Fast Channel Shortening with Polynomial Weighting Functions

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***Abstract*— Channel shortening is often necessary for demodulation of multicarrier signals and complexity reduction of maximum likelihood sequence estimation (MLSE). This paper has two parts: (i) the proposal of complexity-reduction techniques for channel shortening designs with polynomial weighting functions, including the minimum inter-block interference (Min-IBI) and minimum delay spread (MDS) designs, and (ii) the evaluation of several standard channel shorteners, including Min-IBI and MDS, in terms of bit error rate (BER) for an OFDM system. Previous evaluation of channel shortening designs has focused on the wireline case, for which the performance metric (bit rate for fixed BER) is quite different.**

I. Introduction

Multicarrier modulation has been implemented in an increasing number of applications in recent years, including digital video/audio broadcast (DVB/DAB), asymmetric digital subscriber loops (ADSL), wireless LANs (IEEE 802.11a, IEEE 802.11g, HIPERLAN/2), power line communications (PLC), and satellite radio. The popularity of multicarrier systems is largely due to the fact that if the channel is shorter than the guard time between blocks and is static over each block, then the frequency selective channel appears as a set of flat sub-channels, which can each be equalized by a complex scalar.

Three guidelines dominate the selection of the block size and the length of the guard interval: (i) the block size must be short enough to minimize the channel's time variations within each symbol, (ii) the guard interval must be long enough to exceed the delay spread of the vast majority of channels that will be encountered, and (iii) the throughput loss due to the use of the guard interval must be kept as small as possible. These requirements have generally been met by current standards in the environments for which they have hitherto been deployed. A notable exception is ADSL, for which the channel almost always exceeds the length of the guard inverval. In this case, a time-domain equalizer (TEQ), also called a channel shortening equalizer (CSE) [1], can be employed to shorten the channel to the length of the guard interval.

The main thrust of this paper is to reduce the complexity of the Minimum Inter-Block Interference (Min-IBI) [2] and Minimum Delay Spread (MDS) [3] TEQ designs, to make them feasible for low-cost implementation in a wireless receiver. This is important because for many wireless channels a TEQ is not needed, so its presence should not be obtrusive.

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As current multicarrier standards are deployed in more challenging environments, and as new standards are proposed, it is expected that the guard interval will often be inadequate in wireless multicarrier systems as well. The performance metric for a broadcast wireless system is the bit error rate (BER), yet the performance metric for ADSL is the throughput for a fixed BER. Thus, previous TEQ performance analysis, which has largely been performed for DSL, will not clearly indicate the BER performance in a wireless system. For this reason, a second aspect of this paper is a BER comparison of common TEQ designs, including the Min-IBI and MDS designs.

Section II describes the system model and notation. Sections III and IV propose low-complexity implementations of designs of the Min-IBI and MDS designs, respectively, and Section V generalizes designs with arbitrary polynomial weighting functions. Section VI provides a BER comparison of popular TEQ designs, and Section VII concludes the paper.

II. System model

The multicarrier system model is shown in Fig. 1, and the notation is summarized in Table I. The input stream is divided into blocks of N bins, and each bin is viewed as a QAM signal that will be modulated by a different carrier. An efficient means of implementing the multicarrier modulation in discrete time is to use an inverse fast Fourier transform (IFFT), which mimics a bank of synchronized oscillators. The IFFT converts each bin (which acts as one of the frequency components) into a time-domain signal. After transmission through a frequency-selective channel $\mathbf{h}$, the receiver can use an FFT to recover the data.

If the received data is a circular convolution of the channel and transmitted data, then the received frequency-domain output is a *pointwise* multiplication of the transmitted frequency-domain data with the discrete Fourier transform (DFT) of the channel. Since the convolution is actually linear rather than circular, it is made to appear circular by adding a cyclic prefix (CP, also called a guard interval) to the start of each data block. The CP is obtained by prepending the last ν samples of each

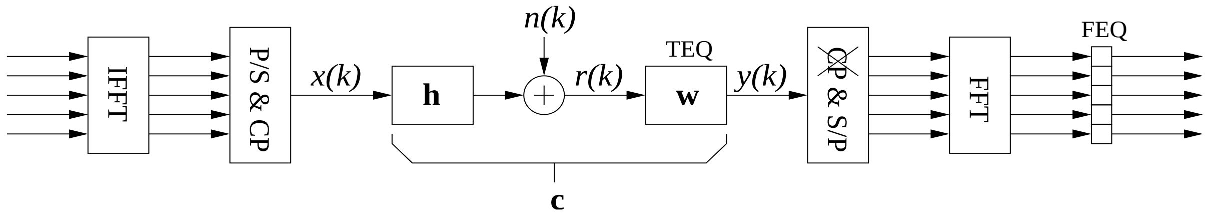

Fig. 1. Multicarrier system model. (I)FFT: (inverse) fast Fourier transform, P/S: parallel to serial, S/P: serial to parallel, CP: add cyclic prefix, and xCP: remove cyclic prefix.

