Spatially variant noise estimation in MRI: A homomorphic approach

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A B S T R A C T
The reliable estimation of noise characteristics in MRI is a task of great importance due to the influence of noise features in extensively used post-processing algorithms. Many methods have been proposed in the literature to retrieve noise features from the magnitude signal. However, most of them assume a stationary noise model, i.e., the features of noise do not vary with the position inside the image. This assumption does not hold when modern scanning techniques are considered, e.g., in the case of parallel reconstruction and intensity correction. Therefore, new noise estimators must be found to cope with non-stationary noise. Some methods have been recently proposed in the literature. However, they require multiple acquisitions or extra information which is usually not available (biophysical models, sensitivity of coils). In this work we overcome this drawback by proposing a new method that can accurately estimate the non-stationary parameters of noise from just a single magnitude image. In the derivation, we considered the noise to follow a non-stationary Rician distribution, since it is the most common model in real acquisitions (e.g., SENSE reconstruction), though it can be easily generalized to other models. The proposed approach makes use of a homomorphic separation of the spatially variant noise in two terms: a stationary noise term and one low frequency signal that correspond to the \( \mathbf{x} \)-dependent variance of noise. The non-stationary variance of noise is then estimated by a low pass filtering with a Rician bias correction. Results in real and synthetic experiments evidence the better performance and the lowest error variance of the proposed methodology when compared to the state-of-the-art methods.

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1. Introduction

Noise is known to be one of the most common sources of deterioration of the quality of Magnetic Resonance Imaging (MRI) data. The principal source of noise in most MR scans is the subject or object to be imaged, followed by electronics noise during the acquisition of the signal in the receiver chain. It is produced by the stochastic motion of free electrons in the RF coil, which is a conductor, and by eddy current losses in the patient, which are inductively coupled to the RF coil. The presence of noisy patterns on the acquired MR signal is a problem that affects not only the visual quality of the images, but also may interfere with further processing techniques such as registration, fMRI analysis or tensor estimation in Diffusion Tensor MRI (McGibney and Smith, 1993; Gudbjartsson and Patz, 1995; Aja-Fernández et al., 2008b; Huang et al., 2004; Noh and Solo, 2011). The accurate modeling of signal and noise statistics in MR data usually underlies the tools for processing and interpretation within MRI.

The stationary Rician distribution (Gudbjartsson and Patz, 1995) has been widely accepted in literature as a suitable model for noise in MR magnitude images. Many authors have precisely introduced the Rician statistics in the estimation of diffusion models (Clarke et al., 2008), curve fitting for quantitative perfusion measurements (Friman et al., 2008; Schmid et al., 2009), hypothesis tests to assess the activation level in functional MRI (Noh and Solo, 2010), in a preprocessing step to remove the bias of the data for the subsequent processing stages (Koay and Basser, 2006) and denoising techniques (Tristán-Vega and Aja-Fernández, 2010; Manjón et al., 2008; Tristán-Vega et al., 2012).

The main assumption for single coil Rician acquisitions is that the noise is stationary, and therefore a single value of \( \sigma \) characterizes the whole data set. However, this premise will mostly fail when considering modern scanners with multiple-coil antennae and scanning software to correct artifacts, and to improve the final appearance of the image. Linear operations carried out over the complex Gaussian data modify the variance of the noise \( \sigma^2 \) differently for each position. As a result, the final magnitude signal will
have an $x$-dependent value of $\sigma$, i.e. $\sigma(x)$, generating a non-stationary (or non-homogeneous), distribution. For instance, this is the case of a Sum-of-Squares (Constantinides et al., 1997) reconstruction of a multiple-coil acquisition, where the composite magnitude signal can be approximated by a non-stationary non-central $\chi$ (nc-$\chi$) distribution (Aja-Fernández and Tristán-Vega, 2012). It is also the case for accelerated acquisitions with parallel MRI (pMRI) reconstruction techniques. If GRAPPA (Generalized Autocalibrating Partially Parallel Acquisition, Criswold et al., 2002) is used for reconstruction, the final noise is known to follow a non-stationary nc-$\chi$ distribution (Aja-Fernández et al., 2011; Thünberg and Zetterberg, 2007), while if SENSE (Sensitivity Encoding for Fast MRI, Pruessmann et al., 1999) is used, the magnitude signal may be considered Rician distributed with a spatially variant value of $\sigma(x)$ (Dietrich et al., 2008; Thünberg and Zetterberg, 2007; Aja-Fernández et al., 2014).

Nowadays, SENSE has become practically a de facto standard in most acquisitions. However, many processing techniques still assume the stationary Rician distribution as a model for the signal and noise, forgetting about the non-stationarity of the data. This is probably due to the fact that most noise estimators in literature are based on a single $\sigma^2$ value for all the pixels in the image, either assuming a Rician model (Sijbers et al., 1998b,a; Brummer et al., 1993; Aja-Fernández et al., 2008a, 2009; Sijbers et al., 2007) or an nc-$\chi$ (Constantinides et al., 1997; Aja-Fernández et al., 2009; Koay and Basser, 2006; Aja-Fernández and Tristán-Vega, 2012). There have also been some proposals to carry out a rough estimation of non-stationary noise maps. However, these approaches require extra information beyond the simple magnitude signal: multiple acquisitions or different signals are required (Veraart et al., 2013; Maximov et al., 2012; Landman et al., 2009), a biophysical model must be defined (Landman et al., 2009), or even acquisition information such the estimated sensitivity of the coils is needed (Aja-Fernández et al., 2014). This need of extra information has supposed a drawback in the usage of more complex noise models.

In this paper we propose a new technique that allows the estimation of the spatially variant maps of noise $\sigma(x)$ from the magnitude signal when only a single image is available and no additional information is required. The estimator is developed for the non-stationary Rician case, and it is complemented with the estimators for Gaussian and Rayleigh cases. The methodology here presented is totally compatible with other noise models, such as the non-stationary nc-$\chi$ distribution, and the extension would be straightforward. The initial assumption needed is that the variability of the map of noise is smaller than the variability of the noise itself, i.e., $\sigma(x)$ is a low frequency signal when compared to the noise, which is a rational assumption in MRI acquisitions. Both sources of variability are separated by using a homomorphic transformation (Oppenheim et al., 1968). This technique allows us to improve the estimation of the map of noise while it avoids the granularity produced by most local methods.

