An Economic Dispatch Model Incorporating Wind Power

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***Abstract*—In solving the electrical power systems economic dispatch (ED) problem, the goal is to find the optimal allocation of output power among the various generators available to serve the system load. With the continuing search for alternatives to conventional energy sources, it is necessary to include wind energy conversion system (WECS) generators in the ED problem. This paper develops a model to include the WECS in the ED problem, and in addition to the classic economic dispatch factors, factors to account for both overestimation and underestimation of available wind power are included. With the stochastic wind speed characterization based on the Weibull probability density function, the optimization problem is numerically solved for a scenario involving two conventional and two wind-powered generators. Optimal solutions are presented for various values of the input parameters, and these solutions demonstrate that the allocation of system generation capacity may be influenced by multipliers related to the risk of overestimation and to the cost of underestimation of available wind power.**

***Index Terms*—Economic dispatch, penalty cost, reserve cost, Weibull probability density function, wind energy.**

I. Introduction

ECONOMIC dispatch (ED) deals with the minimum cost of power production in electrical power system analysis [1], [2]. More specifically, in solving the ED problem, one seeks to find the optimal allocation of the electrical power output from various available generators. Prior to the widespread use of alternate sources of energy, the ED problem involved only conventional thermal energy power generators, which use depletable resources such as fossil fuels. It has become apparent that there is a need for alternatives to thermal energy power generation, and one of the sources that is now seeing more widespread use, particularly outside of the United States, is the wind energy. One of the major benefits of wind energy is that, after the initial land and capital costs, there is essentially no cost involved in the production of power from wind energy conversion systems (WECS). In addition, the impacts of WECS are generally considered to be environmentally friendlier than the impacts of thermal energy sources.

The primary problem associated with the incorporation of wind power into the ED model is the fact that the future wind speed, which is the power source for the WECS, is an unknown at any given time. A similar comment might be made about the volatility of the prices of conventional energy sources, such as coal or oil, or about the future system load; however, even though these inputs, too, are unknowns, their variability is still much lower than that of the future wind speed. Several investigations have looked at the prediction of wind speed for use in determining the available wind power. These investigations have been based on such foundations as fuzzy logic [3], neural networks [4], and time series [5]. Because the focus of this paper is on the ED problem and not on wind power forecasting, fuzzy logic or similar theories to develop the wind speed profile will not be used, but a known probability distribution function (PDF) for the wind speed will be assumed, and then, transformed to the corresponding wind power distribution for use in the ED model.

With the more widespread use of the WECS, the power system operator now faces the problem of not only allocating system power among conventional generators, but also among various available wind-powered generators. The primary characteristic that differentiates wind-powered from conventional generators in the ED problem is the stochastic nature of wind speed. After an ED model that incorporates both thermal and wind energy sources is developed, it is still necessary to characterize the stochastic nature of the wind speed in order to analyze the problem with numerical results.

The objective of this paper is to incorporate wind-powered generators into the classical economic dispatch problem and to investigate the problem via numerical solutions. In Section II, the economic dispatch model, which is fundamentally a classic optimization problem, will be developed to include both conventional generators and wind-powered generators. Section III will discuss the characterization of wind speed as a random variable and will introduce the Weibull probability density function (pdf) as the basis for numerical solutions of the ED model. The WECS power input–output equation and the transformation from the wind speed random variable to the wind power random variable is presented in Section IV. In Section V, the numerical solution to the ED problem using the MATLAB is discussed. Section VI presents a discussion of the numerical results achieved when several different wind and cost scenarios are applied to the ED model. Finally, in Section VII, conclusions are drawn, based on the results found from the numerical analyses in Section VI.

II. Economic Dispatch Model Including the WECS

As applied to electrical power systems, the economic dispatch problem is a classic mathematical optimization problem. The goal is to obtain an optimum allocation of power output among the available generators with given constraints. The sum of the outputs from the available generators must equal the system load plus any system losses. In addition, certain constraints

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may be placed on the generators in the model. These constraints typically take the form of minimum and maximum generator outputs. The ED equations are valid for a given time period within which the generator outputs, loads, and losses are considered constant. From the point of view of the system operator, the economic dispatch problem may take different forms, depending on the extent of ownership by the system operator of the conventional and wind-powered generators.

If the wind-powered generators are owned by the system operator that is performing the economic dispatch, there is little or no incremental cost associated with the wind-powered generators. This incremental cost forms the basis for an economic dispatch; so effectively, the system operator will want to use all available wind energy. On the other hand, because of the uncertainty in the availability of wind energy at any time in the future, even if the system operator owns the wind-powered generators, the ED model must still provide some check on the overscheduling wind power, and this is the reason why some factor in the model must account for the reserve necessary in case the scheduled wind power is not available.

In [6], an economic dispatch model to include wind-powered generators is developed by using concepts from the fuzzy set theory. To the classic ED equations, the authors of [6] add a penalty cost factor for not using the available wind power capacity. A fuzzy wind border is defined, where wind values below a certain minimum are considered fully acceptable based on security concerns and wind values above a certain maximum are considered unacceptable for system security reasons. Values within these two limits form the fuzzy border, where the membership value goes from 1 for the minimum value to 0 for the maximum value.

In a manner somewhat similar to the fuzzy set theory approach, this paper will use probability functions to characterize the wind speed profiles, and an additional factor for overestimation of the available wind power will be added. In general, losses are ignored in the model; however, they could be added in the system load and losses term L, if necessary.

