

Sliding mode synchronization controller design with neural network for uncertain chaotic systems

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Abstract

A sliding mode synchronization controller is presented with RBF neural network for two chaotic systems in this paper. The compound disturbance of the synchronization error system consists of nonlinear uncertainties and exterior disturbances of chaotic systems. Based on RBF neural networks, a compound disturbance observer is proposed and the update law of parameters is given to monitor the compound disturbance. The synchronization controller is given based on the output of the compound disturbance observer. The designed controller can make the synchronization error convergent to zero and overcome the disruption of the uncertainty and the exterior disturbance of the system. Finally, an example is given to demonstrate the availability of the proposed synchronization control method.

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1. Introduction

Many research results have been reported recently about chaos synchronization control [1–6]. Such types of control methods have wide applicability in many practical systems such as secure communication, chemical reactions, biological systems and information processing. Ref. [1] studied synchronization of uncertain chaotic systems with parameter perturbation via the active control. A new adaptive variable structure control method was proposed for chaotic synchronization and applied it to secure communication in [2]. In Ref. [3], an adaptive sliding mode controller was provided for chaos synchronization with uncertainties. Refs. [4,5] studied synchronization of uncertain chaotic systems via backstepping approach. A synchronous control technique was developed based on RBF neural network in [6].

Sliding mode control (SMC), based on the theory of variable structure systems (VSS), has been widely applied to robust control of nonlinear systems due to many attractive advantages, e.g., fast response, good transient performance and insensitivity to variation in plant parameters, external disturbances and unmodeled dynamics. In [7], A stable adaptive fuzzy sliding-mode controller was developed for nonlinear multivariable systems with unavailable states. A practical design was proposed in [8], which combined a fuzzy adaptation technique with sliding mode control to enhance robustness and sliding performance for a class of uncertain MIMO nonlinear systems. In chaos synchronization control area, many chaotic systems are inevitably affected by unknown parameter variations and external disturbances, so the sliding mode in variable structure control is widely used in synchronization controller design. In [9], an active sliding

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mode control method for synchronizing two chaotic systems with parametric uncertainty was studied. Ref. [10] considered the robust adaptive synchronization problem of Rossler and Chen chaotic systems with different time varying unknown parameters. Sliding mode observer based fault diagnosis for nonlinear systems was investigated in [11].

Note that nonlinearity, disturbance, modeling error and inner uncertainty naturally exist in many control systems. Control design has to be considered for uncertain nonlinear systems with different disturbances. However, most existing methods restrict the disturbance in some particular forms (e.g., disturbance has a certain bound or a certain expression), which limit the application of these control methods. To overcome these disadvantages, the disturbance observer design was developed to estimate the uncertain disturbance of the system [12–15].

In this paper, a disturbance observer is proposed to monitor the system disturbance. Then a sliding mode synchronization controller is designed based on the output of the disturbance observer to make two chaotic systems synchronous. The rest of this paper is organized as follows. Section 2 gives the problem formulation for nonlinear chaos synchronization. Section 3 proposes a disturbance observer using RBF neural network. The chaos synchronization control scheme based on disturbance observer is discussed in Section 4. The simulation results are given in Section 5, followed by some concluding remarks in Section 6.

2. Problem formulation

Consider the chaotic systems in the form of

$$\dot{x} = (A + \Delta A)x + f(x) + \Delta f(x) + u + d(t) \quad (1)$$

where $x \in R^n$ is the state vector, $A \in R^{n \times n}$ is a certain constant matrix, $f(x)$ is a smooth nonlinear vector function, and u is the control input. $\Delta A, \Delta f(x)$ are the linear and nonlinear uncertain terms, and $d(t)$ is the external disturbance vector with unknown upper bound.

Definition. Define an expectation chaotic system

$$\dot{x}_m = g(x_m) \quad (2)$$

where $x_m \in R^n$ is the state vector of expectation system, $g(x_m) \in R^n$ is a smooth nonlinear vector function which can be equal to or different from $f(x)$.

