Design of Sliding Mode - Neural Network Controller with Fuzzy Observer for Hyper Chaotic Systems Considering Uncertainty, Disturbance, and Nonlinear Control Inputs

Abstract

The main idea of chaos is defining behavior of certain systems that are highly sensitive to their initial condition. In chaotic systems, using a nonlinear controller is recommended to achieve good performance. Sliding mode controller is a robust state feedback controller to deal with uncertainties and has fast transient response. However, it is not an appropriate controller in the steady state of the system because of the existence of chattering. On the other hand, the neural controller is successfully applied on the control of chaotic systems but transient performance of this controller is not appropriate in the face of uncertainties due to its inevitable learning process.

In this paper, a hybrid sliding mode-neural network controller with variable weights is designed to utilize the advantages of both controllers. A fuzzy observer has the task of optimal tuning of these coefficients. The fuzzy observer also supervises the switching process between the two controllers.

Introduction

While the properties of chaotic systems are almost clear and by observing the behavior of a chaotic system, one can say that if it is chaotic or not, there is no precise definition of chaos. This is partly conceivable since before being a purely mathematical concept, the chaos is a concept that has its origins in the field of human perception and natural language. Proposing a theory that covers all human understanding of such half-objective half-subjective phenomenon is not a simple task. However, several different mathematical definitions for chaos are available that are more or less acceptable.

One of the characteristics of chaotic systems is high sensitivity to initial conditions [1]. A chaotic system, unlike a system with stable equilibrium point or a limit cycle, is sensitive to small changes in its states. Very small changes in initial conditions lead to significant changes in the final conditions. Known examples are weather condition phenomenon and Butterfly phenomenon. Nonlinear dynamics of the atmosphere and not having all initial conditions at the start of calculation (for example, lack of access to and uncertainties in the measurement of temperature, humidity etc. in all parts of the earth) make long-term predication impossible.

In the past, due to complex dynamics and inherent instability of chaotic systems, their control was considered impossible. However, it has been shown that chaotic systems can also be controlled [2] and different control objectives are conceivable for them. Nonlinear control strategies are also applicable for these systems. Moreover, Because of their special features, new control methods can also be applied on these systems [3-5]. One of the first applications of chaos control has been reported as the stabilization of laser output intensity chaos [6]. In this paper, using OPF (Optimal Power Flow), the output power has increased 15 times. For more information on chaos control applications refer to [7]. Other applications are controlling pendulum, gyroscope, bouncy balls, ship fluctuations, turbulence in fluids, multi-mode behavior of laser, deletion, or creation of a chaotic state in a chemical reaction called Belousov-Zhabotinsky, and controlling chaotic mixing to increase the speed of mixing process, insect population with little change in the number of adult insects and removal of epilepsy cases in animals.

The main objective of control engineering is to introduce stable systems. Most designs are done in a way that the signals reach a predetermined value or track other signals. Instability is not accepted in control engineering and for this reason most of the control theory is about ideas on stabilization. Taking this in mind, chaos is undesirable. Several methods for stabilizing a chaotic system, stable equilibrium or stable limit cycle, have been proposed. However, some applications have been introduced recently that suggest that creation of a chaotic signal is not only not harmful, but also desirable. For example, mixing liquids that make them act faster than the periodic motion [8] and generating chaotic signal for safe telecommunications can be mentioned [9].

Methods that have been proposed for the control of chaotic systems to date are classical techniques such as linear control, sliding mode control and intelligent techniques such as fuzzy logic and neural networks. As we know, the linear control is ineffective subject to uncertainties [10]. Sliding mode control is a robust nonlinear control method that is very effective to deal with uncertainties. On the other hand, this method has a very fast transient response. However, it has discontinuous control signal and chattering, which may lead to excitation of high frequency dynamics.

In recent decades, the use of artificial intelligence to control chaotic systems is considered [11, 12]. Common methods of artificial intelligence include neural networks control and fuzzy control. Neural networks have inherent ability to learn and approximate a nonlinear function with arbitrary precision. This property is being used in control to model complex processes and compensating uncertainties. However, it is inevitable learning process reduces its transient performance in the face of disturbances. Superiority of fuzzy control is in utilizing human knowledge in control process. However, the main issue of this method is lack of enough theories to check stability of fuzzy controllers in general. To overcome the disadvantages and utilizing the advantages of intelligent and conventional controllers, a method is proposed in [13] that combines sliding mode control with neural network control with different weights. These weights are determined using a fuzzy observer and are applied successfully on a robotic arm.

In this paper, the objective is to combine sliding mode control with neural network with different weights to control the chaotic system presented in [14] and these weights are tuned using a fuzzy observer. In this method, neural network controller is used in parallel with the sliding mode controller. Sliding mode control is designed to reject disturbances and guarantee the stability of the system. Neural network controller reduces chattering. It also estimates dynamics of the system and using its learning ability overcomes unstructured uncertainties. High gain is assigned in the transition state to the sliding mode control to guarantee system robustness. When approaching a steady state response, the neural network switches to the main controller to overcome the uncertainties and increase tracking accuracy. Fuzzy observer can facilitate this switching between controllers.

