\\i_c\end{bmatrix} - \frac{\pi}{\tau_p}\cdot\frac{dx}{dt}\cdot\Psi_m\begin{bmatrix}\sin\left(\frac{\pi}{\tau_p}x\right)\\\sin\left(\frac{\pi}{\tau_p}x-\frac{2\pi}{3}\right)\\\sin\left(\frac{\pi}{\tau_p}x+\frac{2\pi}{3}\right)\end{bmatrix} \tag{7}$$

In order to simplify the linear motor model writing, (7) can be written in a compacted form (8).

$$\mathbf{u_{abc}} = \mathbf{R_s}\cdot\mathbf{i_{abc}} + \mathbf{L_{abc}}\cdot\frac{d\mathbf{i_{abc}}}{dt} + \frac{dx}{dt}\cdot\frac{d\mathbf{\Psi_{mabc}}}{dx} \tag{8}$$

In (8) $\mathbf{R_s}$, represents the resistances matrix of the linear motor stator windings; $\mathbf{L_{abc}}$, represents the self-induction and mutual induction coefficients matrix of the stator windings; $\mathbf{u_{abc}}$, represents the matrix voltages supplies applied to linear motor; $\mathbf{i_{abc}}$, represents the stator windings currents matrix; $\mathbf{\Psi_{mabc}}$, represents the linkage fluxes matrix established by permanent magnets in stator windings.

The mechanics equation is given by the second Newton's equation, (9).

$$F_e - F_r = I\frac{d^2x}{dt^2} \tag{9}$$

In (9), F_e represents the electromagnetic force of the linear motor in Newton, I the inertia of the linear shaft in kilogram and x the shaft displacement coordinate. The resistant force, F_r, can be translated by (10).

$$F_r = K_d\cdot\frac{dx}{dt} + K_e + F_{load} \tag{10}$$

In (10), K_d represents the dynamic attrition coefficient, K_e the static attrition coefficient and F_{load} represents the force exerted by load connected to the shaft.

The electromagnetic force, F_e, is obtained by calculating magnetic coenergy, W_{cm}, (11).

$$F_e = \frac{\partial W_{cm}}{\partial x} \tag{11}$$

The magnetic coenergy is obtained from (12).

$$W_{cm} = \frac{1}{2}\left[\mathbf{I_{abc}}\right]^T\left[\mathbf{L_{abc}}\right]\left[\mathbf{I_{abc}}\right] + \left[\mathbf{I_{abc}}\right]^T\left[\mathbf{\Psi_{mabc}}\right] + W_{PM} \tag{12}$$

In (12), W_{PM} represents the energy stored in the permanent magnets.

The equation of electromagnetic force, according to (11) and (12), is given by (13).

$$\begin{aligned}F_e = \frac{\partial W_{cm}}{\partial x} = \frac{\pi}{\tau_p}\cdot\Psi_m\cdot\Bigg(-i_a\cdot\sin\left(\frac{\pi}{\tau_p}x\right) - \\ -i_b\cdot\sin\left(\frac{\pi}{\tau_p}x-\frac{2\pi}{3}\right) - i_c\cdot\sin\left(\frac{\pi}{\tau_p}x+\frac{2\pi}{3}\right)\Bigg)\end{aligned} \tag{13}$$

The equations (8), (9), (10) and (13) characterize the PM-TLSM model in *abc* coordinates.

The Park's equation, (14), transforms the stator three phase quantities into a new reference frame which moves with the motor shaft.

$$\mathbf{C}(\theta) = \begin{bmatrix}\cos\theta & -\sin\theta & 0\\\sin\theta & \cos\theta & 0\\0 & 0 & 1\end{bmatrix} \tag{14}$$

In the Park's matrix transformation, $\mathbf{C}(\theta)$, the angular position, θ, is given by rotary-linear geometric motor transformation, (15).

$$\theta = \frac{\pi}{\tau_p} x \qquad (15)$$

Employing the Park's transformation to previous PM-TLSM model, the equivalent model can be expressed as (16).