We define $e = x - x_m$. If

$$\lim_{t \rightarrow \infty} e(t) = 0 \quad (3)$$

then the chaotic system (1) and the chaotic system (2) are synchronous.

From (1) and (2), differentiating $e = x - x_m$ yields

$$\begin{aligned} \dot{e} &= \dot{x} - \dot{x}_m = (A + \Delta A)x + f(x) + \Delta f(x) + u + d(t) - g(x_m) \\ &= A_m e + (A + \Delta A - A_m)x + f(x) + \Delta f(x) + d(t) + u - g(x_m) + A_m x_m \end{aligned} \quad (4)$$

where A_m is the beforehand given matrix whose eigenvalues lie in the left plane. Define

$$F(x) = (A + \Delta A - A_m)x + f(x) + \Delta f(x) + d(t) - g(x_m) + A_m x_m \quad (5)$$

then Eq. (4) can be rewritten as

$$\dot{e} = A_m e + F(x) + u \quad (6)$$

where the unknown term $F(x)$ can be treated as the compound disturbance of the system (6). In the following sections, we will propose a disturbance observer to approximate $F(x)$, and then design a sliding mode synchronization controller to make the chaotic systems (1) and (2) synchronous based on the output of the designed disturbance observer.

3. Design of disturbance observer with RBF neural network

To proceed with the design of synchronization controller, the following lemma and assumption are required.

Lemma. Assuming that X and Y are vectors or matrices with corresponding dimensions, by choosing a constant $\alpha > 0$, we can obtain the following equation:

$$X^T Y + Y^T X \leq \alpha X^T X + \alpha^{-1} Y^T Y$$

Assumption 1. Let $\mathbf{x} \in M_{\mathbf{x}}$, where $M_{\mathbf{x}}$ is a compact set of system states. The optimal weight \mathbf{w}^* of the neural network is defined as

$$\mathbf{w}^* = \arg \min_{\mathbf{w} \in \Omega} \left[\sup_{\mathbf{x} \in M_{\mathbf{x}}} |F - \hat{F}(\mathbf{x}, \hat{\mathbf{w}})| \right] \tag{7}$$

$$\Omega = \{\mathbf{w} : \|\mathbf{w}\| \leq M\} \tag{8}$$

where M is the design parameter, Ω is the valid field of weight parameters, $\hat{\mathbf{w}}$ is the weight of the neural network, and $\hat{F}(\mathbf{x}, \hat{\mathbf{w}})$ is the output of the neural network. In this paper, the nonlinear compound disturbance observer is designed using the RBF neural network which can monitor and represent the disturbance by tuning the design parameter of the RBF neural network.

Assuming that $\gamma_i > 0$ is the learn rate of the RBF neural network, $\sigma > 0$ is the design parameter, $\hat{\mathbf{w}}_i$ is the weight vector of the RBF neural network, $\Phi(\mathbf{x})$ is the base function of the RBF neural network, $\mathbf{w} = [\mathbf{w}_1, \dots, \mathbf{w}_n]^T \in R^{n \times n}$, $\Phi = [\Phi_1, \dots, \Phi_n]^T \in R^{n \times 1}$, $\Phi_i(x) = \exp(-\|x - c_i\|^2 / \delta_i^2)$, c_i and δ_i are the center and width of the neural cell in the i th hidden layer, and $\varsigma = [\varsigma_1, \dots, \varsigma_n]^T \in R^{n \times 1}$, then there yields the following theorem:

Theorem 1. Consider the following dynamic system:

$$\dot{\mathbf{z}} = -\sigma \mathbf{z} + \mathbf{p}(\mathbf{x}, \hat{\mathbf{w}}) \tag{9}$$

where $\mathbf{p}(\mathbf{x}, \hat{\mathbf{w}}) = \sigma \mathbf{e} + A_m \mathbf{e} + \mathbf{u} + \hat{F}(\mathbf{x}, \hat{\mathbf{w}})$ and define a new variable ‘disturbance observation error’ as $\varsigma = \mathbf{e} - \mathbf{z}$. If the adjustable neural network weight vector is tuned by

$$\dot{\hat{\mathbf{w}}}_i = \gamma_i \varsigma_i \Phi(\mathbf{x}) \tag{10}$$

then the disturbance observation error ς is uniformly ultimately bounded.