This paper is organized as follows. In the second part, dynamics of the chaotic system are introduced. The proposed control scheme is presented in the third part. Computer simulation, discussion, and conclusion of it are discussed in the fourth part and finally, the fifth part concludes the paper.

Dynamic equations of hyper chaotic system under study

As mentioned in the introduction, the hyper chaotic system introduced in [14] have been studied. The dynamic equations of the system are expressed in (1).

$$\begin{cases} \dot{\mathbf{x}}_1 = a(\mathbf{x}_2 - \mathbf{x}_1) + b\mathbf{x}_2\mathbf{x}_3^2 \\ \dot{\mathbf{x}}_2 = c\mathbf{x}_1 + d\mathbf{x}_1\mathbf{x}_3^2 + n\mathbf{x}_4 \\ \dot{\mathbf{x}}_3 = f\mathbf{x}_3 + g\mathbf{x}_2^2 + h\mathbf{x}_1\mathbf{x}_4 \\ \dot{\mathbf{x}}_4 = k\mathbf{x}_2 \end{cases} \qquad (1)$$

Where $\mathbf{x}_i \, (i = \mathbf{1,2,3,4})$ represent the system state variables. System parameters are as follows (Table 1):

Table 1 – parameters of dynamic equations of the system

a	*b*	*c*	*d*	*n*	*f*	*g*	*h*	*k*
7/7	-1	8	4	8	-4	1	1	-2

The system exhibits the behavior of a chaotic system with two positive Lyapunov exponent $L_1=2.316$ and $L_2=0.59$. To study different dynamic behaviors of this system one can refer to [14].

Figure 1 displays a number of absorbent hyper chaotic images of system (1) in two-dimensional and three-dimensional space.

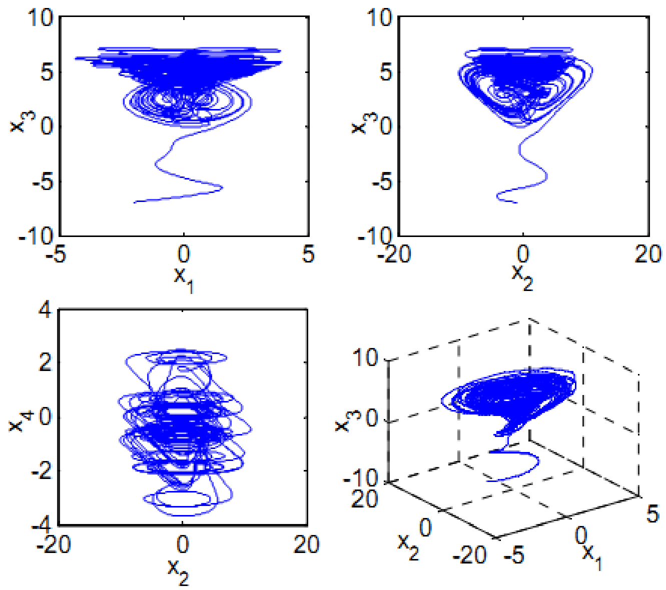

Figure 1: absorbent chaotic images in two-dimensional and three-dimensional space

Suppose that system (1) is under non-structural uncertainties, external disturbances and nonlinear control inputs. Having these in mind, matrix equations of the system can be written as (2).

$$\dot{\mathbf{x}}(\mathbf{t}) = \mathbf{A}\mathbf{x}(\mathbf{t}) + \mathbf{B}\mathbf{f} + \mathbf{B}\phi(\mathbf{u}(\mathbf{t})) + \mathbf{B}\Delta\mathbf{f} + \mathbf{D}\mathbf{w}(\mathbf{t}) \qquad (2)$$

Where $\mathbf{x} = [x_1 \ldots x_4]^{\mathrm{T}} \in \mathbf{R}^{4\times1}$ denotes state variables, $\mathbf{A} = \mathbf{R}^{4\times4}$ is a matrix of linear terms of system dynamics, $\mathbf{f} = [f_1 \quad f_2 \quad f_3]^{\mathrm{T}} \in \mathbf{R}^{3\times1}$ is a matrix of nonlinear terms of system dynamics, $\mathbf{B} \in \mathbf{R}^{4\times3}$ displays a constant matrix, $\phi(\mathbf{u}) = [\phi_1(u_1)\phi_2(u_2)\phi_3(u_3)]^{\mathrm{T}} \in \mathbf{R}^{3\times1}$ represents vector of nonlinear control inputs, $\Delta\mathbf{f} = [\Delta f_1 \quad \Delta f_2 \quad \Delta f_3] \in \mathbf{R}^{3\times1}$ is a vector of non-structured uncertainties, $\mathbf{w}(t) \in \mathbf{R}$ shows external disturbances and $\mathbf{D} \in \mathbf{R}^{4\times1}$ is a constant vector. A, B, F, and D are given in (3).

