

# Velocity Control of DC Motor Based Intelligent methods and Optimal Integral State Feedback Controller

M.B.B. Sharifian, R.Rahnavard and H.Delavari

**Abstract**—Velocity control of DC motors is an important issue also shorter settling time is desired. In this paper at first a PID compensator which adjusted by genetic algorithm is designed then another compensator will be designed by combining two methods, Integral controller and optimal State Feedback controller (I&S.F.). In the second compensator, design specifications, depend on choosing weighting matrices Q and R, we use the Genetic Algorithm (GA) to find the proper weighting matrices. Of course Kalman filter is used as a system observer in order to increasing the system robustness. Then the performance of the control techniques is compared in terms of rise time, settling time, tracking error, and robustness with respect to modeling errors and disturbances. The controller design process and implementation requirements are also discussed. Then the comparison between the PID control and the optimal control shows that the optimal controller significantly reduced the overshoot, settling time and has the best performance encountering with system uncertainties. Also we apply noise and 20% parameters variation for DC motor and compare the results. According to the simulation results, the second controller has better performance than the PID controller.

**Index Terms**—DC Motor, Genetic Algorithm, Kalman Filter, Optimal Control, PID Controller

## I. INTRODUCTION

The parameters of an actual DC servomotor used in this paper are listed in Table I.

TABLE I: MOTOR PARAMETERS

$R_a$	4.67 $\Omega$
$L_a$	170e-3 H
$J$	42.6e-6 Kg.m <sup>2</sup>
$f$	47.3e-6 N.m / rad / sec
$K$	14.7e-3 N.m / A
$K_b$	14.7e-3 V.sec / rad

And the transfer function for this DC Motor is:

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$$G_p = \frac{0.0147}{7.242e-006 S^2 + 0.000207 S + 0.000437} \quad (1)$$

The state space matrices for this system are:

$$\mathbf{A} = \begin{bmatrix} -28.58 & -7.542 \\ 8 & 0 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 16 \\ 0 \end{bmatrix}$$

$$\mathbf{C} = [0 \quad 15.86] \quad \mathbf{D} = [0] \quad (2)$$

By this system definition we design the system controller.

## II. PID CONTROLLER DESIGN

The conventional PID regulator is the most widely used control strategy in the industry process, because of the remarkable effectiveness, simplicity of implementation and broad applicability. Often in practice, tuning is carried out by an experienced operator using a 'trial and error' procedure and some practical rules, this is often a time consuming and difficult activity.

The objective of a PID controller in a velocity control system is to maintain a velocity set point at a given value and be able to accept new set-point values dynamically. At first a PID controller is designed with genetic algorithm then we show although it has good performance but it is not so robust with system parameters deviation and disturbance and noise in plant [1], [2].

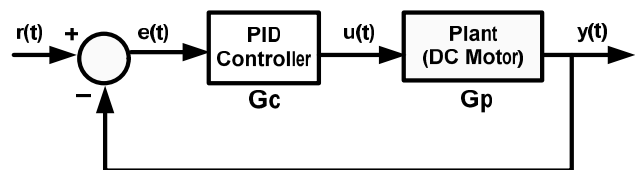


Fig. 1 A Typical PID Controller System

PID controller Parameters are obtained using a novel genetic algorithm based optimization technique. The conventional integral controller was replaced by a PID controller with the following structure.

$$G_c(s) = K_p + \frac{K_i}{s} + K_d s \quad (3)$$

Where  $K_p$  is the proportional gain  $K_i$  and  $K_d$  are integral and derivative time constants respectively. These parameters are selected by genetic algorithm. The fitness function is defined as:

$$FitnessFunction = \int \left( (e(t))^2 + (u(t))^2 dt \right)$$

$$e(t) = y(t) - r(t) \quad (4)$$

Where

$y(t)$  : DC Motor velocity (Step response)

$u(t)$  : DC Motor in-put

$r(t)$  : Reference in-put (Step function)

$e(t)$  : Error between DC Motor velocity and reference input (it means that smaller  $e(t)$  is desired).

By implying  $u(t)$  in this optimization problem we optimized consumption energy. Step response for this PID has shown in Fig. 2.

Optimal parameters after a complete GA running are shown in Table II [6], [7], [8], and [3].