Proof. From the expression of $\mathbf{p}(\mathbf{x}, \hat{\mathbf{w}})$, the dynamic system (9) can be expressed as

$$\dot{\mathbf{z}} = \sigma(\mathbf{e} - \mathbf{z}) + A_m \mathbf{e} + \mathbf{u} + \hat{F}(\mathbf{x}, \hat{\mathbf{w}}) \tag{11}$$

Defining $\varsigma = \mathbf{e} - \mathbf{z}$, from (6) and (11), its derivative along the system trajectory is expressed as

$$\dot{\varsigma} = -\sigma \varsigma + F(\mathbf{x}) - \hat{F}(\mathbf{x}, \hat{\mathbf{w}}) \tag{12}$$

Using Assumption 1 and the universal approximation capability of the neural network, the compound disturbance $F(\mathbf{x})$ can be completely described by the output $\hat{F}(\mathbf{x}, \mathbf{w}^*)$ of a neural network plus a reconstruction error ε as follows:

$$\begin{aligned} F(\mathbf{x}) &= \hat{F}(\mathbf{x}, \mathbf{w}^*) + \varepsilon \\ |\varepsilon_i| &\leq \bar{\varepsilon}_i \end{aligned} \tag{13}$$

where $\varepsilon = [\varepsilon_1, \dots, \varepsilon_n]^T$ is the approximation error of RBF neural network, and $\bar{\varepsilon} = [\bar{\varepsilon}_1, \dots, \bar{\varepsilon}_n]^T$ is the approximation error upon bound of RBF neural network.

Substituting (13) into (12) yields

$$\dot{\varsigma} = -\sigma \varsigma + \hat{F}(\mathbf{x}, \mathbf{w}^*) + \varepsilon - \hat{F}(\mathbf{x}, \hat{\mathbf{w}}) \tag{14}$$

Furthermore, the output of the RBF neural network can also be described as

$$y_{nn} = \hat{\mathbf{w}}^T \Phi(\mathbf{x}) \tag{15}$$

Defining the weight vector error $\tilde{\mathbf{w}} = \mathbf{w}^* - \hat{\mathbf{w}}$ and substituting (15) into (14) yield

$$\dot{\varsigma} = -\sigma \varsigma + \tilde{\mathbf{w}}^T \Phi(\mathbf{x}) + \varepsilon \tag{16}$$

Consider the Lyapunov function as

$$V = \frac{1}{2} \varsigma^T \varsigma + \sum_{i=1}^n \frac{1}{2\gamma_i} \tilde{\mathbf{w}}_i^T \tilde{\mathbf{w}}_i \tag{17}$$

Differentiating (17) yields

$$\dot{V} = \zeta^T \dot{\zeta} + \sum_{i=1}^n \frac{1}{\gamma_i} \tilde{\mathbf{w}}_i^T \dot{\tilde{\mathbf{w}}}_i \tag{18}$$

Since $\dot{\tilde{\mathbf{w}}}_i = -\dot{\hat{\mathbf{w}}}_i$, (18) can be rewritten as

$$\dot{V} = \zeta^T \dot{\zeta} + \sum_{i=1}^n \frac{1}{\gamma_i} \tilde{\mathbf{w}}_i^T \dot{\tilde{\mathbf{w}}}_i = -\sigma \zeta^T \zeta + \zeta^T \tilde{\mathbf{w}}^T \Phi(\mathbf{x}) + \zeta^T \varepsilon - \sum_{i=1}^n \frac{1}{\gamma_i} \tilde{\mathbf{w}}_i^T \dot{\hat{\mathbf{w}}}_i \tag{19}$$