$$\mathbf{A} = \begin{bmatrix} -7/7 & 7/7 & 0 & 0 \\ 8 & 0 & 0 & 8 \\ 0 & 0 & -4 & 0 \\ 0 & -2 & 0 & 0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \\ \mathbf{f}_3 \end{bmatrix} = \begin{bmatrix} -\mathbf{x}_2\mathbf{x}_3^2 \\ 4\mathbf{x}_1\mathbf{x}_3^2 \\ \mathbf{x}_2^2 + \mathbf{x}_1\mathbf{x}_2 \end{bmatrix}, \mathbf{D} = [0 \quad 0 \quad 0 \quad 2]^{\mathrm{T}} \qquad (3)$$

$\dot{\mathbf{x}}(\mathbf{t}) = \mathbf{Ax}(\mathbf{t}) + \mathbf{f} + \mathbf{B}\phi(\mathbf{u}(\mathbf{t})) + \mathbf{\Delta f} + \mathbf{Dw}(\mathbf{t})$ can be used to describe equations of system (1) in matrix form and Δf and f should be modified as $\mathbf{\Delta f} = [\Delta f_1 \quad \Delta f_2 \quad \Delta f_3 \quad \Delta f_4]^{\mathbf{T}}$ and $\mathbf{f} = \left[-x_2 x_3^2 \quad 4x_1 x_3^2 \quad x_2^2 + x_1 x_4 \quad 0\right]^{\mathbf{T}}$. For external disturbance $\mathbf{w}(t) \in \mathbf{R}$, the condition $\|\mathbf{w}(t)\|_2 < \infty$ is considered and (4) defines $\|\mathbf{w}(t)\|_2$ that $\|\mathbf{w}(t)\|$ represents Euclidean norm of $\mathbf{w}(t) \in \mathbf{R}$.

Equation (4) shows the assumed constraint for non-structural uncertainties. According to (4), non-structural uncertainties are bounded and real constants $\eta_3, \quad \eta_2, \quad \eta_1$ are upper bounds of these uncertainties.

$$\left|\Delta f_i (x_1, x_2, x_3, x_4)\right| \le \eta_i, i = 1,2,3 \tag{4}$$

Supposed conditions for nonlinear functions of control inputs are given in (5). These circumstances suggest that $\phi_1(u_1), \phi_2(u_2)$ and $\phi_3(u_3)$ are within the sectors $[\alpha_1, \alpha_2]$, $[\beta_1, \beta_2]$ and $[\zeta_1, \zeta_2]$ respectively [15-16].

$$\begin{cases} \phi_i(0) = 0, & i = 1,2,3 \\ \alpha_1 u_1^2(t) \le \phi_1(u_1(t)) u_1(t) \le \alpha_2 u_1^2(t), & \alpha_1, \alpha_2 > 0 \\ \beta_1 u_2^2(t) \le \phi_2(u_2(t)) u_2(t) \le \beta_2 u_2^2(t), & \beta_1, \beta_2 > 0 \\ \zeta_1 u_3^2(t) \le \phi_3(u_3(t)) u_3(t) \le \zeta_2 u_3^2(t), & \zeta_1, \zeta_2 > 0 \end{cases} \tag{5}$$

Figure 2 depicts a hypothetical schematic of conditions in (5) for $\phi_1(u_1)$. In this paper, two objectives are considered to control the system (2). The first objective is to design a robust controller such that the hyper chaotic system converges to its equilibrium point $\mathbf{e} = (x_{1e}, x_{2e}, x_{3e}, x_{4e}) = (0,0,0,0)$ despite the presence of non-structural uncertainties, external disturbances and nonlinear control inputs, and the second goal is to weaken the disturbance.

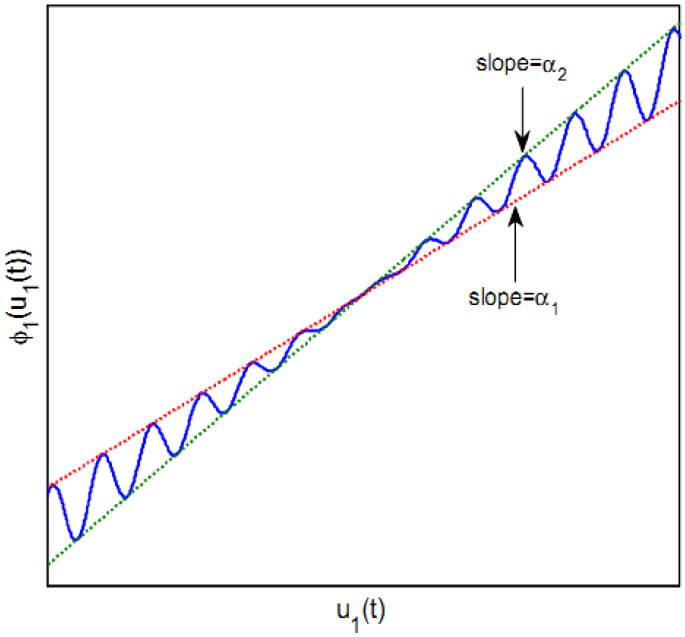

Figure 2: hypothetical schematic of conditions in (5)

The proposed control method

Here, the feedback controller $\upsilon = h(\mathbf{x}, k)$ is determined so that the control system converges to its unstable equilibrium point or its periodic orbits. If we assume $\upsilon = 0$, the controlled system becomes the primary chaotic system.