### 2. Background: non-stationary noise in MRI

When dealing with MRI data, before performing any processing that may involve noise related parameters, it is necessary to identify the specific noise model present in your data. The probability model of noise in the data depends on the coil configuration of the scanner and on the kind of processing the MR data goes through before producing the final magnitude image. The purpose of this section is precisely to provide a general framework of the most usual models of noise in MR data, as well as the most common procedures that originates these distributions.

Most applications dealing with noise in MRI rely on the assumption of a single value of the variance of noise $\sigma^2$ for every pixel within the image, i.e., they assume a stationary noise model: the features of noise do not change with position. However, this is not entirely the case in modern acquisition systems, when pMRI protocols and artifact correction techniques are applied. The linear manipulation of the original Gaussian data, the combination of different coil information and adaptive processing change the features of noise differently in every location of the image. However, due to the kind of processing carried out, most of the times the Rician and nc-$\chi$ assumptions still hold, although the stationarity does not.

In what follows the procedures that lead to non-stationary noise models in MRI will be put together, and the main proposals in literature for non-stationary noise estimation will be reviewed.

#### 2.1. Noise model for each coil

The first step in modeling the final magnitude image in MRI is to model the noise distribution in each of the scanner coils and, then, to propagate the model along the processing pipeline.

Let us assume an $L$-coil antenna configuration, being $L$ the number of coils in the system. We denote $s_l(k)$ the signal in the $k$-space acquired by the $l$-th coil, which corresponds to a complex signal $S_l(x)$ in the image domain. Signals $s_l(k)$ are considered to be corrupted by Additive White Gaussian Noise (AWGN), with zero mean and variance $\sigma_{l,2}^2$:

$$s_l(k) = a_l(k) + n_l(k, \sigma_{l,2}^2), \quad l = 1, \ldots, L,$$

with $a_l(k)$ the noise-free signal and $n_l(k, \sigma_{l,2}^2) = n_l(k, \sigma_{l,2}^2) + j \cdot n_l(k, \sigma_{l,2}^2)$ the AWGN process, which is initially assumed stationary, so that $\sigma_{l,2}^2$ does not depend on $k$. The complex $x$-space is obtained as the inverse Discrete Fourier Transform (iDFT) of $s_l(k)$ for each slice or volume, so the noise in the complex $x$-space is still Gaussian (Thünberg and Zetterberg, 2007):

$$S_l(x) = A_l(x) + N_l(x, \sigma_{l,1}^2), \quad l = 1, \ldots, L,$$

where $N_l(x, \sigma_{l,1}^2)$ is also a complex AWGN process (note that we are assuming no spatial correlations) with zero mean and covariance matrix:

$$\Sigma = \begin{pmatrix}
\sigma_{1,1}^2 & \sigma_{1,2}^2 & \cdots & \sigma_{1,L}^2 \\
\sigma_{2,1}^2 & \sigma_{2,2}^2 & \cdots & \sigma_{2,L}^2 \\
\vdots & \vdots & \ddots & \vdots \\
\sigma_{L,1}^2 & \sigma_{L,2}^2 & \cdots & \sigma_{L,L}^2
\end{pmatrix}. $$

The relation between the noise variances in the $k$- and $x$-domains is given by the number of points used for the iDFT:

$$\sigma_{l,1}^2 = \frac{1}{|\Omega|} \sigma_{l,2}^2,$$

with $|\Omega|$ the final number of pixels in the field of view (FOV).

#### 2.2. Noise models in pMRI

In order to accelerate the acquisition rate in multiple coil systems, pMRI techniques are used. This acceleration is achieved by subsampling the $k$-space data in each coil (Hoge et al., 2005; Larkman and Nunes, 2007), i.e., not all the frequency lines are acquired. The immediate effect of this $k$-space subsampling is the appearance of aliased replicas in the image domain retrieved at each coil. In order to suppress or correct this aliasing, pMRI techniques are used.

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combines the redundant information from several coils to reconstruct a single non-aliased image domain.

The commonly used stationary Rician and nc-\chi models do not necessarily hold in this case. Depending on the way the information from each coil is combined, the statistics of the image follows different distributions. It is therefore necessary to study the behavior of the data for each particular reconstruction method.

Let us call $s_j^l(k)$ to the subsampled signal at the $l$-th coil in the k-space and $S_j^l(x)$ in the image domain. The relation between the noise variances in the k- and x-domains in Eq. (3) depends on the kind of reconstruction used for the iDFT. If a $S_j^l(x)$ is direct iDFT of $s_j^l(k)$, the relation, assuming an acceleration rate of $r$, becomes

$$\sigma_l^2 = \frac{r}{|k|} \sigma_k^2.$$  \hfill{(4)}

On the contrary, if the iDFT is computed after zero-padding, the missing (not sampled) k-space lines, the relation is:

$$\sigma_l^2 = \frac{1}{|k|} \sigma_k^2.$$  \hfill{(5)}

Relations between the variance of noise in complex x-space and k-space for each coil are summarized in Table 1.

Many different methods have been defined to reconstruct the final image from subsampled versions of the signals in each coil, being SENSE and GRAPPA dominant in commercial scanners. However, new reconstruction methods and modifications of the existing ones are continuously proposed. From a statistical point of view, reconstruction methods carry out linear operations over the subsampled signals $S_j^l(x)$, in order to obtain a final reconstructed magnitude image, which is the one of the main causes of the non-stationarity of noise. There are mainly two different approaches for signal reconstruction:

1. **Reconstruction of a single complex image**: the reconstruction process combines the data of the different coils with some extra information (such as the sensitivity map of each coil or the covariance matrix) to obtain a single image:

$$S_j^l(x) = f\left(\left\{S_j^l(x), l = 1, \ldots, L\right\}, \Theta\right).$$  \hfill{(6)}

with $f(\cdot)$ a linear reconstruction function (see some specific functions in Pruessmann et al. (1999) and Blaimer et al. (2004)) and $\Theta$ any additional information needed. The linear operations over the Gaussian data generate correlated Gaussian data. However, the reconstruction affects the stationarity of the noise in the resulting image. Thus, the final signal can be seen as a reconstructed signal corrupted with Gaussian noise whose variance depends on the position:

$$S_j^l(x) = A_j^l(x) + N_j^l(x; \sigma_{j,l}^2(x)),$$  \hfill{(7)}

where $N_j^l(x; \sigma_{j,l}^2(x))$ is a non-stationary complex AWGN process. The final magnitude image is obtained by using the absolute value:

$$M(x) = |S_j^l(x)|$$  \hfill{(8)}

and therefore it follows a non-stationary Rician distribution, with the parameter $\sigma_{j,l}^2(x)$ being spatially variant. This is the case, for instance, of pMRI data reconstructed with SENSE in its original form.