In this paper, the ED model will be developed in the most general case, so that it is adaptable to all situations, regardless of who owns the generation facilities. In the most general form, the system operator will have certain conventional generators and certain wind generators available. Because of the uncertainty of the wind energy available at any given time, factors for overestimation and underestimation of available wind energy must be included in the model. The factor for overestimation is easily explainable in that, if a certain amount of wind power is assumed and that power is not available at the assumed time, power must be purchased from an alternate source or loads must be shed. In the case of the underestimation penalty, if the available wind power is actually more than what was assumed, that power will be wasted, and it is reasonable for the system operator to pay a cost to the wind power producer for the waste of available capacity. The surplus wind power is usually sold to adjacent utilities, or by fast redispatch and automatic gain control (AGC), the output of nonwind generators is correspondingly reduced. Only if this cannot be achieved, then dummy load resistors need to be connected to "waste" the surplus energy. Obviously, these practicalities can be modeled by a simple underestimation penalty cost function.

Putting the aforementioned discussion in the format of an optimization problem, the mathematical model directly follows. This model is valid in any given ED time period; however, to lessen confusion at this point, the time dependence of the equations is suppressed.

Minimize

$$\sum_{i}^{M} C_i(p_i) + \sum_{i}^{N} C_{\mathrm{wj}}(w_i) + \sum_{i}^{N} C_{p,\mathrm{wj}}(W_{i,\mathrm{av}} - w_i) + \sum_{i}^{N} C_{r,w,i}(w_i - W_{i,\mathrm{av}}) \quad (1)$$

subject to

$$p_{i,\min} \le p_i \le p_{i,\max} \quad (2)$$

$$0 \le w_i \le w_{r,i} \quad (3)$$

$$\sum_{i}^{M} p_i + \sum_{i}^{N} w_i = L \quad (4)$$

where

M number of conventional power generators;

N number of wind-powered generators;

p_i power from the ith conventional generator;

w_i scheduled wind power from the ith wind-powered generator;

$W_{i,\mathrm{av}}$ available wind power from the ith wind-powered generator. This is a random variable, with a value range of $0 \le W_{i,\mathrm{av}} \le w_r$ and probabilities varying with the given pdf. We considered Weibull pdf for wind variation;

$w_{r,i}$ rated wind power from the ith wind-powered generator;

C_i cost function for the ith conventional generator;

$C_{w,i}$ cost function for the ith wind-powered generator. This factor will typically take the form of a payment to the wind farm operator for the wind-generated power actually used;

$C_{p,w,i}$ penalty cost function for not using all available power from the ith wind-powered generator;

$C_{r,w,i}$ required reserve cost function, relating to uncertainty of wind power. This is effectively a penalty associated with the overestimation of the available wind power;

L system load and losses.

Taking a closer look at the objective function (1), the first term is the traditional sum of the fuel costs of the conventional generators. The second term is the direct cost for the power derived from the wind-powered generators. The existence and size of this term will depend on the ownership of the wind-powered generators. If the generators are owned by the system operator, this term may not even exist if it accounts only for the incremental fuel cost, which is zero for the wind; however, if the system operator is paying for the wind power from the owner of the wind farm, a direct cost will be involved. The third term, which will be explained in more detail next, accounts for not using all the available wind power. As with the previous term, the costs associated with this term will depend on who owns the wind-powered generators. Finally, the fourth term in the objective function relates to the price that must be paid for overestimation of the available wind power. Without regard to ownership of the wind-powered generators, the ED model must account for the possibility that a reserve would need to be drawn on if all the available wind power is inadequate to cover the amount of the wind power scheduled in a given time period.

For the conventional generators, a quadratic cost function will be assumed, which is practical for most of the cases, and is given by

$$C_i(p_i) = \frac{a_i}{2} p_i^2 + b_i p_i + c_i \tag{5}$$

where a_i, b_i, and c_i are cost coefficients for the *i*th conventional energy source, which are found from the input–output curves of the generators and are dependent on the particular type of fuel used [1], [2].

In the case where the WECS is owned by the system operator, this function may not exist as the power requires no fuel, unless the operator wants to assign some payback cost to the initial outlay for the WECS or unless the system operator wants to assign this as a maintenance and renewal cost. But in a nonutility-owned WECS, the wind generation will have a cost that must be based on the special contractual agreements. The output of the wind generator is constrained by an upper and lower limit, decided by the system operator based on the agreements for the optimal operation of the system [7]. For simplicity, this can be considered to be proportional to the scheduled wind power or totally neglected. We neglected this in all our studies for the system-operator-owned WECS, and considered to be proportional to the scheduled wind power for the nonutility-owned WECS. The ED model is thus developed in the most general sense to make it adaptable to all situations, regardless of who owns the generation facilities.

A linear cost function will be assumed for the wind-generated power actually used as

$$C_{w,i}(w_i) = d_i w_i \tag{6}$$

where d_i is the direct cost coefficient for the *i*th wind generator.

It will be assumed that the penalty cost for not using all the available wind power will be linearly related to the difference between the available wind power and the actual wind power used. The penalty cost function will then take the following form

$$\begin{aligned} C_{p,w,i}(W_{i,\mathrm{av}} - w_i) &= k_{p,i}(W_{i,\mathrm{av}} - w_i) \\ &= k_{p,i} \int_{w_i}^{w_{r,i}} (w - w_i) f_W(w) dw \end{aligned} \tag{7}$$

where

$k_{p,i}$ penalty cost (underestimation) coefficient for the *i*th wind generator;

$f_W(w)$ WECS wind power pdf.