TABLE II: PID PARAMETER WITH GA

$K_p$	$K_i$	$K_d$
0.496639	1.0910	0.00599889

### III. OPTIMAL INTEGRAL STATE FEEDBACK CONTROLLER DESIGN

At first we design integral controller for the DC motor and then state feedback and Kalman filter are added. We will consider a system defined via minimal state space realization:

$$\dot{x}(t) = \mathbf{A}x(t) + \mathbf{B}u(t)$$

$$y(t) = \mathbf{C}x(t) \quad (5)$$

And  $q(t)$  assume by the following definition:

$$\dot{q}(t) = r - y(t) = r - \mathbf{C}x(t) \quad (6)$$

$$\begin{bmatrix} \dot{x}(t) \\ \dot{q}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ -\mathbf{C} & \mathbf{0} \end{bmatrix} \begin{bmatrix} x(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix} u(t) + \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix} r$$

$$y(t) = [\mathbf{C} \quad \mathbf{0}] \begin{bmatrix} x(t) \\ q(t) \end{bmatrix} \quad (7)$$

And the design problem reduces to find the control energy signal  $u(t)$ , that given by:

$$u(t) = [-\mathbf{K}_1 \quad -\mathbf{K}_2] \begin{bmatrix} x(t) \\ q(t) \end{bmatrix} \quad (8)$$

Substituting this equation with equation (7) results in:

$$\begin{bmatrix} \dot{x}(t) \\ \dot{q}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{B}\mathbf{K}_1 & -\mathbf{B}\mathbf{K}_2 \\ -\mathbf{C} & \mathbf{0} \end{bmatrix} \begin{bmatrix} x(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix} r \quad (9)$$

By suitable selection of  $\mathbf{K}_1, \mathbf{K}_2$  matrices, closed loop system will be stable and steady state value of  $x(t), q(t)$  will be constant at steady state condition. We have:

$$\lim_{t \rightarrow \infty} \dot{q}(t) = \lim_{t \rightarrow \infty} (r - y(t)) = 0$$

$$\lim_{t \rightarrow \infty} y(t) = r \quad (10)$$

This controller guaranties reference input tracking, and reduces disturbance effects in closed loop system and eliminates them at steady state condition. Adjusting  $\mathbf{K}_2$  parameter can improve the closed loop system transient performance, but this system hasn't good performance by parameter variation, consequently we combine Kalman filter

to improve it. System output has shown in Fig.4.

Indeed, nonlinear behavior, plant disturbance, sensor noise, and model errors will in variably lead to deviation from the true states unless due to precautions are taken during the observer design. Also the measured noise and process noise can disturb the models characteristics. Then we must design a controller, which can overcome these problems.

State space realization are routinely used in science and engineering to model linear and nonlinear dynamical system. The states are usually not available since rarely one can have a sensor on every state variable, and some form of reconstruction from the available measured output is required. In this case, an observer can be constructed using the mathematical model of the plant, to obtain estimation for  $\hat{x}$  of the true state  $x$ . This estimation can then be used as a substitution for  $x$ , where it is required, then as an optimal observer, Kalman filter is used. The standard approach used to estimate the state, consist of employing the following structure:

$$\dot{\hat{x}}(t) = \mathbf{A}\hat{x}(t) + \mathbf{B}u(t) + \mathbf{L}[y(t) - \mathbf{C}\hat{x}(t)] \quad (11)$$

Equation (11) defines the gain of state observer. And  $u(t)$  assume to be:

$$u(t) = -\mathbf{K}\hat{x}(t) + r \quad (12)$$

Equations (11), (12) with equation (5) result in:

$$\begin{bmatrix} \dot{x}(t) \\ \dot{\hat{x}}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A} & -\mathbf{B}\mathbf{K} \\ \mathbf{L}\mathbf{C} & \mathbf{A} - \mathbf{L}\mathbf{C} - \mathbf{B}\mathbf{K} \end{bmatrix} \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix} r$$

$$y(t) = [\mathbf{C} \quad \mathbf{0}] \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix} \quad (13)$$

The block diagram of this controller has shown in Fig. 3.