2. **Reconstruction of multiple complex images**: the reconstruction process combines the data of the different coils to obtain a reconstructed image per coil:

$$S_j^l(x) = f\left(\left\{S_j^l(x), m = 1, \ldots, L\right\}, \Theta\right), \text{ with } l = 1, \ldots, L.$$  \hfill{(9)}

with $f(\cdot)$ a linear reconstruction function over each coil (see one specific function in Griswold et al. (2002)). As in the previous case, the linear operations over the Gaussian data generate non-stationary Gaussian data in each coil. The final signal in each coil can be seen as a reconstructed signal corrupted with Gaussian noise whose variance depends on the position:

$$S_j^l(x) = A_j^l(x) + N_j^l(x; \sigma_{j,l}^2(x)),$$  \hfill{(10)}

The probability distribution of the final magnitude image depends on the method used to merge the information of the multiple reconstructed coils into one single image. To avoid any extra information, one of the most common approaches is the Sum-of-Squares (SoS):

$$M_{\text{SoS}}(x) = \sqrt{\sum_{l=1}^{L} |S_j^l(x)|^2}.$$  \hfill{(11)}

<table>
<thead>
<tr>
<th>Noise relations</th>
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<tbody>
<tr>
<td><strong>k-Space</strong></td>
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<tr>
<td>Fully sampled, $\sigma_k^2$, k-size: $</td>
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<tr>
<td>Subsampled $r$, $\sigma_k^2$, k-size: $</td>
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<tr>
<td>Subsampled $r$, $\sigma_k^2$, k-size: $</td>
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The distribution of the magnitude signal $M_{\text{SoS}}(x)$ depends on the relation of noise with the reconstructed signal in each coil, $S_i^2(x)$. For a GRAPPA procedure, for instance, it can be approximated by a non-stationary nc-$\chi$ distribution (Aja-Fernández et al., 2011). However, since it is the sum of multiple signals, the Rician distribution does no longer hold. Due to the bias that the method can introduce over the resulting signal, manufacturers are lately trying to avoid the SoS to obtain $M(x)$. Even for GRAPPA, some new approaches use a reconstruction similar to the first case, where data is Rician distributed.

Other common strategy is based on the spatial matched filter approach, which linearly combines the complex signals of each coil and produces voxelwise complex signals (McKenzie et al., 2002). This is the same methodology used by SENSE, and it requires extra information of the sensitivity of each coil. One simple way to implement this filtering is the following:

$$S_i^2(x) = \sum_{l=1}^{L} S_i^2(x) \cdot C_l(x), \quad (12)$$

with $C_l(x)$ an estimation of the sensitivity map in each coil. The complex image $S_i^2(x)$ now follows a complex Gaussian distribution, similar to the previous case. The magnitude signal is then obtained:

$$M_i(x) = |S_i^2(x)|$$

and therefore it follows a non-stationary Rician distribution, with the parameter $\sigma_i^2(x)$ being spatially variant.

2.4. Non-stationary noise estimation

Although most of the noise estimators in literature cope with the problem of a single value of $\sigma$ for the whole image (Sijbers et al., 1998b,a; Brummer et al., 1993; Aja-Fernández et al., 2008a, 2009; Sijbers et al., 2007), there have been some attempts to estimate non-homogeneous maps of noise, not only in the MRI context. The most usual approaches are based on wavelet decomposition or on multiple acquisitions.

One of the first attempts to estimate spatially variant noise in images is due to Goossens et al. (2006). Authors estimate the spatially variant map of noise in images assuming they are corrupted by a non-stationary AWGN process. To separate the signal and the noise, they use a wavelet transform, assuming that the high–high subband is strictly noise. In DeVore et al. (2000), authors propose the joint estimation of non-homogeneous noise and signal in Rician data, using an expectation–maximization (EM) algorithm to find the maximum likelihood (ML) estimate for the parameters in synthetic aperture radar images. To carry out the algorithm, multiple samples of the receiving signal are necessary.

In MRI, Delakis et al. (2007) proposed a method to estimate spatially variant noise by suppressing the signal component. To that end, the stationary wavelet transform of the magnitude image at the scale $s = 1$ is calculated and the low–low subband coefficients are removed. The estimation is done assuming that the remaining signal is only noise following a Rayleigh distribution. This assumption, however, is not entirely true, as can be experimentally proved. A similar approach, also based in image removal to get an only noise image is carried out by Manjón et al. (2010). They model the variance of noise as a minimal distance between local neighborhood (patch) of the current pixel and the remaining patches in the non-local means filtering scheme. The estimate is corrected for low Signal-to-Noise Ratio (SNR) regions applying the technique described in Koay and Basser (2006). Thus, a SNR iterative estimation is needed.

The median absolute deviation (MAD) estimator for stationary Rician noise estimation proposed in Coupé et al. (2010) has been extended by some authors to non-stationary noise using a local version of the MAD, see for instance Maximov et al. (2012) and Liu et al. (2014), with very similar approaches, and Veraart et al. (2013) with a very effective technique that needs information of multiple diffusion weighted images of the same slice to carry out the estimation. All these methods also need an estimation of the SNR.

An alternative technique was proposed in Landman et al. (2009), based in a $Q_{\sigma}$ estimator followed by a regularization procedure using coil sensitivity model. Although this method has been proposed to cope with multiple independent MR scans, in its basic scenario it can be used to estimate the noise map on a single image.

Other significant methods are the following: Guo and Huang (2009) proposed a local variance as a noise level estimator after edges exclusion by means of local mutual information and k-means segmentation. This method suffers from edges overestimation, thus, a mathematical morphology filter is applied to suppress this undesirable effect; Samsonov and Johnson (2004) calculate the noise map from the receiver coil noise matrix, which, in fact, is not always available in a clinical routine; Ding et al. (2010) proposed a method to assess temporary random noise in dynamic MR image series, e.g., cardiac function imaging or blood flow velocity mapping.

Finally, in Aja-Fernández et al. (2014), authors propose a method to estimate the variable noise maps in MRI assuming SENSE and GRAPPA reconstruction. Although the results are precise, the drawback of this method is that some prior information about the reconstruction process is needed: namely the sensitivity map estimated for each coil (for SENSE) or the GRAPPA reconstruction coefficients. Although these parameters can be...
obtained from the scanner, they are not always available in clinical acquisitions. This limitation, present in most of the reviewed methods, is the main motivation of this current work.