TABLE I
CHANNEL SHORTENING NOTATION

Notation	Meaning
N	FFT size
ν	length of CP
Δ	desired delay of effective channel
η	desired centroid of effective channel
γ	maximum allowed Min-IBI weight
$\mathbf{h} = [h_0, \cdots, h_{L_h}]$	channel impulse response
$\mathbf{w} = [w_0, \cdots, w_{L_w}]$	TEQ impulse response
$\mathbf{c} = [c_0, \cdots, c_{L_c}]$	effective channel ($\mathbf{c} = \mathbf{h} \star \mathbf{w}$)
$\tilde{L}_h = L_h + 1$	channel length
$\tilde{L}_w = L_w + 1$	TEQ length
$\tilde{L}_c = L_c + 1$	length of the effective channel
$\mathbf{H}$	$\tilde{L}_c \times \tilde{L}_w$ channel convolution matrix
$\mathbf{R}_n$	$\tilde{L}_w \times \tilde{L}_w$ noise covariance matrix
$\mathbf{0}_{n\times m}$	$n \times m$ matrix of all zeros
$\mathbf{1}_{n\times m}$	$n \times m$ matrix of all ones
$\mathbf{I}_n$	$n \times n$ identity matrix
$\mathbf{A}^*$, $\mathbf{A}^T$, $\mathbf{A}^H$	conjugate, transpose, and Hermitian

block to the beginning of the block. If the CP plus one is at least as long as the channel, then the convolution appears to be circular and the output of each subchannel (i.e. frequency bin) is equal to the input times a scalar complex gain factor. The signals in the bins can then be equalized by a bank of complex gains, referred to as a frequency domain equalizer (FEQ).

The above discussion assumes that CP length + 1 is greater than or equal to the channel length, i.e. $\nu + 1 \geq L_c + 1$. However, transmitting the cyclic prefix wastes time that could be used to transmit data. Thus, the CP is usually set to a reasonably small value, and a TEQ $\mathbf{w}$ is employed to shorten the channel to this length if necessary. Typically, the CP length is $\frac{1}{4}$ to $\frac{1}{16}$ of the block length. TEQ design methods for wireline, point-to-point systems have been well explored [2], [3], [4], [5], [6], [7] [8].

The TEQs that will be discussed in this paper can be considered as special cases of the general form proposed in [7]. Consider minimization of the cost function [7]

$$J = \alpha J_{short} + (1-\alpha) J_{noise} \tag{1}$$

$$= \frac{\alpha \sum_{n=0}^{L_c} f(n-\Delta) \left|c(n)\right|^2 + (1-\alpha)\frac{\sigma_q^2}{\sigma_x^2}}{\sum_{n=0}^{L_c} \left|c(n)\right|^2} \tag{2}$$

where $\sigma_q^2 = \mathbf{w}^H \mathbf{R}_n \mathbf{w}$ is the power of the filtered noise. The function in the numerator is chosen to penalize channel taps in undesired locations, which are then minimized with respect to the entire channel energy (via the denominator). The α and $1-\alpha$ weights allow for variable suppression of the noise as well. Assuming unit signal power, $\sigma_x^2 = 1$, eq. (2) can be rewritten as a generalized Rayleigh quotient, leading to

$$\mathbf{w}_{opt} = \arg\min_{\mathbf{w}} \frac{\mathbf{w}^H \mathbf{A} \mathbf{w}}{\mathbf{w}^H \mathbf{B} \mathbf{w}}, \tag{3}$$

where

$$\mathbf{A} = \alpha \mathbf{H}^H \mathbf{Q} \mathbf{H} + (1-\alpha)\mathbf{R}_n, \tag{4}$$

$$\mathbf{B} = \mathbf{H}^H \mathbf{H}, \tag{5}$$

and where $\mathbf{Q}(\Delta)$ is a diagonal matrix with n^{th} diagonal element equal to $f(n-\Delta)$. Designs that fit into this framework include the maximum shortening SNR (MSSNR) design [4], the minimum MSE (MMSE) design with a white input [5], the Min-IBI design [2], and the MDS design [3]. Complexity reduction of [4] and [5] was addressed in [9]; this paper addresses complexity reduction of the more complicated designs in [2] and [3].

III. MIN-IBI IMPLEMENTATION

The Min-IBI design [2], [10] minimizes the IBI power subject to the constraint that the desired signal energy is held constant. We breifly review the design, then demonstrate how to simplify its computation. As in [4], we define the Toeplitz channel "window" matrix

$$\mathbf{H}_{win}(\Delta) = \begin{bmatrix} h(\Delta) & h(\Delta-1) & \cdots & h(\Delta - L_w) \\ \vdots & & \ddots & \vdots \\ h(\Delta+\nu) & h(\Delta+\nu-1) & \cdots & h(\Delta+\nu-L_w) \end{bmatrix} \tag{6}$$

and the block Toeplitz channel "wall" matrix

$$\mathbf{H}_{wall}(\Delta) = \left[\mathbf{H}_1^T, \mathbf{H}_2^T\right]^T, \tag{7}$$

where the channel "head" is contained in

$$\mathbf{H}_1(\Delta) = \begin{bmatrix} h(0) & 0 & \cdots & 0 \\ \vdots & \ddots & & \\ h(\Delta-1) & h(\Delta-2) & \cdots & h(\Delta-L_w-1) \end{bmatrix}, \tag{8}$$

and the channel “tail” is contained in

$$\mathbf{H}_2(\Delta) = \begin{bmatrix} h(\Delta+\nu+1) & h(\Delta+\nu) & \cdots & h(\Delta+\nu-L_w+1) \\ \vdots & \ddots & & \\ 0 & 0 & \cdots & h(L_h) \end{bmatrix}. \tag{9}$$