As mentioned earlier, in the proposed control method, the combination of sliding-mode controller and neural controller with different weights is used to utilize the advantages of both controllers and avoid their disadvantages. A fuzzy observer has the task of optimal determining of coefficients depending on the current situations. The use of fuzzy observer also prevents from sudden switching between the two controllers and facilitates the transition between the two.

The basic structure of a fuzzy controller is composed of four parts of fuzzifier, fuzzy inference system, fuzzy rules base, and defuzzifier.

In particular, a fuzzy controller with if-then rules base, minimum inference engine, singleton fuzzifier (single) and defuzzifier of mean center of gravity is expressed as (6).

$$f(x) = \frac{\sum_{m=1}^{M} y^m \left(\min_{i=1}^{n} \mu A_i^m (x_i) \right)}{\sum_{m=1}^{M} \min_{i=1}^{n} \mu A_i^m (x_i)} \quad (6)$$

In the current study, this type of fuzzy controller is used as fuzzy observer.

In earlier works, fuzzy supervisory control that ensures system stability is presented. The main idea behind this is that if the system works well under the supervision, the observer will be idle. On the other hand, if the system under supervision tends toward instability, the observer will take action to avoid the instability of the system. This leads to discontinuity in the control function. To achieve continuous control, fuzzy gain-scheduling supervisory control is presented such as fuzzy supervisory PID control, fuzzy supervisory sliding mode and PI control [17]. It is obvious that if the controller is under supervision of a linear controller, it will not be effective enough to deal with uncertainties [18]. Hence, in this study a nonlinear intelligent controller, which is effective in dealing with uncertainties, is used.

One of the advantages of sliding mode control is its fast and robust behavior in dealing with disturbance. Its disadvantages include chattering, the need to estimate the limits and conservative applying of bigger factors. On the other hand, neural network controller is able to overcome the structural uncertainties. However, the main disadvantage of this controller is its undesired transient response due to the process of updating parameters. To counter the disadvantages of the two mentioned methods, a new method is proposed that has been named as supervisory fuzzy sliding mode control and neural networks.

Sliding mode control signal ensures system stability and directs the error to the sliding surface. When the error is closer to the sliding surface, chattering can be reduced with a smaller gain. In this case, the system is still subject to unstructured uncertainties and this causes the system to have inappropriate behavior in steady state. Therefore, the neural network capable of self-learning plays a major role in coping with unstructured uncertainties. On the other hand, delay in calculating control signal causes a discontinuity in the sliding mode control signal. Fuzzy controller acts as a supervisor and specifies gain coefficients of each low-level controller based on system behavior and the rules database. Fuzzy controller can also facilitate the switching of sliding mode controller to neural network controller.

In this study, the fuzzy controller is a supervisory behaviorist controller. As shown in Figure 3, supervisory fuzzy controller is used to obtain the coefficients m and m-1 for sliding mode controller and neural network controller respectively. Sum of gain coefficients of sliding mode controller and neural network controller equals one, so output limit of fuzzy observer (m) is considered to be between 0 and 1. The block diagram of the control system is shown in Figure 3.

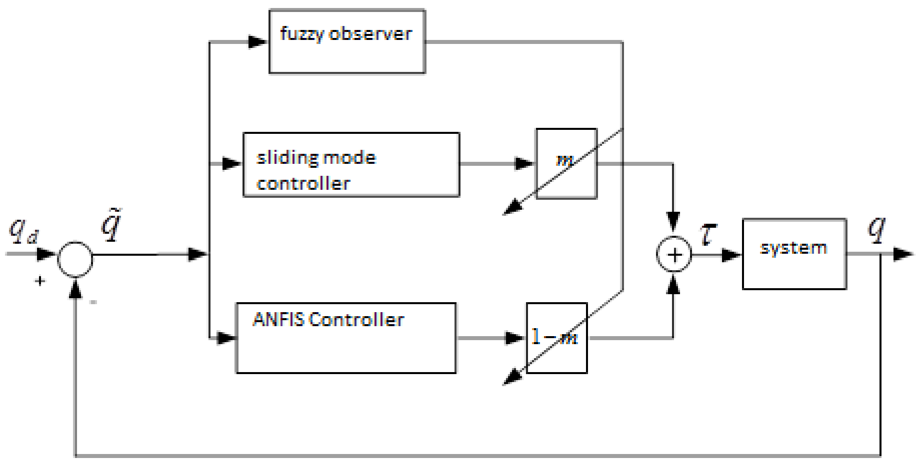

Figure 3: block diagram of the proposed control method

Where $n, \mathbf{q}, \tilde{\mathbf{q}}$ and s are system's order, system's output, reference signal-output error and sliding surface.