As with the direct cost, if the system operator owns the wind-powered generators, the penalty cost may not exist.

The reserve requirement cost will be similar to the penalty cost (7), in that it is an integral over the pdf of the wind power random variable, except that, in this case, it is a cost due to the available wind power being less than the scheduled wind power. Both (7) and (8) can be modeled in the MATLAB using the built-in "quad" function

$$\begin{aligned} C_{r,w,i}(w_i - W_{i,\mathrm{av}}) &= k_{r,i}(w_i - W_{i,\mathrm{av}}) \\ &= k_{r,i} \int_{o}^{w_i} (w_i - w) f_W(w) dw \end{aligned} \tag{8}$$

where $k_{p,i}$ is the reserve cost (overestimation) coefficient for the *i*th wind-powered generator.

To avoid unnecessary complexity in the model, it is assumed that the difference between the available wind power and the scheduled wind power, multiplied by the wind power output probability function is linearly related to the reserve cost.

To obtain a numerical value for the reserve and penalty costs, it is necessary to find or assume the pdf for the wind power output. In general, of course, the wind speed is an unknown at any future time; however, in order to obtain some quantitative results, some known probability function for the wind speed will be assumed. This leads to the next section.

III. Wind Speed Characterization

In order to be able to rationally approach the economic dispatch with WECS problem, some characterization of the uncertain nature of the wind speed is needed. Prior research [8] has shown that the wind speed profile at a given location most closely follow a Weibull distribution over time. The pdf for a Weibull distribution is given by

$$f_v(v) = \left(\frac{k}{c}\right)\left(\frac{v}{c}\right)^{(k-1)} (e)^{-(vc)^k}, \quad 0 < v < \infty \tag{9}$$

where

V wind speed random variable;
v wind speed;
c scale factor at a given location (units of wind speed);
k shape factor at a given location (dimensionless).

For later use in conjunction with the wind power probability function, the Weibull PDF is given by

$$F_V(v) = \int_0^v f_V(\tau)\, d\tau = 1 - e^{-(v/c)^k}. \tag{10}$$

The Weibull distribution function with a shape factor of 2 is also known as the Rayleigh distribution. In [9], the advantages of the Weibull distribution are noted as follows: 1) it is a two-parameter distribution, which is more general than the single-parameter Rayleigh distribution, but less complicated than the five-parameter bivariate normal distribution; 2) it has been previously shown to provide a good fit to observed wind speed data; and 3) if the *k* and *c* parameters are known at one height, a methodology exists to find the corresponding parameters at another height. The characteristics of the wind depend on various factors like geography, topography, etc., and can be estimated by the observed frequency of wind speed in the target region.

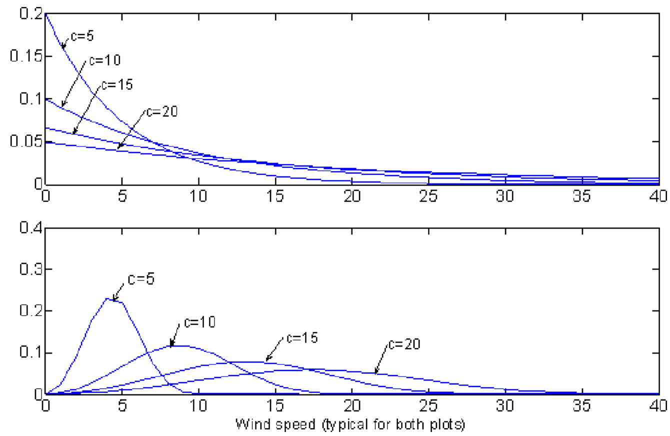

Fig. 1. Weibull probability density functions for k factors of 1 (top) and 3 (bottom), each with c factors of 5, 10, 15, and 20.

Methods of estimating the Weibull shape and scale factors using the available wind speed data are given in [9] and [10]. The shape parameter varied from 1.0 to 3.0 and the scale parameter ranged from 5 to 20 in [8], which are used in this paper for analysis.

It will be shown later that, due to the characteristics of WECS generators, the continuous wind speed distributions will become mixed discrete and continuous distributions in the transformation to wind power distributions.

Fig. 1 shows the Weibull pdf with shape factors of 1and 3. Within each of these plots, curves of scale factor 5, 10, 15, and 20 are indicated.

Before moving on to look at another potential wind speed probability distribution to be used with the ED model, some comments on Fig. 1 may be made. The mean of the Weibull function is

$$\mu = c\Gamma(1 + k^{-1})$$

and the variance is

$$\sigma_v^2 = c^2\Gamma(1 + 2k^{-1}) - \mu^2$$

and where the gamma function is

$$\Gamma(x) = \int_0^\infty y^{x-1} e^{-y} dy, \quad y > 0.$$

For the Rayleigh distribution, $k = 2$ and

$$\mu = \frac{\sqrt{\pi}}{2} \quad \text{and} \quad \sigma_v^2 = C^2(1 - \frac{\pi}{4}).$$

From the aforementioned, it is seen that, as the c factor of the Weibull function increases, the mean and standard deviation also increase in a linear relationship. The topic of wind speed prediction is outside the scope of this paper; however, in the analysis section of this paper, some assumptions about possible outcomes of prediction algorithms will be made, and it will be seen how the potential outcomes will affect the allocation of system generation capacity among available generators.