Together, $\mathbf{H}_{win}$ and $\mathbf{H}_{wall}$ partition the $(L_c+1)\times(L_w+1)$ Toeplitz channel convolution matrix $\mathbf{H}$. Thus, $\mathbf{c}_{win} = \mathbf{H}_{win}\mathbf{w}$ yields a length $\nu+1$ window of the effective channel, and $\mathbf{c}_{wall} = \mathbf{H}_{wall}\mathbf{w}$ yields the remainder of the effective channel. Define the IBI weighting matrices as [2]

$$\mathbf{Q}_{ibi}(\Delta) = \text{diag}[\Delta, \cdots, 2, 1, \underbrace{0, \cdots, 0}_{\nu+1}, 1, 2, \cdots, L_c-\nu-\Delta],$$

$$\widetilde{\mathbf{Q}}_{ibi}(\Delta) = \text{diag}\left[\Delta, \cdots, 2, 1, 1, 2, \cdots, L_c-\nu-\Delta\right] \tag{10}$$

where “diag $[\cdot]$” is a diagonal matrix with the elements of the argument along the main diagonal. The Min-IBI design uses $\mathbf{Q}$ to suppress the taps of the effective channel outside of the desired window, with linearly increasing weights at further distances from the edge of the window. Thus,

$$\begin{aligned}\mathbf{A} &= \alpha\mathbf{H}^H\mathbf{Q}_{ibi}\mathbf{H} + (1-\alpha)\mathbf{R}_n \qquad (11)\\ &= \alpha\mathbf{H}_{wall}^H\widetilde{\mathbf{Q}}_{ibi}\mathbf{H}_{wall} + (1-\alpha)\mathbf{R}_n.\end{aligned}$$

The optimization problem is then given by[1] (3), (5), and (11). The solution is the generalized eigenvector of $(\mathbf{A}, \mathbf{B})$ corresponding to the smallest generalized eigenvalue [11].

The only differences between the MSSNR design [4] and the Min-IBI design are that the MSSNR design sets $\widetilde{\mathbf{Q}} = \mathbf{I}$, the identity matrix, and the MSSNR design assumes $\alpha = 1$. The MSSNR design places equal weight on all taps in the channel tails, even though the more distant taps contribute more to the IBI. However, with $\widetilde{\mathbf{Q}} = \mathbf{I}$, the $\mathbf{A}$ matrix for delay $\Delta+1$ can be obtained almost entirely from the $\mathbf{A}$ matrix for delay Δ [9], which is important because computing $\mathbf{A}$ is computationally expensive, i.e. $\mathcal{O}\left(L_w^2(L_h-\nu)\right)$ per delay. The method of performing this feat for the Min-IBI matrix is not so apparent, and this is the focus of the remainder of this section.

Define the error matrices

$$\widehat{\mathbf{E}}(\Delta) = \text{diag}\left[\mathbf{1}_{1\times\Delta}, \mathbf{0}_{1\times\nu}, -\mathbf{1}_{1\times(L_c+1-\nu-\Delta)}\right] \tag{12}$$

$$\begin{aligned}\mathbf{E}(\Delta) &= \mathbf{H}^H\ \widehat{\mathbf{E}}(\Delta)\ \mathbf{H} \qquad (13)\\ &= \mathbf{H}_{wall,\nu}^H(\Delta)\ \widetilde{\mathbf{H}}_{wall,\nu}(\Delta)\end{aligned}$$

where $\widetilde{\mathbf{H}}_{wall,\nu} = \left[\mathbf{H}_1^T, -\mathbf{H}_2^T\right]^T$, and the subscript ν denotes the fact that these particular matrices only eliminate ν rows from $\mathbf{H}$ rather than $\nu+1$. With this definition of the error matrix $\mathbf{E}$, we have

$$\mathbf{Q}_{ibi}(\Delta+1) = \mathbf{Q}_{ibi}(\Delta) + \widehat{\mathbf{E}}(\Delta+1), \tag{14}$$

$$\mathbf{A}(\Delta+1) = \mathbf{A}(\Delta) + \mathbf{E}(\Delta+1). \tag{15}$$

[1]Celebi's Min-IBI design [2] uses $\alpha = 1$ and $\mathbf{B} = \mathbf{H}_{win}^H\mathbf{H}_{win}$ rather than $\mathbf{B} = \mathbf{H}^H\mathbf{H}$, but we prefer Tkacenko's formulation [7].

where (15) follows from (14) by left- and right-multipliying (14) by $\mathbf{H}^H$ and $\mathbf{H}$, respectively. The intuition behind (15) is that $\mathbf{A}(\Delta)$ linearly weights the channel tails outside of a length-$(\nu+1)$ window, and $\mathbf{A}(\Delta)$ does the same thing for a window that is shifted over by one sample. Thus, the effect of incrementing the delay is to increment all of the weights on the channel “head” (up to tap $\Delta+1$) by one and to decrement the weights on the channel “tail” (starting at tap $\Delta+\nu$) by one.