3. A homomorphic approach to non-stationary noise estimation

In what follows we will assume that, due to the processing done in the scanner, the final image \(I(x)\) is corrupted with noise whose variance is \(x\)-dependent. For the sake of generality and simplicity, no specific pMRI reconstruction method will be considered. We will also assume that only one single 2D image is available and no extra parameters are known. Note that, unlike our proposal, some of the estimation methods proposed in the literature are based on the availability of multiple repetitions or information about the reconstruction process. The methodology presented is based on the initial assumption that the spatial dependent variance \(\sigma^2(x)\) shows a low variability, i.e., it can be considered a low pass signal. Therefore, it can be separated from the noise pattern that multiplies it. This is a rational assumption in MRI acquisitions.

In the following study, three different cases will be considered: Gaussian, Rayleigh and Rician. Although the latest is the most suitable model for MRI, the first one provides a good alternative for high SNR. It also presents a solid alternative to an automatic estimator built-in inside the scanning software before the magnitude is calculated. Finally, for the sake of completeness, the Rayleigh case is also considered since it is the lower boundary for the Rician case when the SNR tends to zero.

3.1. The Gaussian case

Let us assume a simple case in which an image \(A(x)\) is corrupted with additive Gaussian noise with zero mean and spatially-dependent variance \(\sigma^2(x)\):
\[
I(x) = A(x) + N(x, 0, \sigma^2(x)) = A(x) + \sigma(x) \cdot N(x, 0, 1). \tag{13}
\]

Our purpose is to estimate \(\sigma(x)\) from the final image \(I(x)\). To that aim, we use a homomorphic filtering that will extract the spatially variant pattern of noise.

Let us assume that the variance of noise \(\sigma^2(x)\) slowly varies across the image, i.e. it is a low frequency signal. We remove the mean of the image to avoid any contribution of \(A(x)\):
\[
I_d(x) = I(x) - E[I(x)] = \sigma(x) \cdot N(x) \tag{14}
\]

where \(E[I(x)]\) denotes the expectation value in each point of the image. To avoid the local mean, so that \(E[I(x)] = A(x)\). Next, we separate signals \(\sigma(x)\) and \(N(x)\) by applying the logarithm:
\[
\log[N(x)] = \log[\sigma(x)] + \log[N(x)].
\]

The noise term \(\log[N(x)]\) has its energy distributed all along frequencies, while the term \(\log[\sigma(x)]\) is a low frequency signal by hypothesis. The latest can be recovered using a low pass filtering of \(\log[I_d(x)]\):
\[
\text{LPF}[\log[I_d(x)]] \approx \log[\sigma(x)] + \delta_N. \tag{15}
\]

with \(\delta_N\) being a low pass residue of \(\log[N(x)]\). This residue must be calculated to remove it from the estimation. If we assume that the LPF has a small passband, the \(\text{LPF}[\log[N(x)]]\) is a good estimator of the local mean. By hypothesis, \(N(x)\) is stationary, and therefore the mean is the same for all pixels. Thus, we can consider the LPF as a good approximation of the mean of the signal:
\[
\text{LPF}[\log[N(x)]] \approx E[\log[N(x)]].
\]

Since we know that \(N(x)\) follows a Gaussian distribution (zero mean and unitary variance), then \(N(x)\) follows a half-normal distribution, and the mean of \(\log[N(x)]\) can be written as:
\[
E[\log[N(x, 0, \sigma^2)]] = \int_0^\infty \log(x) \frac{\sqrt{2}}{\sqrt{\pi e}} e^{-x^2/2} dx = \log \sigma - \log \sqrt{2 - \gamma^2/2}, \tag{16}
\]

where \(\gamma\) is the Euler–Mascheroni constant. With this solution, and with \(\delta_N = E[\log[N(x, 0, 1)]]\), Eq. (14) becomes:
\[
\text{LPF}[\log[I_d(x)]] \approx \log[\sigma(x)] - \log \sqrt{2 - \gamma^2/2}.
\]

And taking the exponential of each term:
\[
e^{\text{LPF}[\log[I_d(x)]]} \approx \sigma(x) \frac{e^{-\gamma^2/2}}{\sqrt{2}}. \tag{17}
\]

Thus, we can define an estimator for \(\sigma(x)\) as
\[
\hat{\sigma}(x) = \sqrt{2} e^{\text{LPF}[\log[I_d(x)]] + \gamma^2/2}. \tag{18}
\]

The whole estimation pipeline for the Gaussian case is depicted in Fig. 1. Note that a practical problem may arise when estimating the local mean. This estimation is usually carried out by local sample moments under the assumption of local stationarity. This assumption is not valid in regions with more than one tissues, particularly on the edges, and therefore the estimation can be biased. The proposed methodology overcomes this problem, even when the estimation of \(E[I(x)]\) is not perfectly achieved. Note that the edges within the image are high frequency areas. The low pass filtering used for the homomorphic separation will remove the effect of edges in the calculation of local moments.

3.2. The Rayleigh case

We know that in those areas of MRI data where the signal is absent, under certain conditions, the noise is Rayleigh distributed (Macovski, 1996; Aja-Fernández et al., 2009). The Rayleigh distribution in the background of the acquisitions has traditionally been used for noise estimation in the stationary case. However, once \(\sigma(x)\) becomes \(x\)-dependent, the estimation over the background might not be related to the estimation over the signal areas. Nevertheless, for the sake of completeness, we add the Rayleigh case here as a previous step for the Rician case. In addition, note that it can be also used in a calibration step or to design coil configuration attending to the generated noise map. Furthermore, note that the spatially variable noise here proposed is similar to some speckle models in literature, and results can be easily extrapolated.