Note that $\zeta^T \tilde{\mathbf{w}}^T \Phi(\mathbf{x}) = \sum_{i=1}^n \frac{1}{\gamma_i} \tilde{\mathbf{w}}_i^T \gamma_i \zeta_i \Phi(\mathbf{x})$. Substituting (10) into (19) yields

$$\dot{V} = -\sigma \zeta^T \zeta + \zeta^T \varepsilon = -\sigma \zeta^T \zeta + \frac{1}{2} \zeta^T \varepsilon + \frac{1}{2} \varepsilon^T \zeta \tag{20}$$

Using the Lemma, we can obtain

$$\frac{1}{2} \zeta^T \varepsilon + \frac{1}{2} \varepsilon^T \zeta \leq \frac{\sigma}{2} \zeta^T \zeta + \frac{1}{2\sigma} \varepsilon^T \varepsilon \tag{21}$$

Substituting (21) into (20) yields

$$\dot{V} \leq -\sigma \zeta^T \zeta + \frac{\sigma}{2} \zeta^T \zeta + \frac{1}{2\sigma} \varepsilon^T \varepsilon \leq -\frac{\sigma}{2} \zeta^T \zeta + \frac{1}{2\sigma} \bar{\varepsilon}^T \bar{\varepsilon} \tag{22}$$

When $\zeta^T \zeta > (\frac{1}{\sigma} \bar{\varepsilon}^T \bar{\varepsilon})$, $\dot{V} < 0$. Thus the disturbance observation error ζ is uniformly ultimately bounded.

According to (12), we know that when ζ is uniformly ultimately bounded, then $F(\mathbf{x}) - \hat{F}(\mathbf{x}, \hat{\mathbf{w}})$ is uniformly ultimately bounded as well. So under the adaptive law of weight, the disturbance observer which is based on RBF neural network can approximate the compound disturbance [12].

4. Chaos sliding mode synchronization control based on disturbance observer

The task of this section is to design a synchronization controller which makes the systems (1) and (2) synchronous.

Theorem 2. Consider the following dynamic system:

$$\dot{z} = -\sigma z + p(\mathbf{x}, \hat{\mathbf{w}}) \tag{23}$$

where $p(\mathbf{x}, \hat{\mathbf{w}}) = \sigma e + A_m e + \mathbf{u} + \hat{F}(\mathbf{x}, \hat{\mathbf{w}})$ and define a new variable ‘disturbance observation error’ as $\zeta = \mathbf{e} - \mathbf{z}$. If the adjustable weight vector of the RBF neural network is tuned by

$$\dot{\hat{\mathbf{w}}}_i = \gamma_i e_i \Phi_i(\mathbf{x}) + \gamma_i \zeta_i \Phi_i(\mathbf{x}) \tag{24}$$

then the disturbance observation error ζ is uniformly ultimately bounded. If the synchronization controller is designed as follows:

$$\mathbf{u} = \mathbf{u}_1 + \mathbf{u}_2 \tag{25}$$

where

$$\begin{aligned} u_1 &= -\hat{F}(x) \\ u_2 &= -\text{sgn}(e)\bar{\varepsilon} \end{aligned}$$

then synchronization controller can make the system (1) and the system (2) synchronous.

Proof . Let the Lyapunov function be defined as

$$V_m = \frac{1}{2} e^T e + \sum_{i=1}^n \frac{1}{2\gamma_i} \tilde{\mathbf{w}}_i^T \tilde{\mathbf{w}}_i + \frac{1}{2} \zeta^T \zeta \tag{26}$$