The objective is to form an efficient update rule for $\mathbf{E}(\Delta)$, then use (15) to update $\mathbf{A}$. Since $\mathbf{E}(\Delta)$ is very similar to $\mathbf{H}_{wall}^H\mathbf{H}_{wall}$, we can use techniques similar to those used for the MSSNR design [9], [12]. For $i \geq j$, element (i,j) of $\mathbf{E}(\Delta)$ is given by

$$[\mathbf{E}(\Delta)]_{(i,j)} = \sum_{l=0}^{\Delta-1-i} h_l^* h_{(l+i-j)} - \sum_{l=\Delta+\nu-j}^{L_h} h_{(l+j-i)}^* h_l. \tag{16}$$

Throughout, matrix and vector indexing starts at zero, rather than at one. By incrementing Δ,

$$\begin{aligned}[\mathbf{E}(\Delta+1)]_{(i,j)} = &\sum_{l=0}^{\Delta-1-(i-1)} h_l^* h_{(l+(i-1)-(j-1))}\\ &- \sum_{l=\Delta+\nu-(j-1)}^{L_h} h_{(l+(j-1)-(i-1))}^* h_l\\ = &[\mathbf{E}(\Delta)]_{(i-1,j-1)}. \qquad (17)\end{aligned}$$

In block form,

$$\boxed{[\mathbf{E}(\Delta+1)]_{(1:L_w,1:L_w)} = [\mathbf{E}(\Delta)]_{(0:L_w-1,0:L_w-1)}} \tag{18}$$

By keeping Δ fixed and incrementing i and j instead,

$$\begin{aligned}[\mathbf{E}(\Delta)]_{(i+1,j+1)} &= \sum_{l=0}^{\Delta-1-i-1} h_l^* h_{(l+i-j)} - \sum_{l=\Delta+\nu-j-1}^{L_h} h_{(l+j-i)}^* h_l\\ &= [\mathbf{E}(\Delta)]_{(i,j)} - h_{(\Delta-1-i)}^* h_{(\Delta-1-j)}\\ &\quad - h_{(\Delta+\nu-1-i)}^* h_{(\Delta+\nu-1-j)} \qquad (19)\end{aligned}$$

Or, equivalently,

$$\boxed{\begin{aligned}[\mathbf{E}(\Delta)]_{(i,j)} = &[\mathbf{E}(\Delta)]_{(i+1,j+1)} + h_{(\Delta-1-i)}^*\ h_{(\Delta-1-j)}\\ &+ h_{(\Delta+\nu-1-i)}^*\ h_{(\Delta+\nu-1-j)}\end{aligned}} \tag{20}$$

By using (18) we can obtain all of $\mathbf{E}(\Delta+1)$ except the first row and column from $\mathbf{E}(\Delta)$. The first column of $\mathbf{E}(\Delta+1)$ can be efficiently obtained via (20), and its first row can be obtained by symmetry. Finally, $\mathbf{E}(\Delta_{min})$ must be computed explicitly, but this can also be done efficiently using (20). An outline of an efficient Min-IBI algorithm is given in Fig. 2.

Note that $\mathbf{Q}_{ibi}$ may assign very large weights to the extreme edges of the effective channel impulse response. If the channel estimate is imperfect, these large weights will amplify the errors. The solution proposed in [2] is to limit the maximum

1) For Δ_{min}, compute $\mathbf{A}$, $\mathbf{B}$ (using the efficient methods of [12]), and $\mathbf{E}$ (using (20)).
2) Solve $\mathbf{Aw} = \lambda \mathbf{Bw}$ for the generalized eigenvector corresponding to the smallest eigenvalue, as in [4].
3) For $\Delta = \Delta_{min} + 1 : \Delta_{max}$, do the following:
   a) $[\mathbf{E}]_{(1:L_w,1:L_w)} = [\mathbf{E}]_{(0:L_w-1,0:L_w-1)}$
   b) $[\mathbf{E}]_{(0:L_w-1,0)} = [\mathbf{E}]_{(1:L_w,1)} + h(\Delta-1)\cdot h(\Delta-1-[0:L_w-1])^* + h(\Delta+\nu-1)\cdot h(\Delta+\nu-1-[0:L_w-1])^*$
   c) Compute $[\mathbf{E}]_{(L_w,0)}$ from (13)
   d) $[\mathbf{E}]_{(0,1:L_w)} = [\mathbf{E}]^H_{(1:L_w,0)}$
   e) $\mathbf{A} = \mathbf{A} + \mathbf{E}$
   f) Solve $\mathbf{Aw} = \lambda \mathbf{Bw}$ for the generalized eigenvector corresponding to the smallest eigenvalue.
   g) If this delay produces a smaller λ [equal to the ratio in (3)] than the previous delay, save $\mathbf{w}$.