Let us assume a complex Gaussian noise with zero mean and spatially-dependent variance \(\sigma^2(x)\):
\[
N_0(x) = N_0(x, 0, \sigma^2(x)) + j \cdot N_0(x, 0, \sigma^2(x)). \tag{19}
\]

The module of \(N_0(x)\) follows a Rayleigh distribution
\[
R(x; \sigma(x)) = |N_0(x)| = \sigma(x) \cdot \sqrt{N_0^2(x, 0, 1) + N_0^2(x, 0, 1)} = \sigma(x) \cdot R_1(x, 1). \tag{20}
\]

As in the Gaussian case, our purpose is to estimate \(\sigma(x)\) from the Rayleigh noise \(R(x; \sigma(x))\). To that aim, we use again a homomorphic filtering:
\[
\log[R(x; \sigma(x))] = \log[\sigma(x)] + \log[R_1(x, 1)].
\]

The term \(\log[\sigma(x)]\) is a signal with lower frequency components than \(\log[R_1(x, 1)]\). We apply the low pass filtering:
\[
\text{LPF}[\log[R(x; \sigma(x))]] \approx \log[\sigma(x)] + \text{LPF}[\log[R_1(x, 1)]]. \tag{21}
\]

Let us assume again that the LPF is equivalent to a local averaging:
\[
\text{LPF}[\log[R(x; \sigma(x))]] \approx \log[\sigma(x)] + E[\log[R_1(x, 1)]].
\]
For the sake of simplicity, the dependence of $\sigma(x)$ on $x$ is neglected. As before, we apply the homomorphic filtering, where we first apply the logarithm:

$$\log |l(x)| = \log \sigma(x) + \log |\mathcal{G}(s_0(x))|$$

and afterwards a low pass filtering:

$$\text{LPF} \{ \log |l(x)| \} = \log \sigma(x) + \delta_k,$$

with $\delta_k$ being a low pass residue of $\log |\mathcal{G}(s_0(x))|$. Again, the LPF behaves as the expected value of the signal:

$$\mathbb{E}[\sigma(x)] = \log \mathcal{G}(s_0(x)),$$

as shown in Fig. 2. The whole estimation pipeline for the Rayleigh case is depicted in Fig. 2.

### 3.3. The Rician case

Finally, we consider the Rician case. Let us assume a signal $A(x)$ corrupted with complex Gaussian noise with zero mean and spatially-dependent variance $\sigma^2(x)$, whose module follows a non-stationary Rician distribution:

$$l(x; A(x), \sigma(x)) = |A(x) + N_r(x; 0, \sigma^2(x)) + j \cdot N_i(x; 0, \sigma^2(x))|$$

For the sake of simplicity, the dependence of $l(x)$ on $A(x)$ and $\sigma(x)$ will be removed. As in the Gaussian case, our purpose is to estimate $\sigma(x)$ from the Rician signal $l(x)$. However, in this case, the signal and noise are not totally separable. Nevertheless, following the assumption of slow varying $\sigma(x)$, the homomorphic filter can be used to extract the spatially variant pattern of noise.

First, the data is centered by subtracting the local mean of the image.

$$I_\text{c}(x) = l(x) - E[l(x)] = l(x) - \sigma(x) \sqrt{\frac{\pi}{2}} \frac{1}{\sigma^2(x)}$$

Taking the exponential of each term, we can define an estimator for $\sigma(x)$ as

$$\sigma(x) = \frac{1}{\sqrt{2}} \text{LPF} \{ \log |l(x)| \} + \gamma/2.$$  (24)

The whole estimation pipeline for the Rayleigh case is depicted in Fig. 2.

![Figure 2. Pipeline of $\sigma(x)$ estimation assuming Rayleigh noise.](image)

![Figure 3. Mean of the log centered Rician distribution.](image)

![Figure 3. Mean of the log centered Gaussian distribution.](image)
To derive an expression of \( r(x) \) from Eq. (26), assuming a generic Rician random variable \( I_2(x; A, \sigma) \), the expected value of \( \log |I_2(x; A, \sigma)| \) must be studied:

\[
E[\log |I_2(x; A, \sigma)|] = \frac{1}{\sigma^2} \int_0^\infty \log|x-a_1| \cdot x \cdot e^{-\frac{x^2}{2\sigma^2}} \cdot I_0 \left( \frac{Ax}{\sigma} \right) \, dx.
\] (28)

The integral has been numerically solved for \( A = s_0(x) \), \( \sigma = 1 \) and \( a_1 = \sqrt{2}I_{1/2} \left( \frac{\varphi(s_0(x))}{\sigma} \right) \) and depicted in Fig. 3(a), together with the mean of the log Gaussian in Eq. (15). Note that the value of the mean in Eq. (28) depends on the SNR of the signal and, for larger values of SNR, the value approximates to the Gaussian case. In this case, the expected value can be approximated as the mean value obtained for the Gaussian case plus a correction factor as follows:

\[
LPF\{\log |G(s_0(x))|\} \approx E[\log |G(s_0(x))|].
\]

With \( \varphi(s_0(x)) \) a Rician/Gaussian correction function, depicted in Fig. 3(b). Thus, the estimator for \( \sigma(x) \) can be defined as

\[
\hat{\sigma}(x) = \sqrt{2}e^{LPF[\log |I(x)| - E(\{I(x)\})]}e^{\frac{\varphi(s_0(x))}{2}}.
\] (29)

Note that an estimate of the SNR in each pixel is necessary. This requirement is common to other estimators in literature like Manjón et al. (2008) and Maximov et al. (2012).

The whole estimation pipeline for the Rician case is depicted in Fig. 4.

4. Materials and methods

4.1. Materials

For the validation of the proposed estimation methodology, the following data sets are considered:

1. **MR synthetic Images**: five different slices from the Brainweb database (Collins et al., 1998) are used. These slices are shown in Fig. 5(a). All the images are noise free and the background value is zero.

2. **Noise maps**: the noise is artificially added to the previous MR images. Four different spatially variant patterns are considered, as depicted in Fig. 5(b). These patterns are coherent with noise patterns that can be found in real acquisitions. The first pattern is derived from a real GRAPPA acquisition (Aja-Fernández et al., 2014); the second is a linear combination of the first pattern with a rotation of itself; the third pattern is a second order polynomial and the fourth is an isotropic Gaussian function.

3. **Real SENSE acquisitions**: a doped ball phantom is scanned at a 3T Philips Achieva scanner, TFE Pulse Sequence, 224 × 224 × 59, TR/TE = 5.264/2.569, slice thickness 3.20 mm, SENSE with acceleration 2× (reduction factor \( r = 2 \)). 20 repetitions of the same volume were acquired.

4.2. Methods

For the sake of comparison, the proposed method will be compared to some of the proposals in the literature for the estimation...
of non-stationary noise in MRI. Only those capable to carry out the estimation over a single image are considered:

1. Goossens et al. (2006): originally proposed for Gaussian noise. A good behavior for high SNR is expected, when the Gaussian assumption becomes reasonable. The estimator is defined as:

$$\sigma^2(\mathbf{x}) = \left( \frac{1}{N} \sum_{\mathbf{p} \in \Omega(\mathbf{x})} (I(\mathbf{p}) - \bar{I}(\mathbf{x}))^2 \right)_{\mathbf{x}}$$

with $I(\mathbf{p})$ the high–high subband coefficients of the stationary wavelet transform (SWT) of the image $I(x)$ at the scale $s = 1$. The operator $(\bar{I}(\mathbf{x}))_{\mathbf{x}}$ stands for the local sample estimator of the mean:

$$\bar{I}(\mathbf{x}) = \frac{1}{|\eta(\mathbf{x})|} \sum_{\mathbf{p} \in \eta(\mathbf{x})} I(\mathbf{p})$$

with $\eta(\mathbf{x})$ a neighborhood centered in $\mathbf{x}$.