From (6), (16) and (25), differentiating the above equation yields

$$\begin{aligned} \dot{V}_m &= e^T A_m e + e^T (F(x) - \hat{F}(x)) - e^T \text{sgn}(e)\bar{\varepsilon} + \sum_{i=1}^n \frac{1}{\gamma_i} \tilde{\mathbf{w}}_i^T \dot{\tilde{\mathbf{w}}}_i + \zeta^T \dot{\zeta} \\ &= e^T A_m e + e^T \tilde{\mathbf{w}}^T \Phi(x) + e^T \varepsilon - e^T \text{sgn}(e)\bar{\varepsilon} + \sum_{i=1}^n \frac{1}{\gamma_i} \tilde{\mathbf{w}}_i^T \dot{\tilde{\mathbf{w}}}_i - \sigma \zeta^T \zeta + \zeta^T \tilde{\mathbf{w}}^T \Phi(\mathbf{x}) + \zeta^T \varepsilon \end{aligned} \tag{27}$$

Note that $\dot{\hat{w}}_i = -\dot{\hat{w}}_i$, substituting (24) into (27) yields

$$\dot{V}_m = e^T A_m e + e^T \varepsilon - e^T \text{sgn}(e) \bar{\varepsilon} - \sigma \zeta^T \zeta + \zeta^T \varepsilon \tag{28}$$

It is clear that

$$e^T \varepsilon - e^T \bar{\varepsilon} \text{sgn}(e) = \sum_{i=1}^n e_i \varepsilon_i - \sum_{i=1}^n |e_i| \bar{\varepsilon}_i \leq \sum_{i=1}^n |e_i| |\varepsilon_i| - \sum_{i=1}^n |e_i| \bar{\varepsilon}_i \leq 0 \tag{29}$$

Eq. (28) can be written as

$$\dot{V}_m \leq e^T A_m e - \sigma \zeta^T \zeta + \zeta^T \varepsilon \tag{30}$$

According to (21), the inequality (30) can be rewritten as

$$\dot{V}_m \leq e^T A_m e - \frac{\sigma}{2} \zeta^T \zeta + \frac{1}{2\sigma} \bar{\varepsilon}^T \bar{\varepsilon} \tag{31}$$

When $\zeta^T \zeta > (\frac{1}{\sigma^2} \bar{\varepsilon}^T \bar{\varepsilon})$, then

$$\dot{V}_m < 0$$

Therefore, under the designed synchronization controller, the states of the system (6) are stable. Apparently, choosing appropriate design parameters γ_i, σ and $\bar{\varepsilon}$ can make e converge to zero, namely $x \rightarrow y$. So the system (1) and the system (2) are synchronous.

5. Simulation example

Consider the Lorenz systems in the form of [6]

$$\begin{aligned} \dot{x}_1 &= (a + \delta a)(x_2 - x_1) + d_1 + u_1 \\ \dot{x}_2 &= (b + \delta b)x_1 - x_2 - x_1 x_3 + d_2 + u_2 \\ \dot{x}_3 &= x_1 x_2 - (c + \delta c)x_3 + d_3 + u_3 \end{aligned}$$