Considering the above figure, the control signal can be stated as follows:

$$u(t) = u_{pd} + m(t)\,u_{ad}(t) + (1 - m(t)).u_{ls}(t) \quad (7)$$

Since the system is stable with the mentioned conditions, there is no need to use a linear controller. Therefore, the control law can be written as follows:

$$u(t) = m(t)\,u_{ad}(t) + (1 - m(t)).u_{ls}(t) \quad (8)$$

m(t) can be determined using a fuzzy observer considering the error (mean square sliding surfaces) and the amount of error changes.

Simulation

In order to design the neural network controller, inverse identification method is used. This means that first, the inverse system model is identified by neural network and then the inverse model is used as the controller [19]. For identification, system outputs, x, and delayed inputs and outputs are used as neural network input and the system input u is used as the output of the neural network. To have precise identification, a separate network is used for each of the control inputs. Considered Structure for inverse identification of each of the control inputs is depicted in Figure 4.

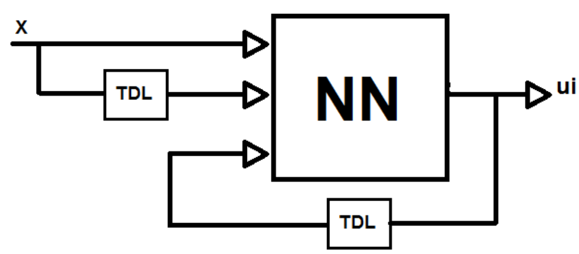

Figure 4: considered structure for identification

The input of the neural network is system's states with their 1-unit and 2-unit delays and 1-unit delay of control inputs. Since each MLP network with one hidden layer and a sufficient number of neurons is able to estimate any nonlinear function, here a neural network with 1 hidden layer and 100 neurons is used. Through a series of Pre- and post-processing operations, a neural network with better performance can be created.

As mentioned earlier, an identified neural network in series or system can be used to control it. The control structure is shown in Figure 5.

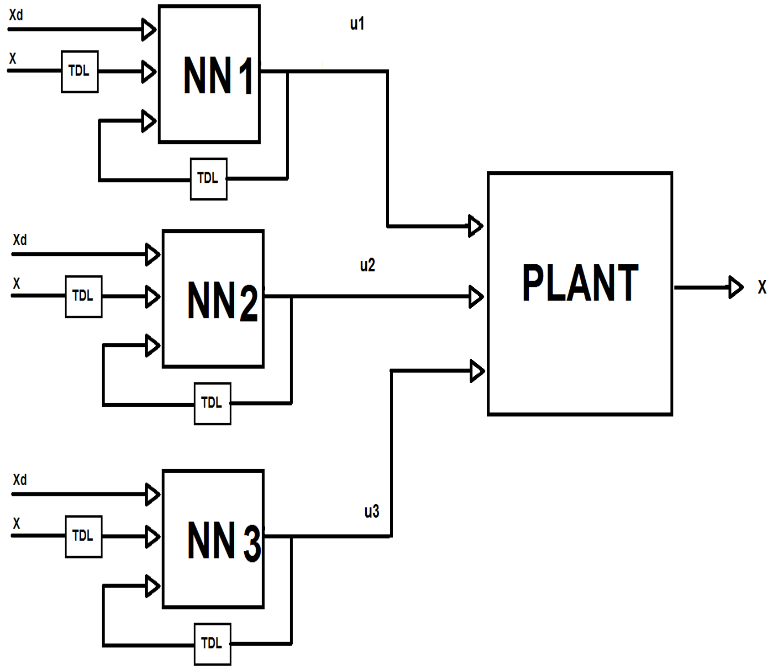

Figure 5: control structure using neural networks

To design sliding-mode controller, equations presented in [20] are used.

In order to design fuzzy observer, error signal and its derivatives is used to calculate the output. Three membership functions i.e. zero, small and large are considered for error and three membership functions i.e. negative, zero and positive are used for error changes. For m, the fuzzy membership functions are considered as fuzzy singletons that are named as zero, small and large. It should be noted that input ranges for error, error changes and output are [0, 15], [-1500 1500] and [0, 1], respectively. Fuzzy rules are as follows: (er and de indicate error and error changes, respectively.)

1- If er is **zero** and de is **negative** then m is **large**
2- If er is **zero** and de is **zero** then m is **large**
3- If er is **zero** and de is **positive** then m is **small**
4- If er is **small** and de is **negative** then m is **large**
5- If er is **small** and de is **zero** then m is **small**
6- If er is **small** and de is **positive** then m is **zero**
7- If er is **positive** and de is **negative** then m is **zero**

8- If er is **positive** and de is **zero** then m is **zero**
9- If er is **positive** and de is **positive** then m is **zero**

After applying control law, the following results are obtained for system states and control input:

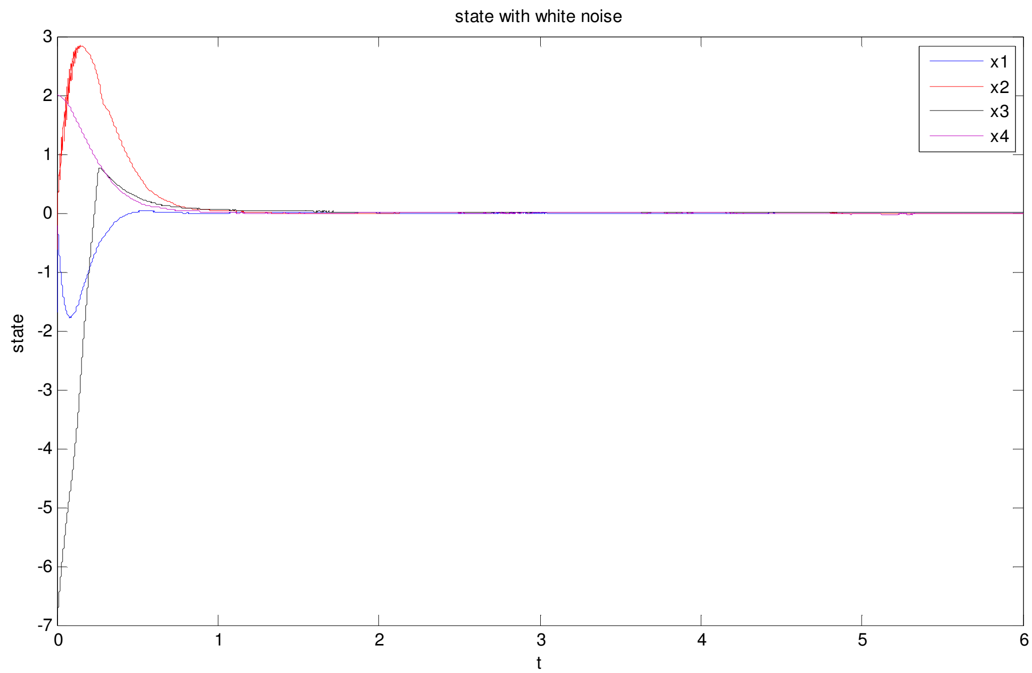

Figure 6: system states

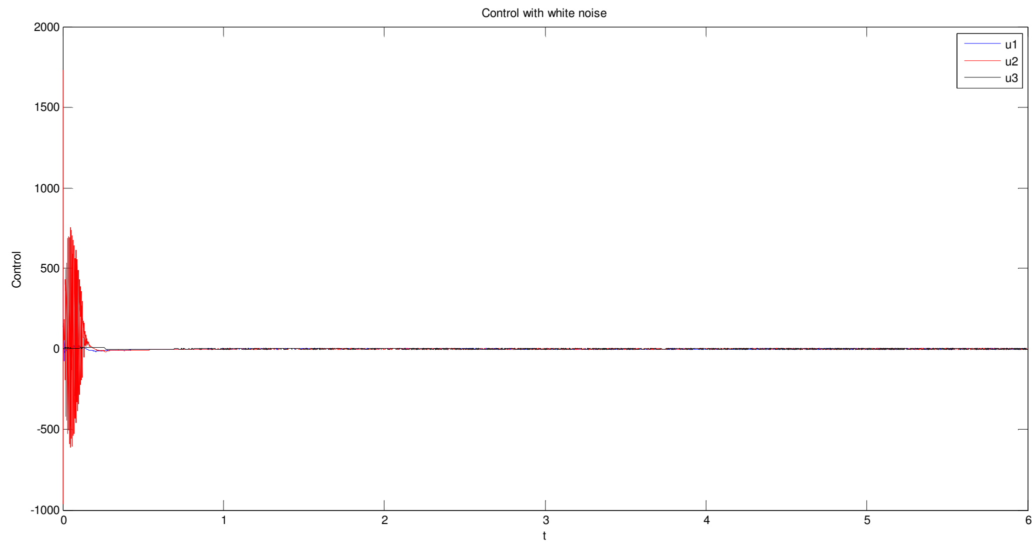

Figure 7: control signal

As can be seen, system states have been improved and control signal chattering has been eliminated in steady state. For better comparison of system states, system outputs when the sliding mode controller is used alone and when a neural network controller is used in parallel with it are saved and are depicted in the following figure simultaneously.