IV. WECS Input/Output and Probability Functions

Once the uncertain nature of the wind is characterized as a random variable, the output power of the WECS may also be characterized as a random variable through a transformation from wind speed to output power. Ignoring minor nonlinearities, the output of the WECS with a given wind speed input may be stated as [11]

$$w = 0, \quad \text{for} \quad v < v_i \quad \text{and} \quad v > v_o \tag{11}$$

$$w = w_r \frac{(v - v_i)}{(v_r - v_i)}, \quad \text{for} \quad v_i \le v \le v_r \tag{12}$$

$$w = w_r, \quad \text{for} \quad v_r \le v \le v_o \tag{13}$$

where

- w WECS output power (typical units of kilowatt or megawatt);
- w_r WECS rated power;
- v_i cut-in wind speed (typical units of miles/hour or miles/second);
- v_r rated wind speed;
- v_o cut-out wind speed.

Thus, it is seen that the WECS has: 1) no power output up to cut-in wind speed (11); 2) a linear power output relationship between cut-in and rated wind speed (12); 3) a constant rated power output between the rated wind speed and cut-out wind speed (13); and 4) once again has no power output with wind speeds greater than the cut-out wind speed (11).

Due to the fact that the WECS power output has a constant zero value below the cut-in wind speed and also above the cut-out wind speed, and due to the fact that the power output is constant between rated wind speed and cut-out wind speed, the power output random variable will be discrete in these ranges of wind speed. The WECS power output is a mixed random variable, which is continuous between values of zero and rated power, and is discrete at values of zero and rated power output.

If it is assumed that the wind speed has a given distribution, such as the Weibull, it is then necessary to convert that distribution to a wind power distribution. This transformation may be accomplished in the following manner, with V as the wind speed random variable and W as the wind power random variable. For a linear transformation, in general [12], such as that described in (12)

$$W = T(V) = aV + b \tag{14}$$

and

$$f_W(w) = f_V[T^{-1}(w)] \left[\frac{dT^{-1}(w)}{dw}\right] = f_V\left(\frac{w-b}{a}\right)\left|\frac{1}{a}\right| \tag{15}$$

where

- T a transformation, in general;
- W wind power random variable;
- V wind speed random variable;
- w wind power (a realization of the wind power random variable);

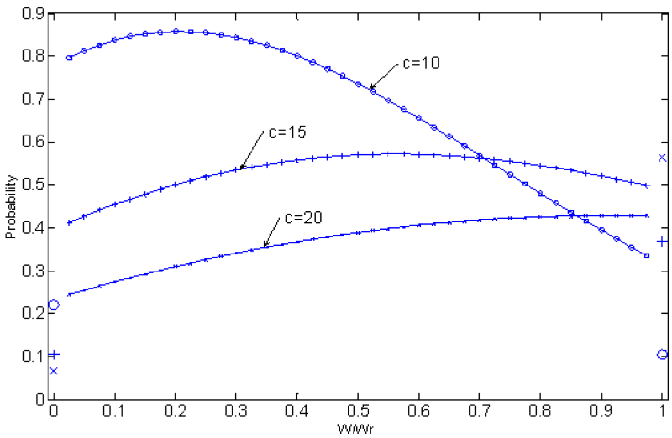

Fig. 2. Wind power output mixed probability function for the Weibull wind speed distribution. Discrete probabilities at 0 and 1; continuous probability function between 0 and 1.

v wind speed (a realization of the wind speed random variable).

For the Weibull function, the discrete portions of the WECS power output random variable will have the following values, found directly from the Weibull PDF

$$\Pr\{W = 0\} = F_V(v_i) + (1 - F_V(v_o)) = 1 - \exp\left(-\left(\frac{v_i}{c}\right)^k\right) + \exp\left(-\left(\frac{v_o}{c}\right)^k\right) \quad (16)$$

and

$$\Pr\{W = w_r\} = F_V(v_o) - F_V(v_r) = \exp\left(-\left(\frac{v_r}{c}\right)^k\right) - \exp\left(-\left(\frac{v_o}{c}\right)^k\right). \quad (17)$$

To make the transformation from the wind speed random variable to the WECS power output random variable in the linear portion of the curve a bit less cumbersome, the following ratios are defined:

$\rho = \frac{w}{w_r}$ ratio of wind power output to rated wind power; and

$l = \frac{(v_r - v_i)}{v_i}$ ratio of linear range of wind speed to cut-in wind speed.

Using (14), the Weibull PDF of the WECS power output random variable in the continuous range then takes the form

$$f_W(w) = \frac{klv_i}{c}\left(\frac{(1+\rho l)v_i}{c}\right)^{k-1} \exp\left(-\left(\frac{(1+\rho l)v_i}{c}\right)^k\right). \quad (18)$$

Before proceeding to the numerical solutions of the ED problem, it may be instructive to look at the relationship between the critical wind speed values related to the WECS generator power output—cut-in, rated, and cut-out—and the critical values that define the wind speed probability profiles – the c and k values in the case of the Weibull distribution function. For Fig. 2, $v_i = 5$, $v_r = 15$, and $v_o = 45$ will be used. These numbers are not for any particular wind turbine; however, they are reflective of general numbers on a mile per hour unit basis (see [13]). Although the plot between 0 and 1 is a continuous pdf, for clarity, individual markers are shown along the continuous line so that these continuous portions of the probability function may be associated with the corresponding discrete probability markers shown at both ends of the probability function.