2. Delakis et al. (2007):

$$\sigma^2(\mathbf{x}) = \left( 2 - \frac{\pi}{2} \right) \left( \bar{I}(\mathbf{x})_{\mathbf{x}} - \left( \bar{I}(\mathbf{x})_{\mathbf{x}} \right)^2 \right),$$

where $\bar{I}(\mathbf{x})$ is the image after removing low frequencies (low–low subband) by using a SWT.

3. Liu et al. (2014):

$$\sigma^2(\mathbf{x}) = 1.4826 \cdot \text{MAD}_n \left( I_{11HH}(\mathbf{x}) \right)$$

where $I_{11HH}(\mathbf{x})$ is again the high–high subband coefficients of the SWT of $I(\mathbf{x})$ and $\text{MAD}_n(\cdot)$ is the (local) median absolute deviation defined as:

$$\text{MAD}_n(\mathbf{x}) = \text{median}_{\mathbf{p} \in \eta(\mathbf{x})} |I(\mathbf{p}) - \text{median}_{\mathbf{q} \in \eta(\mathbf{x})} |I(\mathbf{q})|.$$%

For low SNR, a correction is needed:

$$\hat{\sigma}(\mathbf{x}) = \frac{\sigma(\mathbf{x})}{\sqrt{\zeta(\theta)}},$$

where function $\zeta(\theta)$ is defined in Koay and Basser (2006) and SNR parameter $\theta$ is estimated iteratively:

$$\theta_{k+1} = \sqrt{\zeta(\theta_k) \left( 1 + \frac{\left( \bar{I}(\mathbf{x})_{\mathbf{x}} \right)^2}{\sigma^2(\mathbf{x})} \right)^2 - 2}.$$

4. Maximov et al. (2012): the Gaussianian estimator is defined as

$$\sigma^2(\mathbf{x}) = 1.4826 \cdot \text{MAD}_n(\bar{I}(\mathbf{x})).$$

For low SNR, the correction in Eq. (34) is applied.

5. Manjón et al. (2010):

$$\sigma^2(\mathbf{x}) = \min_{\mathbf{p} \in \eta(\mathbf{x})} \| R(\mathbf{x}) - R(\mathbf{p}) \|^2_2,$$

where $R(\mathbf{x}) = I(\mathbf{x}) - \bar{I}(\mathbf{x})$ and $\bar{I}(\mathbf{x})$ is the low-pass filtered data. For low SNR, the correction in Eq. (34) is also applied.

6. Landman et al. (2009) for one single image:

$$\sigma(\mathbf{x}) = \text{Q}_s(\mathbf{p} \in \eta(\mathbf{x}) : \epsilon(\mathbf{p})).$$

where $\epsilon$ is the difference (residual) between noisy data and a biophysical model projection onto data, see Landman et al. (2009) for further details. For the sake of simplicity, and due to the lack of an available biophysical model, we considered the sample mean:

$$\epsilon(\mathbf{x}) = \bar{I}(\mathbf{x}) - \left( \bar{I}(\mathbf{x})_{\mathbf{x}} \right).$$

The $\text{Q}_s$ scale estimator is defined by Roussieeuw and Croux (1993) as

$$\text{Q}_s(x_1, \ldots, x_n) = 2.2219 \cdot \left( \left| x_i - x_j \right| : i < j \right)_{\text{Q}_s},$$

where symbol $\left( \cdot \right)_{\text{Q}_s}$ denotes $k$-th element in the ascending ordered data (order statistics) and here $k = \left( \frac{n}{2} + 1 \right) / 4$. To mitigate the impact of the outliers, Landman proposes the re-estimation of the noise map by removing observations with lower SNR value than an adaptive computed threshold:

$$\hat{r}(\mathbf{x}) = \min \left\{ 5. \text{median}_{\mathbf{p} \in \eta(\mathbf{x})} \left\{ I(\mathbf{p}) / \sigma(\mathbf{x}) \right\} - 3 \right\}.$$

7. The EM estimator in DeVore et al. (2000): the estimator is initially defined for working with multiple samples. To adapt it for a single image, we replace the estimation along samples by a local estimation:

$$\hat{A}_{k+1}(\mathbf{x}) = \frac{1}{I_0} \sum_{i=1}^{n} \frac{\Delta_i(\mathbf{x}) |I_i(\mathbf{x})|}{\sigma_i^2(\mathbf{x})} \cdot \hat{I}(\mathbf{x}),$$

$$\sigma^2_{k+1}(\mathbf{x}) = \max \left\{ \frac{1}{2} \left( \bar{I}(\mathbf{x})_{\mathbf{x}} - \hat{A}_0(\mathbf{x}) \right)^2, 0 \right\},$$

where $I_0(\cdot)$ is the modified Bessel function of the first kind and $n$-th order. The initialization process of the EM algorithm is obtained by the method of the moments

$$\hat{A}_0(\mathbf{x}) = \frac{1}{2} \left( \bar{I}(\mathbf{x})_{\mathbf{x}} - \hat{A}_0(\mathbf{x}) \right)^2,$$

$$\sigma^2_0(\mathbf{x}) = \frac{1}{2} \left( \bar{I}(\mathbf{x})_{\mathbf{x}} - \hat{A}_0(\mathbf{x}) \right)^2.$$%

For all the methods a $5 \times 5$ window has been considered for the local operators, except for the EM, where the optimal behavior was empirically observed for $3 \times 3$ neighborhoods. In all the cases where the SWT is needed, the Daubechies (db7) wavelet was used. The proposed homomorphic algorithm was implemented using the following methods:

1. For the initial estimation of the SNR, the output of the local EM estimator for signal and noise in Eqs. (38) and (39) has been considered, using a $3 \times 3$ estimation window:

$$\text{SNR}(\mathbf{x}) = \frac{\hat{A}_0(\mathbf{x})}{\sigma_0(\mathbf{x})},$$

with $k = 10$ iterations.