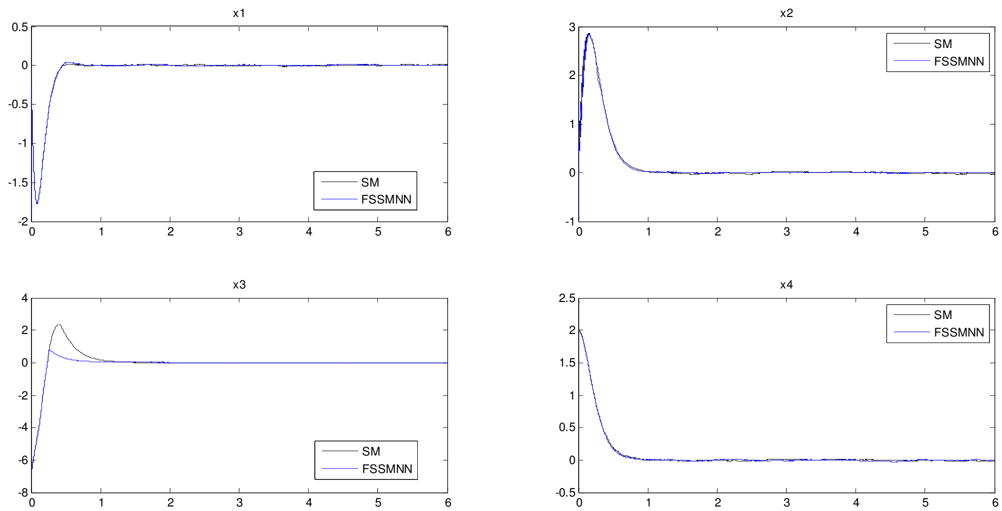

Figure 8: comparison of system states using both controllers

In the above figures, the black figures are related to sliding mode controller and blue figures are related to sliding mode-neural network control with fuzzy observer. As can be seen, the behavior of the system with sliding mode-neural network controller with fuzzy observer has improved in steady state and fluctuations are reduced dramatically. For a better view of the above figure, the following figure is presented.

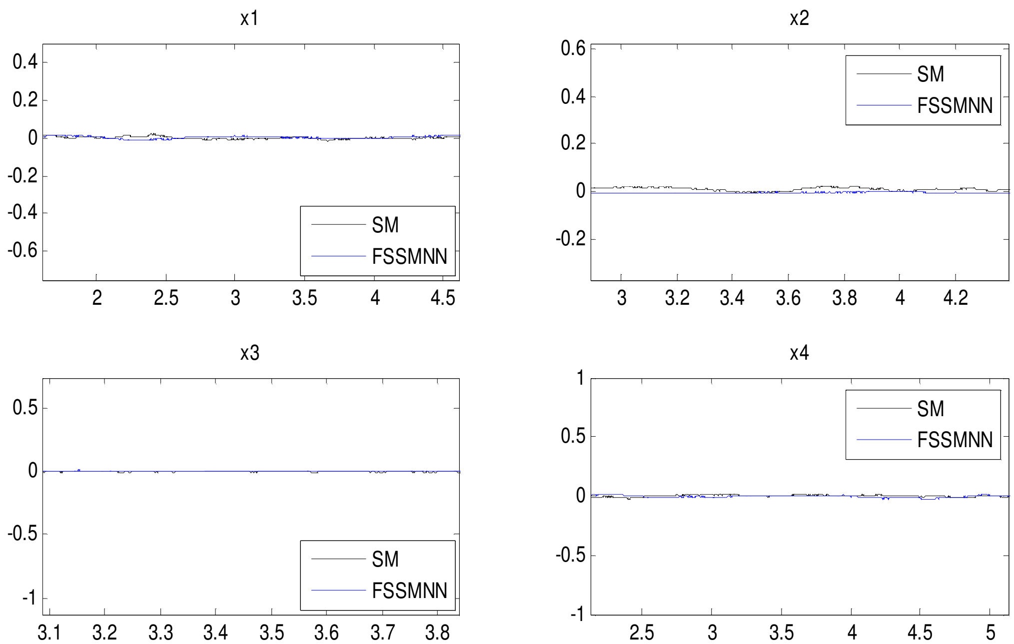

Figure 9: comparison of system states using the two controllers

The sum of squared errors for both methods is shown in the below figure.

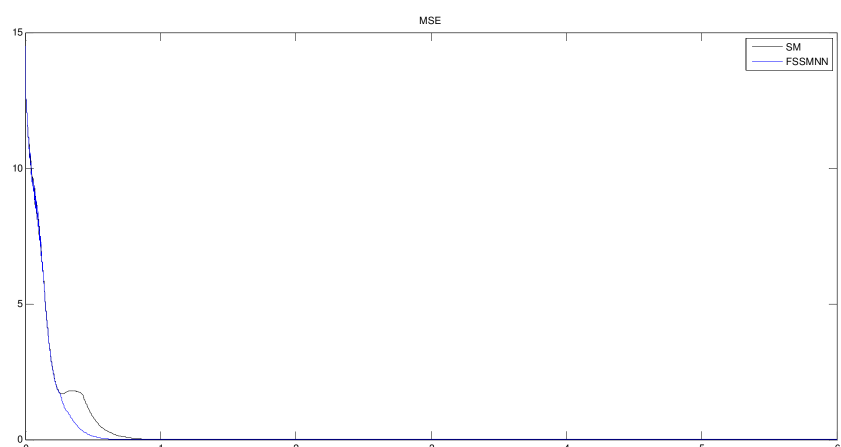

Figure 10: comparison of sum of squared errors

As can be seen, the sum of squared errors is smaller when using sliding mode-neural network controller with fuzzy observer.