In Fig. 2, the discrete and continuous portions of the wind power output probability function based on the Weibull wind speed pdf with $k = 2$ and c factors of 10, 15, and 20 are plotted. As the c factor in the Weibull distribution function is increased, a greater proportion of the wind speed profile will be located at higher values of wind speed. This translates to a lower discrete probability of zero power, a higher discrete probability of rated power, and less power in the continuous portion of the plot. As with any other mixed discrete and continuous probability function, the sum of the discrete probabilities at zero and rated power, plus the integral from 0 to 1 of the continuous function will sum to 1.

V. Numerical Solution

The classic solution of the ED problem without the inclusion of the WECS is to take the partial derivatives of the objective (cost) function with respect to each generator output. Except for the generators operating at a fixed minimum, the solution is found where the partial derivatives, also known as incremental costs, are equal for all generators. In addition, other constraints of the ED problem, most importantly, the load balance equation must be satisfied. This method could potentially be used with the inclusion of the WECS generators; however, the difficulty arises in that the derivatives with respect to the generator outputs for the objective cost components (7), (8) are not as easily found as those for the objective cost components (5), (6), due to the fact that the solutions to the integrals cannot be derived in the closed form.

Given this background, the optimization problem will be solved numerically for the case of two conventional and two WECS generators. Using the model stated in (1)–(4), this numerical method demonstrates solutions for the case of two conventional generators and two wind-powered generators. The input for the conventional generator and direct WECS costs are rather straightforward applications of (5) and (6). As for (7) and (8), the wind power probability functions (16)–(18) for the cases of the Weibull distribution are set up. These functions are then used as inputs for (7) and (8). The constraints (2)–(4) are set up, and then, the optimum minimal solution of an objective function subject to linear and/or nonlinear constraints is found by using the MATLAB optimization toolbox.

As the optimization itself is a challenging topic to be explored, this paper focuses only on the results of ED problem with the WECS but not on the optimization technique itself. However, several optimization algorithms applicable to the ED problem based on classical calculus or modern stochastic searching optimization techniques, including the Lagrangian relaxation (LA) [2], direct search method (DSM) [7], evolution programming (EP) [14], particle swarm optimization (PSO) [15], genetic algorithm (GA) [16], and simulated annealing (SA) [17] are well documented in references to name a few.

VI. Analysis

In the preceding discussion, the model that incorporates both conventional and wind-powered generators into the ED problem has been developed. The use of the Weibull probability distribution to model the wind speed has been explained, and the wind speed distributions have been transformed to wind power distributions using the linear wind power equations. A MATLAB program based on the ED model with two conventional and two WECS generators was developed to provide a numerical tool to investigate how variations in the wind speed profiles and variations in the many different cost coefficients in the model will affect the optimum solution of the ED problem. Because of the number of variables in the model and the need to provide an analyzable graphic output, in general, all factors must be held constant except the one under investigation. In both the text and figures that follow, the abbreviations CG1 and CG2 for the conventional generators, and WG1 and WG2 for the wind-powered generators will be used.

Regarding the values of the parameters that are used later, because the primary focus is on the effects that changes to various parameters have on the optimum scheduled outputs for the generators and on the relationships among the various factors in the ED model, specific dimensional unit values, such as miles/hour, meters/sec, or dollars/kilowatthour will not be assigned to the values. The relationships among the factors are the important aspects and these relationships may be more easily studied without the use of a specific system of units.

A. Optimal Outputs as a Function of the Weibull c Factor

To begin some systematic investigation of the model, it may be reasonable at first to look at the effect of the change in the wind speed profile on the outputs of the generators. Initially, the penalty cost coefficient in the ED model will be zero. This could model the case where the wind-powered generators are owned by the power system operator, and where assessing a cost for nonuse of the available wind power may not be reasonable. Because the focus is not, in general, on the conventional generators, unless otherwise stated, the following constant inputs to the model for the conventional generators will be assumed: $a_1 = 1$, $b_1 = 1.25$, $c_1 = 1$; $a_2 = 1.25$, $b_2 = 1$, and $c_2 = 1.25$.

For the wind-powered generators, $d_1 = 1$ and $d_2 = 1.25$ will be assumed. The study begins without a penalty cost coefficient, but even if the WECS generators are owned by the system operator, providing a direct cost advantage to WG1 over WG2 is useful for comparison purposes, and may even reflect an actual cost advantage due to capital costs.