Fig. 2. Fast Min-IBI TEQ design algorithm.

weight value by redefinining the weighting matrix as

$$\overline{\mathbf{Q}}_{ibi}(\Delta) = \min\{\mathbf{Q}_{ibi}(\Delta), \gamma\} = \text{diag}[\overbrace{\gamma,\gamma,\cdots,\gamma}^{\Delta-\gamma+1}, \gamma-1, \cdots, 2, 1, \overbrace{0,\cdots,0}^{\nu}, 1, 2, \cdots, \gamma-1, \underbrace{\gamma,\gamma,\cdots,\gamma}_{L_c-\nu-\Delta-\gamma+1}] \quad (21)$$

Then the error weighting matrix of (12) becomes

$$\widehat{\mathbf{E}}(\Delta) = \text{diag}\left[\mathbf{0}_{1\times(\Delta-\gamma)}, \mathbf{1}_{1\times\gamma}, \mathbf{0}_{1\times\nu}, -\mathbf{1}_{1\times\gamma}, \mathbf{0}_{1\times(L_c-\nu-\Delta-\gamma+1)}\right]. \quad (22)$$

In this case, (15) and (18) still hold, but (20) requires four update terms rather than two.

IV. MDS IMPLEMENTATION

The MDS design [3] is similar to the Min-IBI design in that is also uses a diagonal weighting matrix. However, the main diagonal is quadratic rather than piecewise linear. The result is that the delay spread of the effective channel is minimized, though the size of the cyclic prefix is not explicitly taken into account.

To formally describe the MDS design, first define the weighting matrix $\mathbf{Q}_{mds}$ as

$$\mathbf{Q}_{mds}(\eta) = \left[\eta^2, (\eta-1)^2, \cdots, 4, 1, 0, 1, 4, \cdots, (L_c-\eta)^2\right]. \quad (23)$$

Then the MDS design is given by (3), (4), (5), and (23). The parameter η is the desired centroid of the effective channel. It replaces the delay parameter Δ as the parameter to be searched over.

Since $\mathbf{B}$ is not a function of η, it only has to be computed once. Moreover, it is Hermitian and Toeplitz, and thus is easily computed. On the other hand, $\mathbf{A}(\eta)$ must be computed once per value of η if a full search is made. We now propose an efficient recursive method for computing $\mathbf{A}(\eta+1)$ from $\mathbf{A}(\eta)$.

1) For η_{min}, compute $\mathbf{A}$, $\mathbf{B}$ (exploiting the Hermitian Toeplitz structure), and $\mathbf{E}_1$.
2) Solve $\mathbf{Aw} = \lambda \mathbf{Bw}$ for the generalized eigenvector corresponding to the smallest eigenvalue, as in [4].
3) For $\eta = \eta_{min} + 1 : \eta_{max}$, do the following:
   a) $\mathbf{E}_1 = \mathbf{E}_1 + 2\mathbf{B}$ (where multiplication by 2 is performed by a shift in binary representation)
   b) $\mathbf{A} = \mathbf{A} + \mathbf{E}_1$
   c) Solve $\mathbf{Aw} = \lambda \mathbf{Bw}$ for the generalized eigenvector corresponding to the smallest eigenvalue.
   d) If this delay produces a smaller λ [equal to the ratio in (3)] than the previous delay, save $\mathbf{w}$.

Fig. 3. Fast MDS TEQ design algorithm.

Define the first-order error matrices

$$\widehat{\mathbf{E}}_1(\eta) = \text{diag}[(2\eta-1), (2\eta-3), \cdots, 3, 1, -1, -3, \cdots, (-2(L_c-\eta)-1)], \quad (24)$$

$$\mathbf{E}_1(\eta) = \mathbf{H}^H\ \widehat{\mathbf{E}}_1(\eta)\ \mathbf{H}. \quad (25)$$

The difference of two monic quadratic polynomials is a linear polynomial, so we have

$$\mathbf{Q}(\eta+1) = \mathbf{Q}(\eta) + \widehat{\mathbf{E}}_1(\eta+1), \quad (26)$$

$$\mathbf{A}(\eta+1) = \mathbf{A}(\eta) + \mathbf{E}_1(\eta+1), \quad (27)$$

where (27) follows from (26) by left- and right-multiplying by $\mathbf{H}^H$ and $\mathbf{H}$, respectively. As with the Min-IBI design, we would like to effciently update $\mathbf{E}_1$ and use it to update $\mathbf{A}$. However, $\mathbf{E}_1$ is now linear rather than peicewise constant. The procedure can be iterated by implicitly defining the second-order error matrices $\widehat{\mathbf{E}}_2$ and $\mathbf{E}_2$ such that we will have

$$\widehat{\mathbf{E}}_1(\eta+1) = \widehat{\mathbf{E}}_1(\eta) + \widehat{\mathbf{E}}_2(\eta+1), \quad (28)$$

$$\mathbf{E}_1(\eta+1) = \mathbf{E}_1(\eta) + \mathbf{E}_2(\eta+1). \quad (29)$$

Inspection of (24) reveals that $\widehat{\mathbf{E}}_2 = 2\mathbf{I}_{L_c+1}$ for all η, and thus

$$\mathbf{E}_2 = \mathbf{H}^H\,(2\mathbf{I}_{L_c+1})\,\mathbf{H} = 2\mathbf{B}. \quad (30)$$