$$\begin{cases} u_d = R_s \cdot i_d + \dfrac{d\psi_d}{dt} - \dfrac{dx}{dt} \cdot \dfrac{\pi}{\tau_p} \cdot \psi_q \\ u_q = R_s \cdot i_q + \dfrac{d\psi_q}{dt} + \dfrac{dx}{dt} \cdot \dfrac{\pi}{\tau_p} \cdot \psi_d \end{cases} \qquad (16)$$

The equation of the electromagnetic force, F_e, is obtained in *dq* referential quantities and can be given from (17) to (19).

$$F_e = \frac{\partial W_{cm}}{\partial x} = \mathbf{i_{dq}}^T \cdot \frac{d\mathbf{\Psi_{mdq}}}{dx} \qquad (17)$$

$$F_e = \frac{\pi}{\tau_p} \cdot \Psi_M \cdot \begin{bmatrix} i_d & i_q \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad (18)$$

$$F_e = \frac{\pi}{\tau_p} \cdot \Psi_M \cdot i_q \qquad (19)$$

Thus, the model of the PM-TLSM in the *dq* coordinates is therefore described by (20).

$$\begin{cases} u_d = R_s \cdot i_d + \dfrac{d\psi_d}{dt} - \dfrac{dx}{dt} \cdot \dfrac{\pi}{\tau_p} \cdot \psi_q \\ u_q = R_s \cdot i_q + \dfrac{d\psi_q}{dt} + \dfrac{dx}{dt} \cdot \dfrac{\pi}{\tau_p} \cdot \psi_d \\ F_e = \dfrac{\pi}{\tau_p} \cdot \Psi_M \cdot i_q \\ F_e - \left(K_d \cdot \dfrac{dx}{dt} + K_e + F_{load} \right) = I \dfrac{d^2 x}{dt^2} \end{cases} \qquad (20)$$

4 Simulation

The PM-TLSM model is synthesized on MatLab/Simulink® simulation software, as shows in Figure 7.

The simulations results are presented in Figure 8 to 11.