Unless otherwise stated, critical wind speed parameters for the WECS generators of $v_i = 5$, $v_r = 15$, and $v_o = 45$ will be assumed. For the initial studies, the Weibull distribution c factor will be varied from 5 to 25. Although no penalty cost coefficient is assumed, a reserve cost coefficient of 1 will be assumed. A minimum power output from the conventional generators

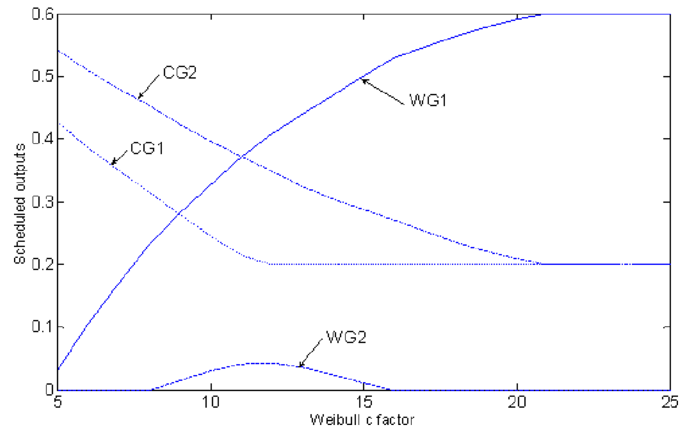

Fig. 3. Generator outputs as a function of Weibull c factor. $k_{r1} = k_{r2} = 1$ and $k_{p1} = k_{p2} = 0$.

of 20% of maximum output from these generators will also be assumed. Fig. 3 follows from using all the aforementioned assumptions.

In may be noted that in some cases, the output from the wind generators exceeds that from the conventional generators. This may not be an ideal situation in terms of security and stability; however, the focus of the simulations in this paper is on the relationship among the wind speed variables and the generator outputs, and not on system security and stability.

At this point, some observations regarding Fig. 3 may be made. As expected from the incremental cost advantage of CG2 over CG1, when not at their minimal outputs, the scheduled output from CG2 will be favored over the output from CG1. Also as expected from the previous discussion and plots (Figs. 1 and 2) based on the Weibull distribution, as the c factor increases, a greater proportion of higher wind speeds will be probable. Combined with the lower direct cost of WG1 as compared to WG2, the scheduled power output of WG1 will be favored over WG2. The small amount of power scheduled from WG2 in the range of Weibull c factor between about 8 and 16 can be explained in that as the c factor increases, more wind power is available and even with the higher direct cost of power from WG2, some of the wind power is economically used by WG2. At a certain point, however, as the amount of power scheduled from WG2 increases, the direct cost also increases, offsetting the benefit, and consequently, decreasing the total amount of power scheduled from WG2 until it again reaches zero. If the direct cost coefficient of WG2 is decreased from 1.25 to 1.10, the scheduled output from WG2 will increase while the output from WG1 will decrease. This change can be seen in Fig. 4. In addition, the conventional generators go to their minimum values at a lower c factor than that in Fig. 3. If the direct cost coefficient for WG2 were decreased further, it would be expected that the scheduled output curve for WG2 would approach that of WG1.

To continue to investigate the effect of the reserve cost coefficient, all other factors are left as in Fig. 4; however, the reserve cost coefficient for WG1 is increased to 2 and the reserve cost coefficient for WG2 is left at a value of 1. These changes lead to Fig. 5.

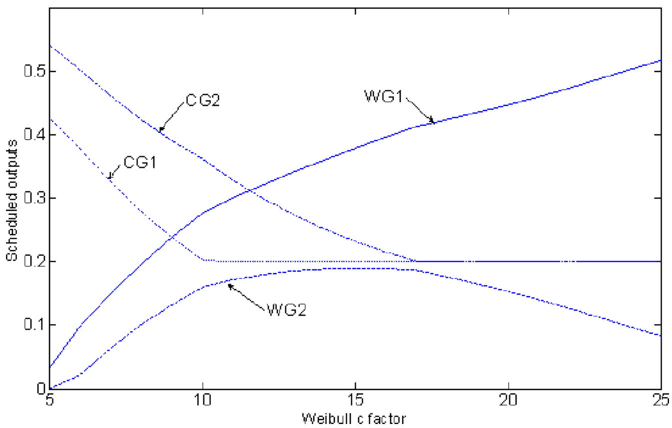

Fig. 4. Same parameters as Fig. 3, except that WG2 direct cost is reduced to 1.10.

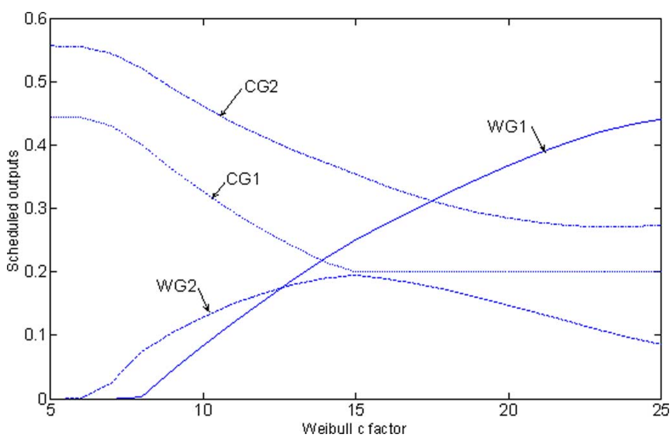

Fig. 5. Same as Fig. 4, except that WG1 reserve cost is increased to 2.

In general, the trends in Fig. 5 are similar to those in Fig. 3, except that, now the increased reserve cost coefficient for WG1 has forced the scheduled output from WG1 to decrease. This effect has even gone so far that the scheduled output from WG2 is greater than that from WG1 at low values of the Weibull c factor. As in Fig. 3, the greater direct cost coefficient for WG2 dominates at larger values of the c factor and causes the scheduled output from WG2 to decrease.