Note that $\mathbf{B}$ has already been computed, and multiplication by two is simply a shift in binary representation. In summary, (29) is used to update $\mathbf{E}_1$, then (27) is used to update $\mathbf{A}$. This procedure only requires $(L_w+1)^2$ extra memory locations and $(L_w+1)^2$ extra additions, with the savings of not having to recompute $\mathbf{A}$ at all. For comparison, computing $\mathbf{A}$ normally takes $\mathcal{O}\left(L_w^2(L_h-\nu)\right)$ multiply-adds for *each* of the L_c+1 possible values of η. An outline of an efficient Min-IBI algorithm is given in Fig. 3.

The MDS penalty function leading to (23) is $f(n) = n^2$. Tkacenko and Vaidyanathan [13] also considered a linear MDS penalty function, $f(n) = |n|$. This leads to the Min-IBI design of Section III with $\nu = 0$. In Section VI we will refer to the original MDS design with a quadratic penalty function as MDS-Q, and the alternate MDS design with a linear penalty function as MDS-L.

V. Arbitrary polynomials

To generalize to arbitrary polynomial weighting functions, let the diagonal elements of $\mathbf{Q}$ be defined by an M^{th} order polynomial. The coefficients may be defined peice-wise over the regions $[0, \Delta-1]$, $[\Delta, \Delta+\nu]$, and $[\Delta+\nu+1, L_c]$, or a single polynomial may be used. For example, the MSSNR TEQ uses three zeroth order (constant) polynomials, the Min-IBI TEQ uses three first order (linear) polynomials, and the MDS design uses a single second order (quadratic) polynomial.

To generalize (15), (27), and (29), up to M error matrices $\mathbf{E}_m$, $1 \leq m \leq M$, may be needed for efficient updating of the $\mathbf{A}$ matrix. Each error matrix will be of size $(L_w+1)\times(L_w+1)$, but it may turn out that some error matrices are independent of the delay and/or already computed, as with $\mathbf{E}_2$ in the MDS technique described in Section IV. Even when an error matrix has not already been computed, it may be obtained almost entirely from the error matrix for the previous delay, as in (18). Moreover, since $\mathbf{A}$ is Hermitian, so is each $\mathbf{E}_m$, and only half of the coefficients must be stored. Thus, the additional memory requirements are at most $\frac{M}{2}(L_w+1)(L_w+2)$ storage words. The computational savings of not recomputing $\mathbf{A}$ are $\mathcal{O}\left(L_w^2(L_h-\nu)\right)$ multiplications saved per delay for up to L_c+1 delays, which more than balances out the extra memory use for reasonably small values of M.

VI. Simulations

This section applies various channel shortening designs to Rayleigh fading channels, and evaluates the resulting bit error rates for a multicarrier system. The designs that will be compared are the Min-IBI design [2], the MDS design with quadratic [3] and linear [13] weights, the MSSNR design [4], and the MMSE design [5]. The MDS and Min-IBI designs use $\alpha = \frac{1}{2}$ in (2), thus penalizing undesired channel taps and noise gain equally.

The parameters were chosen similar to the IEEE 802.11a wireless LAN standard [14]: an FFT size of $N = 64$, a CP of length $\nu = 16$, and QPSK modulation on each tone. The channel shorteners will each have 48 taps, full knowledge is assumed regarding the channel and noise statistics, and all TEQs will be designed using a full search over the delay Δ or the centroid η.

The channel model consists of three parts [15]: $\mathbf{h}_{local,1}$, scatterers near the transmitter; $\mathbf{h}_{mid}$, remote scatterers; and $\mathbf{h}_{local,2}$, scatterers near the receiver. The channel is then

$$\mathbf{h} = \mathbf{h}_{local,1} \star \mathbf{h}_{mid} \star \mathbf{h}_{local,2}, \qquad (31)$$

where $\star$ denotes convolution. For a wireless LAN system, $\mathbf{h}_{local,1}$ and $\mathbf{h}_{local,2}$ can be thought of as the "room response" at the transmitter and the reciever, and $\mathbf{h}_{mid}$ can be thought of as multipath between the two rooms. $\mathbf{h}_{mid}$ consists of 32 uncorrelated Rayleigh fading taps with an exponential delay profile, and $\mathbf{h}_{local,1}$ and $\mathbf{h}_{local,2}$ each consist of 6 uncorrelated Rayleigh fading taps with a uniform delay profile. An example of such a channel is depicted in Fig. 4.