2. The low-pass filter was designed as a Gaussian filter in the Fourier domain with $H(0,0) = 1$ and standard deviation $\sigma_r$. In all the experiments considered, $\sigma_r$ is always in the range 3–5. To avoid the border effects due to the implicit periodicity of the DFT, the equivalent filter in the Discrete Cosine Transform domain was used.

5. Experiments and results

For the sake of validation of the proposed methodology, some illustrative experiments are carried out.

5.1. Noise separability

Before testing the estimation methods, we check the initial assumption that the non-stationary noise can be separated into different components using a homomorphic filtering. To that end, we consider two simple but illustrative examples for the Gaussian and Rayleigh scenarios. In this experiment we want to...
show that the proposed methodology can be affectively applied to separate the stationary component of noise from the spatially variant component.

First, we assume a 256 × 256 stationary Gaussian noise image, \(N(x, 0.1)\), with zero mean and unitary variance. The image is multiplied by the first noise map from Fig. 5, obtaining a non-stationary noise image \(N(x, 0, \sigma(x)) = \sigma(x) \cdot N(x, 0, 0.1)\), where \(\sigma(x)\) is the considered noise map. The image is then processed using the pipeline in Fig. 1 but using two different filters: a low pass filter to obtain the noise map estimation and its high pass counterpart to obtain the stationary noise component. The high pass filter is defined in the frequency domain as
\[
H_{HP}(f) = 1 - H_{LP}(f).
\]

This way, the output of the low pass filter provides the estimator \(\hat{\sigma}(x)\), while the high pass filter should provide the stationary component of noise \(N_{SP}(x)\). Due to the absolute value in the pipeline, \(|N(x, 0.1)|\) follows a half normal distribution and, therefore, \(N_{SP}(x)\) should also follow that distribution. In order to check this assumption, we consider the transformation \(N_{SP}(x)\), which follows a \(\chi^2\) square distribution with 1 degree of freedom. This transformation leads to easily check the hypothesis of stationarity by means of a Chi-squared goodness-of-fit test,\(^1\) where the null hypothesis is: the data follows a \(\chi^2\) square distribution with 1 degree of freedom.

All the aforementioned assumptions were checked throughout the pipeline shown in Fig. 1 for a set of 100 repetitions. First, we tested the Normality assumption of \(N(x, 0.1)\) and \(N(x, 0, \sigma(x))\) where the null hypothesis is the stationary Gaussian distribution. As was expected the goodness-of-fit test for \(N(x, 0.1)\) obtained an average of 0.4851 for the \(p\)-value, and the stationarity of the Gaussian is accepted for all cases. In the case of \(N(x, 0, \sigma(x))\), the stationarity of the data is discarded in all repetitions, where a \(p\)-value close to 0 was obtained for every case. This result shows that the noise map applied from Fig. 5 causes a statistically significant non-stationarity in the data. The goodness-of-fit test performed over \(N_{SP}(x)\) was applied for different values of the filter bandwidth \(\sigma_f\) to check its influence in the stationarity. The averages of \(p\)-values for each \(\sigma_f\) are depicted in Fig. 6(a), where the null hypothesis is accepted, i.e. the stationarity of the high pass component of noise is accepted. Note that these results confirm that the output noise \(N_{SP}(x)\) is stationary for a bandwidth between 3 and 11. These results show the suitability of the proposed methodology for separating the stationary and non-stationary components of noise in the Gaussian scenario.

The same experiment is repeated for the Rayleigh scenario. We assume now a 256 × 256 Rayleigh image, \(R(x, 1)\), with unitary parameter. The image is also multiplied by the first noise map from Fig. 5, obtaining the non-stationary noise image \(R(x, \sigma(x)) = \sigma(x) \cdot R(x, 1)\). The image is then processed using the pipeline in Fig. 2 with a high pass filter in order to obtain the high pass noise component. The output of the low pass filter is again considered as the estimator \(\hat{\sigma}(x)\) and the high pass filter output is supposed to be stationary noise \(N_{SP}(x)\). If the separation is successfully performed, this noise must follow a Rayleigh distribution with unitary parameter.

The goodness-of-fit test was performed for 100 repetitions of the experiment. We first tested the stationarity of \(R(x, 1)\) and \(R(x, \sigma(x))\) as in the previous scenario. Results obtained for \(R(x, \sigma(x))\) showed an average \(p\)-value of 0.5183, and the null hypothesis (stationarity of the Rayleigh distribution) was confirmed for all the cases. The test for the non-stationary noise \(R(x, \sigma(x))\) discarded the null hypothesis for all the cases, obtaining a negligible \(p\)-value for all of them. This result also confirms that the non-stationary noise map applied is statistically significant. The separated high pass noise \(N_{SP}(x)\) obtained after the homomorphic filtering was also tested for different values of the filter bandwidth \(\sigma_f\). The averages of \(p\)-values with respect \(\sigma_f\) is depicted in Fig. 6(b) where the stationarity hypothesis is accepted for a similar range as in the Gaussian scenario. In the light of these results we can conclude that the proposed methodology also separates the stationary and non-stationary components of noise in the Rayleigh scenario.

For the sake of illustration, the separation process of the non-stationary Rayleigh noise is depicted in Fig. 7. Note that, due to the presence of a spatial-dependent \(\sigma(x)\), noise image \(R(x, \sigma(x))\) in Fig. 7(b) shows a spatially variant pattern. The homomorphic filtering process is able to recover the two original components of noise: in Fig. 7(c) a stationary noisy pattern \(N_{SP}(x)\), and in Fig. 7(d) the noise maps corresponding to \(\sigma(x)\).

5.2. Synthetic experiments

First, synthetic images and synthetic noise maps shown in Fig. 5 were used. The noise is generated as a Rician noise with \(x\)-dependent variance \(\sigma^2(x)\) following the generic model in Eqs. (7) and (8). Two different cases are considered: (a) one single image (first slice in Fig. 5) and one single noise map (first one in Fig. 5); and (b) all combinations of the five images and the four noise maps (20

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\(^1\) The Chi-squared goodness-of-fit test evaluates the discrepancies between the observed frequency distribution and a particular theoretical distribution. The \(p\)-value obtained from this test is interpreted based on the significance level (we consider 0.05 in this study) in such a way that if \(p > 0.05\) the null hypothesis is accepted, i.e. there are no statistically significant differences between the frequency distribution and the theoretical distribution.
different combinations). All the noise and signals were normalized to have the same maximum SNR. In order to generate different SNR values, in each case, the noise-free signal is divided by a constant and the noise is generated using the original map. A set of 100 repetitions of each configuration is considered. Noise is estimated using the aforementioned state-of-the-art methods and the relative error for each pixel is calculated as:

\[
\text{error}(x) = \frac{|\hat{\sigma}(x) - \sigma(x)|}{\hat{\sigma}(x)}.
\]

(40)

The average of all the points, repetitions and configurations is considered to achieve a single value for each SNR. For the single image case, the variance of the error along the samples is also considered. Results are depicted in Fig. 8. For the sample moments estimation a window of $5 \times 5$ size is used. The homomorphic methods use a low pass filter set to $\hat{\sigma}_r = 3.4$. This value has been selected following results in Fig. 6.