B. Optimal Outputs as a Function of the Reserve Cost Coefficient k_r

In Section VI-A, three different scenarios were plotted, all looking at the effect on the scheduled generator outputs as a function of the Weibull distribution c factor. Although the changes in the direct and reserve cost coefficients were investigated, all of these plots were with a value of zero for the penalty cost coefficient. Before moving on to investigate the effects of the penalty cost coefficient, it may be instructive to look at the changes in optimal scheduled generator outputs as a function of the reserve cost coefficient, so in Section VI-A, the optimal generator outputs as a function of the reserve (overestimation) cost coefficient k_r will be plotted, and the following values for the remaining parameters in the model will be assumed: $a_1 = 1$, $b_1 = 1.25$, $c_1 = 1$; $a_2 = 1.25$, $b_2 = 1$, $c_2 = 1.25$, $d_1 = 1$, $d_2 = 1.05$, $v_i = 5$; $v_r = 15$, $v_o = 45$, and $c = 15$. Notice that WG2 has been made more competitive with respect to WG1 than in the previous plots. The scheduled outputs are now shown in Fig. 6.

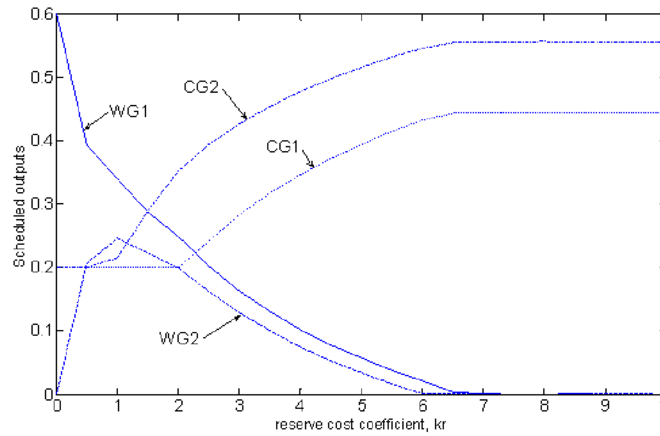

Fig. 6. Generator outputs as a function of the reserve cost coefficient.

The first observation that may be made in looking at Fig. 6 is that, for a reserve cost coefficient of 0, the conventional generators will operate at minimum outputs and the least expensive WECS generator will operate at maximum. As the reserve cost coefficient is increased from 0 to 0.5, it is seen that the scheduled output from WG1 decreases and this decrease is compensated for by an output from WG2. Without the effect of the conventional generators in this section of the plot, it can be observed that the reserve cost increase for WG1 is dominating the cost increase for WG2. Very quickly, however, at a reserve cost coefficient value of 1, both outputs from WG1 and WG2 begin to fall. This is as expected from the model, in that, increasing the reserve cost coefficient effectively counsels the system operator to be more conservative in scheduling, as a greater cost will be paid for overestimating the amount of wind power to be scheduled in the given time period under consideration.

Before moving on to look at the effect of the penalty cost coefficient in the model, it may be interesting to combine the effect of a change in the Weibull c factor by increasing the c factor from 15 to 20. The direct cost for WG1 is returned to 1.05. The results are shown in Fig. 7.

It can be seen that Fig. 7 is very similar to Fig. 6, except that the scheduled outputs from WG1 and WG2 fall at a lower rate. This is as expected, in that with a higher c factor, there is a greater amount of higher wind speed, causing more available power from the wind generators. However, higher reserve cost makes higher wind speed less attractive, and causes both WG1 and WG2 to fall at a lower rate.

C. Optimal Outputs as a Function of the Penalty Cost Coefficient k_p

Having looked at the effect on the ED model of changes in the Weibull c factor and in the reserve cost coefficient, the effect changes that the penalty (underestimation) cost coefficient have on the model will now be investigated. The same general

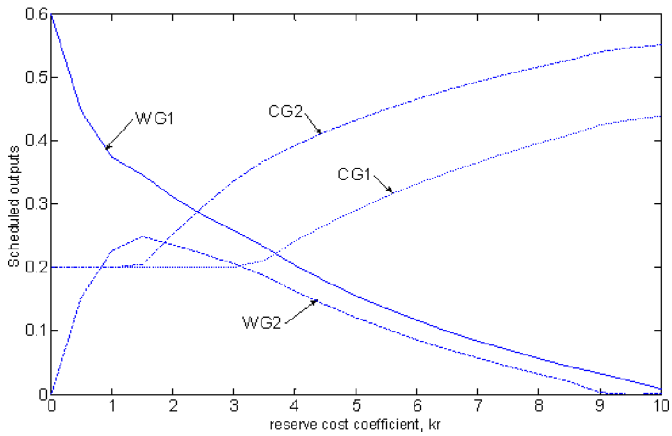

Fig. 7. Same as Fig. 6, except that Weibull c factor is increased to 20.

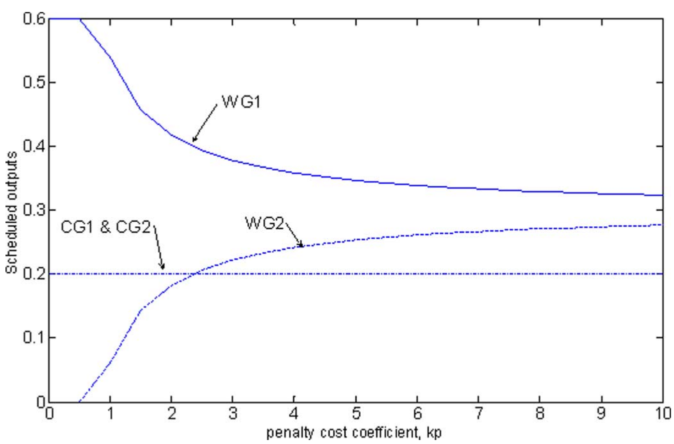

Fig. 8. Generator outputs as a function of the penalty cost coefficient.

parameters as in Sections VI-A and -B will be used; however, the reserve cost coefficients will be set to zero, so that changes to the penalty cost coefficient may be isolated. Fig. 8 illustrates the effect on the scheduled output from various sources caused by the difference in the penalty cost.