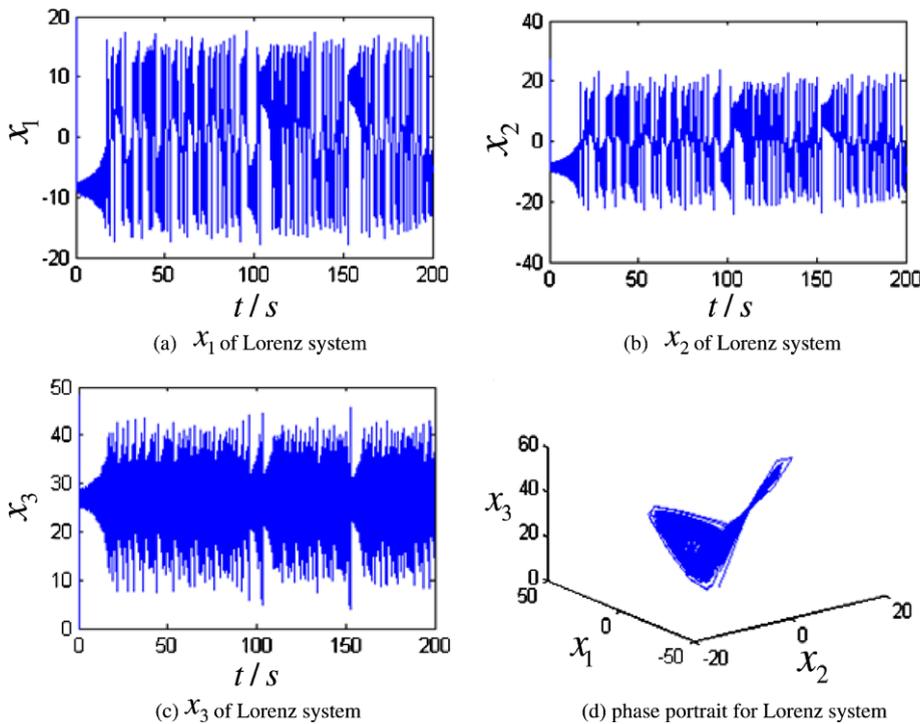


Fig. 1. The state and deformation of the attractor of the uncontrolled Lorenz systems with disturbance.

where x_1 is proportional to the intensity of the fluid motion, x_2 is proportional to the lateral temperature difference in the fluid, and x_3 is proportional to the vertical temperature difference in the fluid. a, b, c are Prandtl number, Rayleigh number and geometry number which is related to the size of the considered region, and all variables had been non-dimensionalized. $\delta a, \delta b, \delta c$ are the corresponding perturbations of the parameters a, b, c . d_1, d_2 and d_3 are the corresponding perturbations of the three states. u is the control input. Choosing $a = 10, b = 28, c = 8/3, \delta a = \delta b = 0.1, \delta c = 0.2, d_1 = 0.3 \sin x_2, d_2 = 0.1 \cos x_1, d_3 = 0.2 \sin(3x_2)$, the state $\mathbf{x} = [x_1, x_2, x_3]^T$ and deformation of the attractor of the uncontrolled Lorenz systems with disturbance are shown in Fig. 1.

Consider the Rössler systems in the form of

$$\begin{aligned} \dot{x}_{m_1} &= -(x_{m_2} + x_{m_3}) \\ \dot{x}_{m_2} &= x_{m_1} + 0.2x_{m_2} \\ \dot{x}_{m_3} &= 0.2 + x_{m_3}(x_{m_1} - 5.7) \end{aligned}$$

The state $\mathbf{x} = [x_{m_1}, x_{m_2}, x_{m_3}]^T$ and deformation of the attractor of the uncontrolled Rössler systems are shown in Fig. 2.

Now, the task is to design a synchronization controller which makes the above Lorenz systems with uncertainties and disturbance and the above Rössler systems synchronous. The synchronization controller is designed according to (25). Choosing $A_m = \text{diag}(-10, -10, -10), \sigma = 350, \gamma_i = 0.01, \bar{e}_i = 0.1$, the number of the hidden layer cell is eight. The synchronization errors under the designed controller are shown in Fig. 3.

From Fig. 3, we can see that the Lorenz systems with disturbance and the Rössler systems can become synchronous, which shows the effectiveness of the proposed control method.