Although a single value is not representative for the estimation of the spatial pattern, this experiment gives a quantitative insight of the behavior of some of the methods in literature and the proposed one. Note that, for the whole considered SNR range, the proposed method clearly outperforms those in literature. For high SNR results are depicted in Fig. 8. The sample moments estimation a window of $5 \times 5$ size is used. The homomorphic methods use a low pass filter set to $\hat{\sigma}_r = 3.4$. This value has been selected following results in Fig. 6.

In the second experiment we search for visual comparison of the different estimation methods. The first image and the first noise map from Fig. 5 were used to generate a single MRI slice corrupted by non-stationary Rician noise. The noise map $\sigma(x)$ is estimated from the magnitude image. Visual results for the considered methods are depicted in Fig. 9, together with the estimation error, calculated by using Eq. (40). To ease the visualization, the image has been saturated to the maximum value of the original map. The maximum value of the error was also limited to 1, although some of the methods showed much greater errors.

This experiment clearly shows the suitable behavior of the homomorphic approach when compared to local estimators. The use of a LPF to extract the noise map leads to avoid the granular pattern and to eliminate the influence of the edges. Note that those methods based on local estimation precisely show such a granular pattern due to the use of local neighborhoods with small size. This implies that the number of points used for estimation is low and, conversely, the variance of the estimation is high. Another problem with local estimators is related to the edges of the image. In those areas where two different tissues or regions lay inside the same window, the estimation becomes highly biased and causes the miss-estimation of local moments. The proposed methodology is able to overcome these local problems by properly estimating a smooth and reliable noisy pattern. Even when the Rician hypothesis is relaxed, as in the Gaussian assumption, the method still outperforms the other methods.

In Fig. 10, we repeat the experiment for the second image and the second noise maps of Fig. 5. Results are totally coherent with the previous experiment. Once more, the proposed method shows the best estimation with both assumptions (Rice and Gaussian). Note that this time, all the results show some influence of the high signal areas over the estimated noise map, though it is more subtle with the homomorphic approach, Fig. 10(i) and (j).

5.3. Real MR acquisition

Finally, an experiment with real acquisitions is carried out. Since a golden standard is not available, we consider the original (multi-sample) EM estimation of $\sigma(x)$ using the 20 samples as a reference (silver standard). We compare it to the average of the estimation of the map of noise in each of the acquisitions by using the different estimators. Results are gathered in Fig. 11. Note that only the local EM scheme, the Landman’s method and the homomorphic approaches are able to follow the variation pattern in the map of noise. Note also that some of the methods based on local estimation show a border effect in the boundaries of the slice. This effect is caused by the larger dimension of the object compared to the FOV, which is not the case of brain acquisitions, but it is in other areas such as imaging of body or knee. Thus, it must be taken into consideration.

The proposed methods show a good estimation of the map of noise, coherent with that considered as silver standard. The local EM approach in Fig. 11(g) shows here a very good behavior, which differs from results of previous experiments, where the worse results were related to those areas with borders and transitions between different tissues. In this case, the phantom has not inner edges and, thus, the EM estimator is able to provide a proper estimate of noise in all the signal area. This different behavior evidences the influence of transitions between different tissues in...
local estimation. A problem that the proposed methodology successfully overcomes.

6. Conclusions

A new methodology to estimate spatially variant noise in MR has been presented. It is based on the homomorphic approach that allows to separate the contribution of the pattern of noise and the noise itself. The estimator has proved to be robust, easy to implement and it does not require additional information about the acquisition, nor multiple repetitions to carry out the estimation. It also reduces the influence of borders in the final estimated map, and avoids the traditional granular pattern shown by methods relying on local estimation. Additionally, results have shown

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Fig. 10. Noise estimation (top) and estimation error (bottom), using second image and second noise map. (a) Original noise map; (b) Goossens; (c) Delakis; (d) Liu; (e) Manjón; (f) Maximov; (g) EM; (h) Landman; (i) Rician Homomorphic; (j) Gaussian Homomorphic.

Fig. 11. Noise estimation using a real SENSE acquisition. (a) EM along 20 samples (Silver standard); (b) Goossens; (c) Delakis; (d) Liu; (e) Manjón; (f) Maximov; (g) Local EM; (h) Landman; (i) Rician Homomorphic; (j) Gaussian Homomorphic.
that the proposed methodology presents the lowest error variance and its independence to the SNR. This result evidences the suitability of the proposed estimators.

Three different estimators for $\sigma(x)$ were presented: for the Gaussian, Rayleigh and Rician cases. While the latter would be the most suitable for MRI, the first one presents a good alternative, with the advantage of not requiring a prior estimation of the SNR. The Gaussian estimator could be used as an approximation of the Rician case that will fail in the low SNR areas: mostly the background, where a proper estimation is not so relevant. Additionally, it also presents a solid alternative for a built-in automatic estimator inside the scanning software, which just needs to access to the final complex image in the processing pipeline just before the absolute value is taken. Using this complex image, the variable noise pattern can be accurately estimated without the need of any additional information or SNR estimation.

The proposed methodology shows also a great potential when is jointly used with other estimation methods. In this paper, we have carried out the estimation over the $x$-space using a simple low-pass filtering. However, the homomorphic separation of noise can also be applied to other methods in the literature, particularly those based on wavelets. With the proper adjustments, these other methods can improve in robustness and presumably more accurate results can be achieved.

Finally, in this paper we have only focused on the Rician noise, leaving aside the $nc$-$\gamma$ model, so popular lately in the literature. It was done under the assumption that nowadays most acquisitions are based on single coil systems or in pMRI-SENSE reconstruction. Even those scanning softwares using GRAPPA or similar algorithms (like Siemens) give also the option of a matched filter reconstruction that lately will produce Rician data. Nevertheless, the Gaussianian approach for high SNR is also valid for non-stationary $nc$-$\gamma$ data, and a correction like the one here proposed for Rician can be easily derived.

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