The output of the system when a pulse disturbance is applied at 3.5 to 4 seconds is as follows:

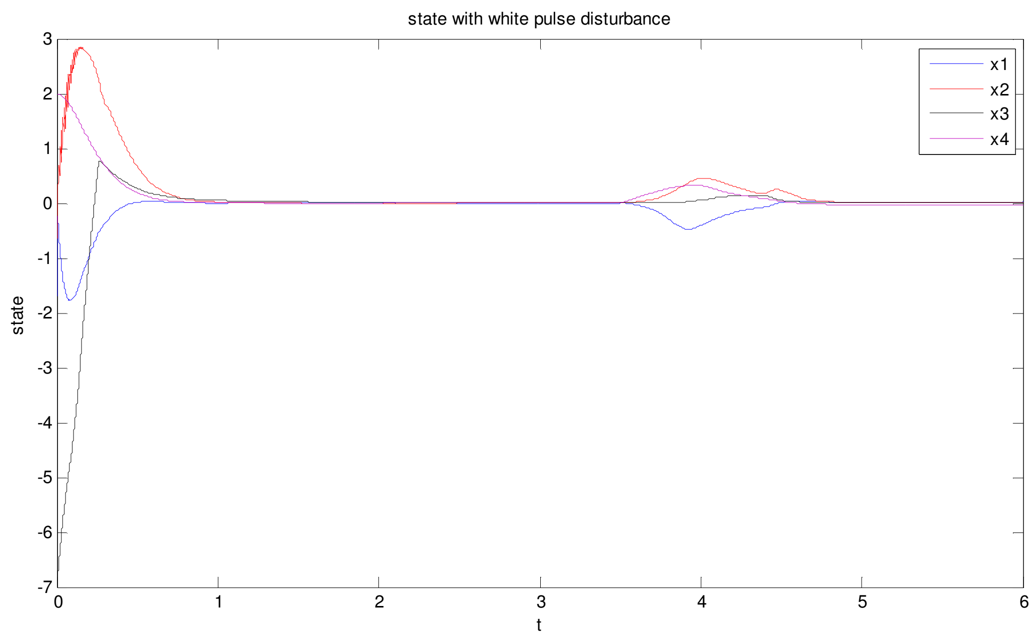

Figure 11: system states in the presence of a pulse noise applied at 3.5 to 4 seconds

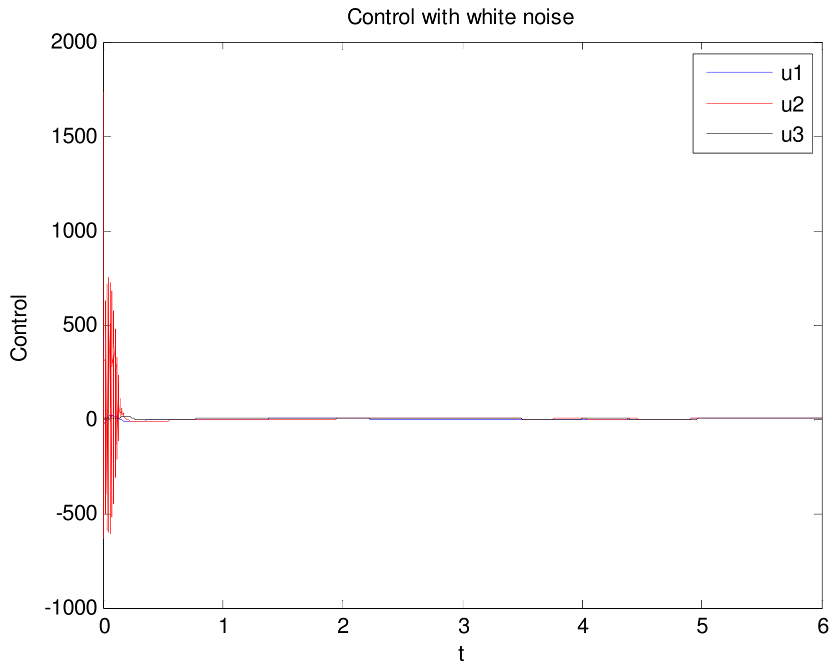

Figure 12: control signal in the presence of a pulse noise applied at 3.5 to 4 seconds

As can be seen, the system is able to cope well with the applied disturbance and the control signal does not have much variations.

Conclusion

As we know, chaotic systems are very complex and demonstrate an intricate behavior subject to uncertainties and various disturbances. Hence, in order to overcome these uncertainties and reject disturbances, a sliding mode controller was first designed that showed a good response in transient working conditions. Then, a neural network controller was designed that demonstrated a good steady state behavior. Finally, these controllers were utilized in parallel and a fuzzy observer was used to tune their coefficients. Finally, parameters of fuzzy observer were tuned in such a way that yielded a response without chattering and with good transient response.

The results clearly showed that the system had desired response and steady state error with designed controller and could efficiently reject chaos. Compared with the sliding mode controller, states and input signal chattering was completely disappeared and disturbance is rejected well. In short, the designed controller took the advantages of both sliding mode and neural network controller to compensate for disadvantages and improved system behavior better than the two controllers.

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