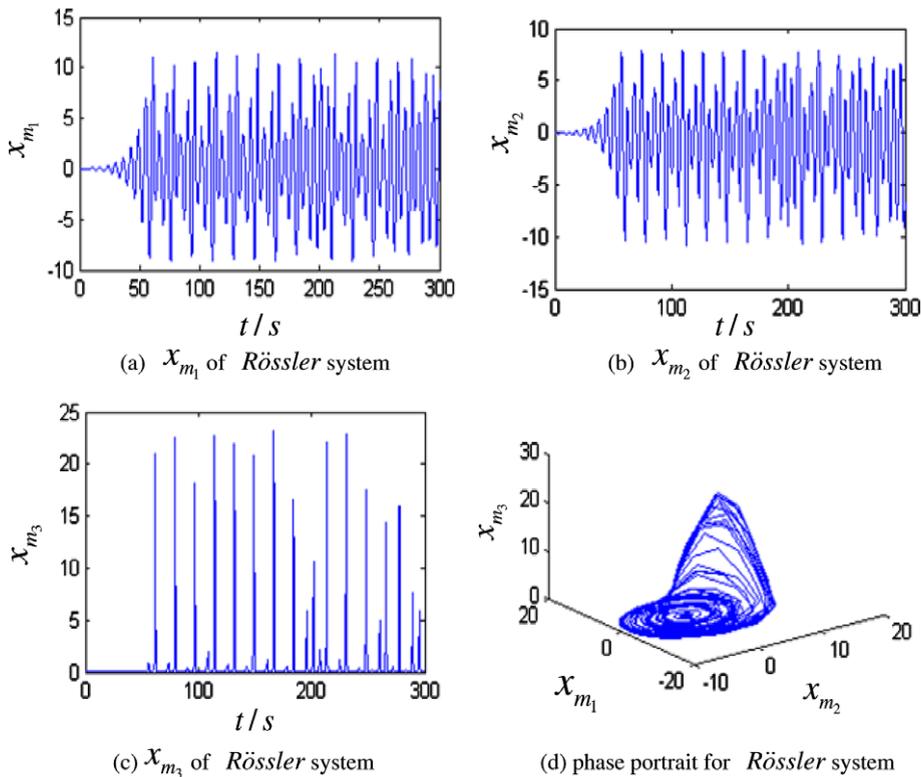


Fig. 2. The state and deformation of the attractor of the uncontrolled Rössler systems.

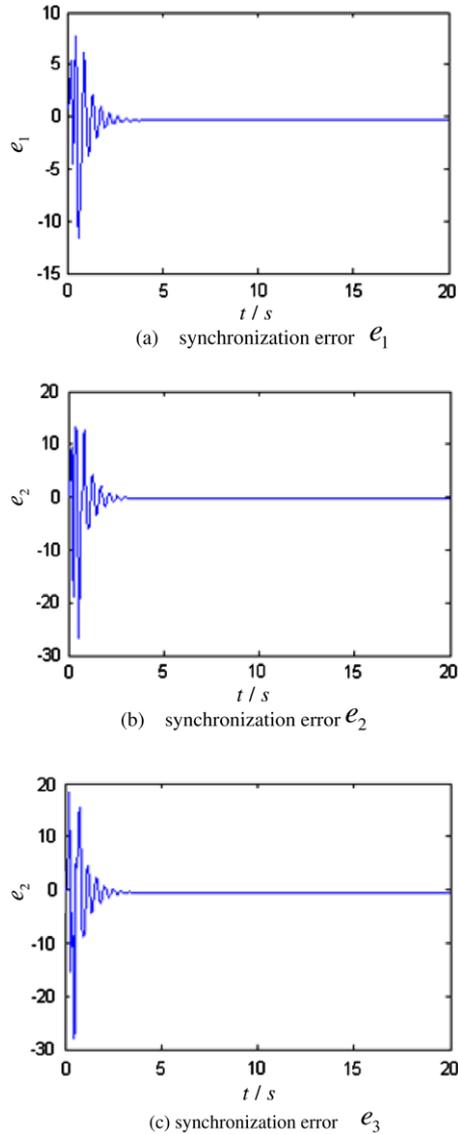


Fig. 3. the synchronization error of the Lorenz systems with disturbance and the *Rössler* systems.

6. Conclusion

This paper proposes a disturbance observer based on RBF neural network and applies it to design a sliding mode synchronization controller for a class of uncertain chaotic systems. Under the given parameter updating law, the error of the compound disturbance observer is bounded. The developed synchronization controller can make the synchronization error convergent to zero. The simulation results demonstrate the availability of the proposed synchronization control method. In our future work, the proposed synchronization method can also be used to control other chaotic systems. Further, we can introduce this proposed synchronization scheme to secure communication systems.

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