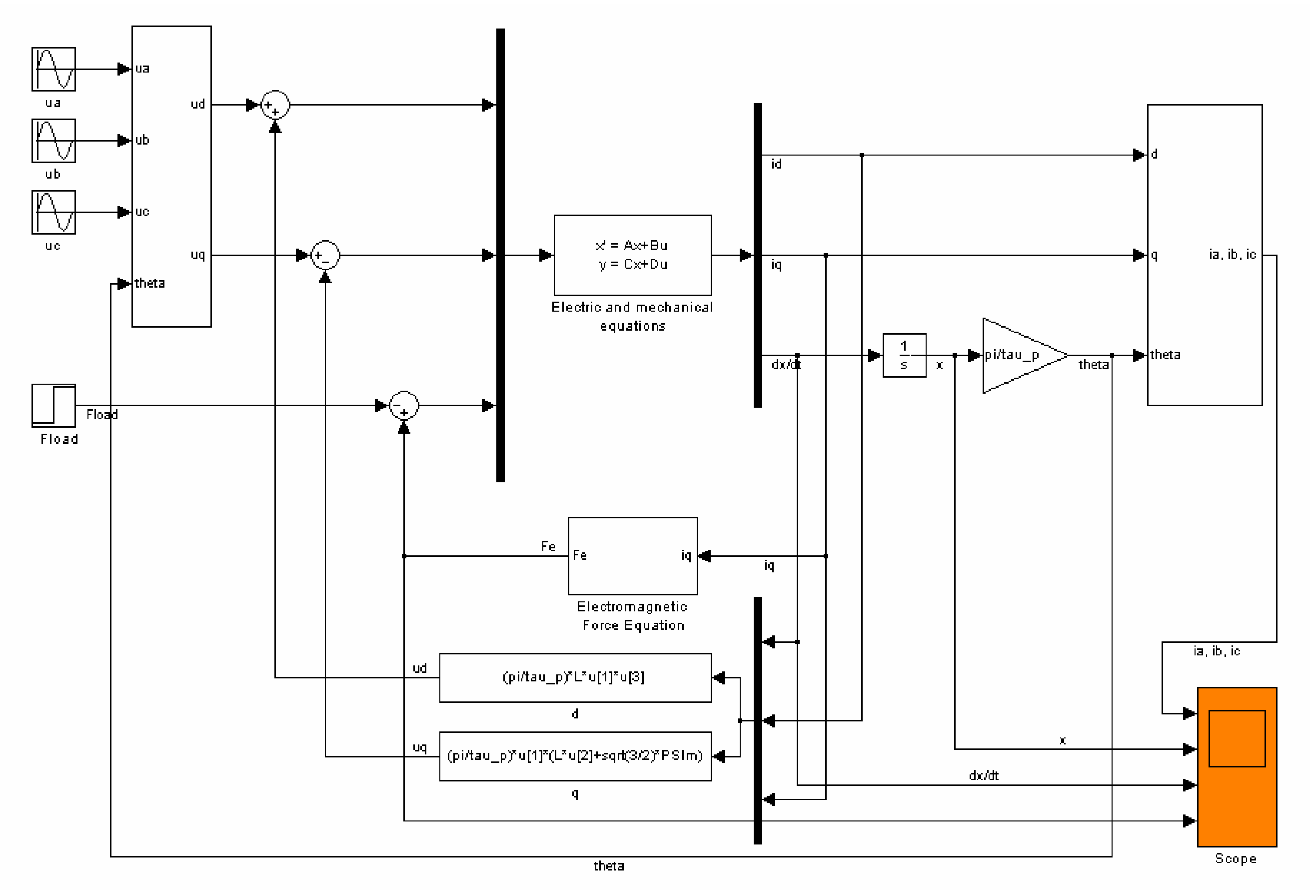

Fig.7: SIMULINK PM-TLSM model.

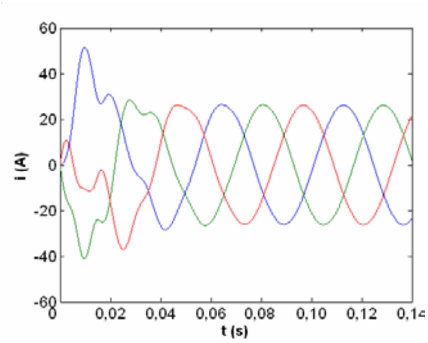

Fig.8: Calculated three phase currents.

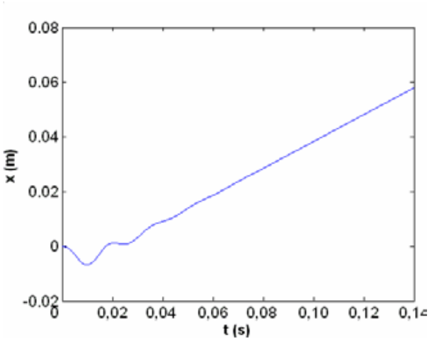

Fig.9: Calculated position.

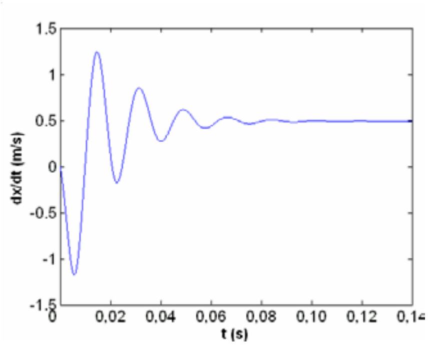

Fig.10: Calculated speed.

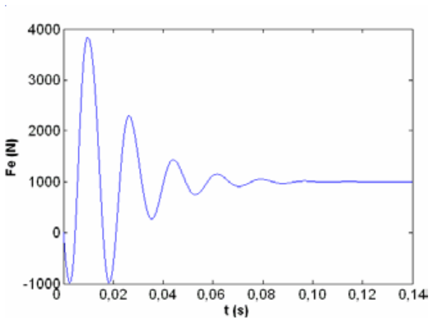

Fig.11: Calculated electromagnetic force.

The analysis of the Figure 8 to 11, shows the force generated by PM-TLSM as a function of the current and the position, which validates the behaviour of this machine.

4 Conclusion

In this paper a PM-TLSM dynamic model without magnet saturation and end effects is described. This model can be applied to other LSM topologies if the parameters are known.

To a several applications that require high accelerations, the resulting currents involve magnetic saturation. In this conditions the electromagnetic force is not easily predicted which leads to complex control requirements.

Future work includes experimental validation, and improvements of the PM-TLSM model, e.g. the magnetic saturation and end effects.

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