Fig. 5 shows BER curves for systems employing the five TEQ designs as well as a system without a TEQ. The data point for each SNR value was obtained by averaging over all carriers for 100 blocks, and then repeating for a total of 100 channel realizations. Surprisingly, none of the TEQ designs improves the BER for SNR values below 25 dB. The MDS-Q design actually degrades the BER values and the MDS-L design hardly changes it. This is not unexpected, since neither MDS design takes the length of the CP into account. The Min-IBI design performs well, but not quite as well as the MMSE and MSSNR designs. This is unexpected, since the Min-IBI design maximizes the total time-domain SINR. This suggests that the *frequency-domain* BER values are not directly related to the total *time-domain* SINR.

In contrast, in an ADSL system, for which the throughput (bit rate) is maximized for a fixed BER, the MDS design frequently out-performs the MMSE and MSSNR designs [16]. Similarly, the Min-IBI design has been shown to perform well in an ADSL system [2]. We wish to test whether it is the

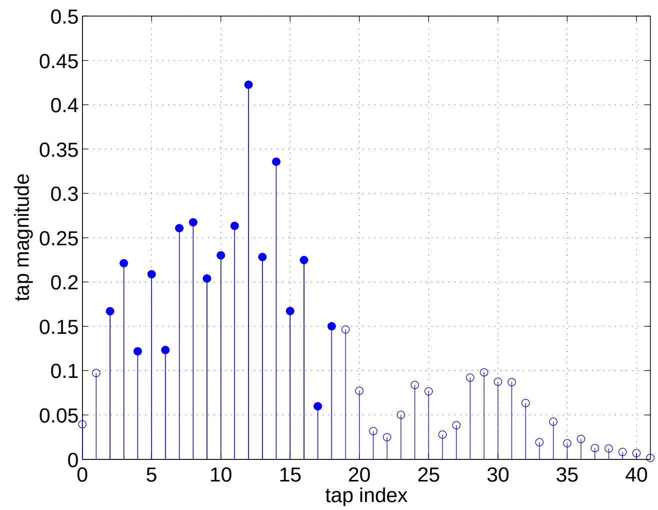

Fig. 4. An example of a random channel as in (31). The filled stems indicate the window of $\nu+1$ consecutive taps with the largest energy. Taps outside this window lead to inter-symbol and inter-carrier interference.

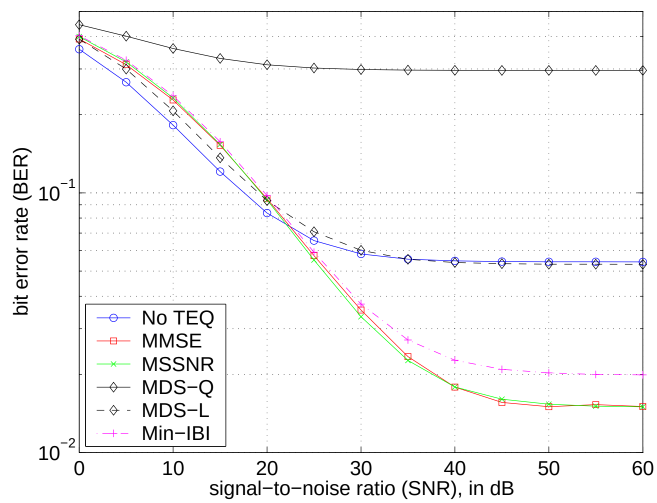

Fig. 5. BER versus SNR for a wireless channel model.

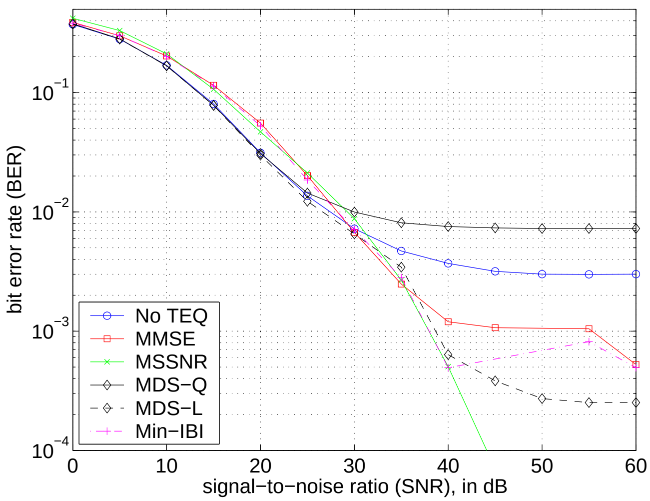

Fig. 6. BER versus SNR for wireline channel models.

channel models (models of telephone lines vs. Rayleigh fading channels) or the performance metric (bit rate vs. BER) that causes the MDS design to perform better in an ADSL system. Fig. 6 shows BER curves for the 8 synthetic carrier serving area (CSA) channels commonly used in ADSL simulations [6], which have a small number of zeros and one or two poles. That is, the performance metric (BER) was retained but the channel models are now wireline rather than wireless. The CSA channels were downsampled by 4 to yield channels about 50 taps long, the CP length was reduced from the downstream ADSL value of 32 to 8, and the TEQ length was 8 taps. The FFT size was 64. The data was averaged over 350,000 blocks per channel and then averaged over the 8 channels. For these parameters and channels, the MDS-L design is competitive with the other designs, and the MDS-Q design is still poor but not so astoundingly poor. These results suggest that the discrepancy between the wireline results and the wireless results are in part due to the channel model and in part due to the performance metric.