Although it is slightly difficult to see because both CG1 and CG2 are at the minimum output, from Fig. 8, as the penalty cost coefficient k_p is increased, the model will indicate that more wind power should be used. Effectively, the increase in the penalty cost coefficient is telling the system operator to take more risk and increase the scheduled amount of wind power. Because the direct cost for wind power from WG2 is greater than that for WG1, the amount of wind power scheduled from WG2 will always be less than that from WG1; however, the influence of the penalty cost coefficient is such that as it increases, it tends to diminish the influence of the greater direct cost for WG2, thereby increasing the scheduled wind power from WG2.

With respect to CG1 and CG2, because the power necessary to meet the load is being provided by the wind generators under the assumption of zero reserve cost, both CG1 and CG2 remain at their minimal levels as the scheduled power increases for WG2 and decreases for WG1.

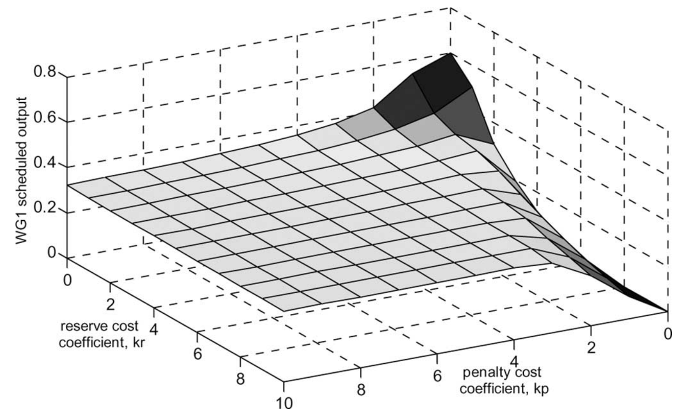

Fig. 9. WG1 output as a function of the reserve and penalty cost coefficients; Weibull c factor equals 15.

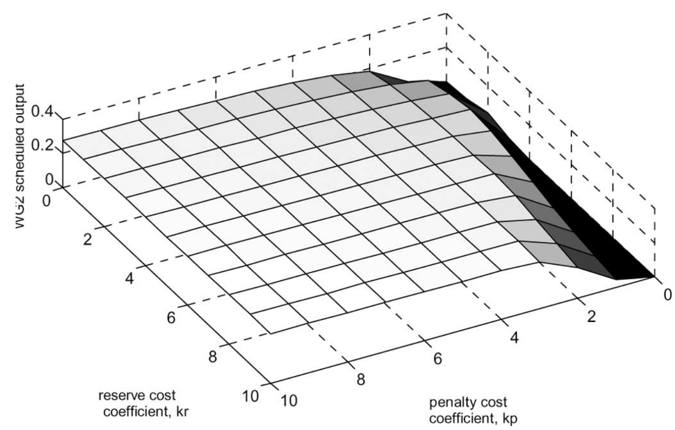

Fig. 10. WG2 output as a function of the reserve and penalty cost coefficients; Weibull c factor equals 15.

D. Optimal Outputs as a Function of k_r and k_p With Weibull pdf

Having looked at the effect of changes in the reserve cost coefficient and in the penalty cost coefficient independently, in this section, the outputs of the ED model with both the reserve and penalty cost coefficients changes will be investigated.

Plotted next as Fig. 9 is the surface of the scheduled output from WG1 as a function of the penalty and reserve cost coefficients. This surface is the result of 121 optimizations at all combinations of k_p and k_r between 0 and 10. A similar surface for WG2 is plotted in Fig. 10.

These two figures confirm the trends found earlier, in that an increase in the reserve cost coefficient tends to decrease the scheduled amount of wind power, whereas an increase in the penalty cost coefficients tends to increase the scheduled amount of wind power.

VII. Conclusion

This paper develops a model to include wind farms in the economic dispatch problem. The uncertain nature of the wind speed is represented by the Weibull pdf. In addition to the classic economic dispatch factors, also included are factors to account for both overestimation and underestimation of available wind power. The optimization problem is then numerically solved for a scenario involving two conventional and two wind-powered generators.

The solution of the ED problem via the model presented is dependent on the values of many coefficients, such as the c factor in the Weibull distribution function, the reserve cost for overestimating the wind energy, and the penalty cost for underestimating the wind energy.

The results also show that the level of wind power scheduled from a particular WECS is strongly dependent on the values of the reserve and penalty cost factors associated with the WECS. If the reserve cost coefficient is increased, the scheduled amount of wind power will be reduced, because it becomes more costly to overestimate the amount of wind power available. Conversely, if the penalty cost coefficient is increased, it becomes more costly to underestimate the amount of wind power available, and the system operator has an incentive to increase the scheduled amount of wind power.

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