VII. Conclusions

Channel shortening designs with polynomial weighting functions (e.g. the Min-IBI and MDS designs) require expensive matrix computations. This paper developed efficient recursive techniques for computing these matrices. This greatly reduced the complexity at the cost of at most $\mathcal{O}\left(\frac{M}{2}L_w^2\right)$ extra memory words, where M is the polynomial order and L_w is the order of the channel shortening filter. The analysis was performed for the specific cases of Min-IBI and MDS designs, then generalized.

The Min-IBI and MDS designs were compared in terms of bit error rate to other channel shortening designs, in the context of a wireless multicarrier system. The results suggest that the MDS design is not well-suited to channel shortening for wireless systems, although it has been shown to perform well in an ADSL system. The Matlab code for producing the figures in this paper is available at [17].

References

[1] N. Al-Dhahir, "FIR Channel-Shortening Equalizers for MIMO ISI Channels," *IEEE Trans. on Comm.*, vol. 49, no. 2, pp. 213–218, Feb. 2001.

[2] S. Celebi, "Interblock Interference (IBI) Minimizing Time-Domain Equalizer (TEQ) for OFDM," *IEEE Signal Processing Letters*, vol. 10, no. 8, pp. 232–234, Aug. 2003.

[3] R. Schur and J. Speidel, "An Efficient Equalization Method to Minimize Delay Spread in OFDM/DMT Systems," in *Proc. IEEE Int. Conf. on Comm.*, Helsinki, Finland, June 2001, vol. 5, pp. 1481–1485.

[4] P. J. W. Melsa, R. C. Younce, and C. E. Rohrs, "Impulse Response Shortening for Discrete Multitone Transceivers," *IEEE Trans. on Comm.*, vol. 44, pp. 1662–1672, Dec. 1996.

[5] N. Al-Dhahir and J. M. Cioffi, "Efficiently Computed Reduced-Parameter Input-Aided MMSE Equalizers for ML Detection: A Unified Approach," *IEEE Trans. on Info. Theory*, vol. 42, no. 3, pp. 903–915, May 1996.

[6] G. Arslan, B. L. Evans, and S. Kiaei, "Equalization for Discrete Multitone Receivers To Maximize Bit Rate," *IEEE Trans. on Signal Processing*, vol. 49, no. 12, pp. 3123–3135, Dec. 2001.

[7] A. Tkacenko and P. P. Vaidyanathan, "A Low-Complexity Eigenfilter Design Method for Channel Shortening Equalizers for DMT Systems," *IEEE Trans. on Comm.*, vol. 51, no. 7, July 2003.

[8] K. Vanbleu, G. Ysebaert, G. Cuypers, M. Moonen, and K. Van Acker, "Bitrate Maximizing Time-Domain Equalizer Design for DMT-based Systems," to appear in *IEEE Trans. on Comm.*, 2004.

[9] R. K. Martin, M. Ding, B. L. Evans, and C. R. Johnson, Jr., "Efficient Channel Shortening Equalizer Design," *EURASIP Journal on Applied Signal Processing*, vol. 2003, no. 13, pp. 1279–1290, Dec. 2003.

[10] S. Celebi, "Interblock Interference (IBI) and Time of Reference (TOR) Computation in OFDM Systems," *IEEE Trans. on Comm.*, vol. 49, no. 11, pp. 1895–1900, Nov. 2001.

[11] G. H. Golub and C. F. Van Loan, *Matrix Computations*, The Johns Hopkins University Press, Baltimore, MD, 1996.

[12] J. Wu, G. Arslan, and B. L. Evans, "Efficient Matrix Multiplication Methods to Implement a Near-Optimum Channel Shortening Method for Discrete Multitone Transceivers," in *Proc. IEEE Asilomar Conf. on Signals, Systems, and Computers*, Pacific Grove, CA, Nov. 2000, vol. 1, pp. 152–157.

[13] A. Tkacenko and P. P. Vaidyanathan, "Noise Optimized Eigenfilter Design of Time-domain Equalizers for DMT Systems," in *Proc. IEEE Int. Conf. on Comm.*, New York, NY, Apr.–May 2002, vol. 1.

[14] The Inst. of Electrical and Electronics Engineers, "Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications. IEEE Std. 802.11a," 1999 Edition.

[15] A. J. Paulraj and B. C. Ng, "Space-time Modems for Wireless Personal Communications," *IEEE Personal Communications*, vol. 5, pp. 36–48, Feb. 1998.

[16] R. K. Martin, K. Vanbleu, M. Ding, G. Ysebaert, M. Milosevic, B. L. Evans, M. Moonen, and C. R. Johnson, Jr., "Multicarrier Equalization: Unification and Evaluation. Parts I and II," Submitted to *IEEE Transactions on Signal Processing*, Dec. 14, 2003.

[17] R. K. Martin, "Matlab code for papers by R. K. Martin," [Online]. Available: http://bard.ece.cornell.edu/matlab/martin/index.html.