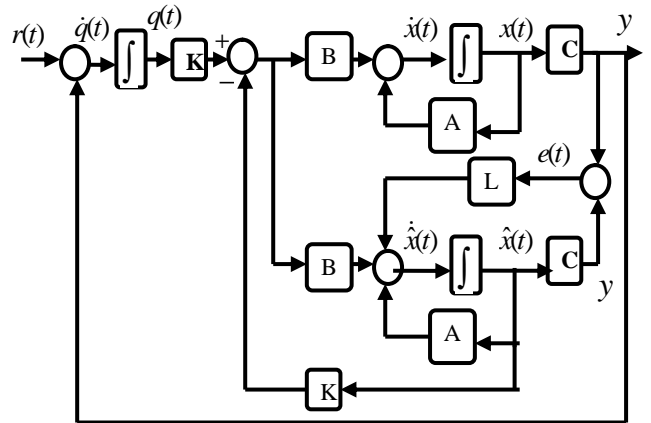


Fig. 3 The Block Diagram of Integral State Feedback Controller with Kalman Filter

We want to design a compensator in order to set-point tracking, and Cost Function will be defined by:

$$\int \left( (y(t) - r(t))^T \mathbf{Q} (y(t) - r(t)) + u(t)^T \mathbf{R} u(t) \right) dt \quad (14)$$

$y(t)$ ,  $u(t)$ ,  $r(t)$  have defined already. And  $\mathbf{Q}, \mathbf{R}$  are the weighting matrices,  $\mathbf{R}$  is a positive definitive symmetric matrix and  $\mathbf{Q}$  is a semi-positive definitive symmetric matrix. In this paper Optimal weighting matrixes  $\mathbf{Q}, \mathbf{R}$  will be found by Genetic Algorithms in order to reduce the settling time and overshoot. Consequently this strategy can reduce control

energy signal consumption and make tradeoff between energy consumption and system characteristics [4].

#### IV. GENETIC ALGORITHM APPROACH

The aim of the optimal Linear Quadratic (LQ) tracking is to obtain an optimal control law that minimizes the performance index and forces the plant to track a desired trajectory over a specified time interval. The closed loop performance is related to these weighting matrices. One way to find the proper weighting matrices is a trial- and- error process.

GA as a powerful and broadly applicable stochastic search and optimization techniques is perhaps the most widely known types of evolutionary computation method today [3].

The control law that minimizes Cost Function is given by linear state feedback [5].

$$\mathbf{k} = (\mathbf{R} + \mathbf{B}^T \mathbf{S} \mathbf{B})^{-1} \mathbf{B}^T \mathbf{S} \mathbf{A} \quad (15)$$

$$\mathbf{v}(k) = (\mathbf{I}_{n \times n} - \mathbf{A}^T + \mathbf{A}^T \mathbf{S} \mathbf{B} (\mathbf{B}^T \mathbf{S} \mathbf{B} + \mathbf{R})^{-1} \mathbf{B}^T)^{-1} \mathbf{C}^T \mathbf{Q} r(k) \quad (16)$$

$$\mathbf{u}(k) = (\mathbf{B}^T \mathbf{S} \mathbf{B} + \mathbf{R})^{-1} \mathbf{B}^T (-\mathbf{S} \mathbf{A} x(k) + \mathbf{v}(k)) \quad (17)$$

$$\mathbf{0} = \mathbf{A}^T \mathbf{S} + \mathbf{S} \mathbf{A} - \mathbf{S} \mathbf{B} \mathbf{R}^{-1} \mathbf{B}^T \mathbf{S} + \mathbf{C}^T \mathbf{Q} \mathbf{C} \quad (18)$$

Weighting matrices  $\mathbf{Q}, \mathbf{R}$  are found by GA, and then  $\mathbf{u}$  can be obtained by solving the Riccati equation (18) and obtaining  $\mathbf{v}$  in (16). To apply the GA, we constitute a chromosome with diagonal elements of  $\mathbf{Q}, \mathbf{R}$  matrices.

The population size was chosen to be 50 and the probability of the crossover and mutation are  $P_c=0.8, P_m=0.03$  respectively.

The best weighting matrices are:

$$\mathbf{Q} = \begin{bmatrix} 8.0135 & 0 \\ 0 & 5.2530 \end{bmatrix} \quad \mathbf{R} = [0.00243] \quad (19)$$

Results show that it has good performance, encountering disturbance, noise and parameters variation in the plant. We combined kalman filter with this controller and results show more robustness performance. System performance for designed controllers has shown in Table III.

Also we apply noise and 20% variation in DC motor parameters and compare the results for two controllers in Fig. 5 and Fig. 6. Error between original plant out-put and plant out-put by applying noise and 20% variation for the PID controller is more than second controller.

#### V. CONCLUSION

The design and implementation of a PC-based dc motor velocity system using both special optimal control and PID have been presented in this paper, the simulation results reveal that using second controller to the velocity control application, has shorter settling time, and smaller over shoot amplitude. Comparisons of simulation results show that the integral state feedback controller combined by kalman filter due to better robustness performance than the PID controller.

We find the Cost Function optimal weighting matrices  $\mathbf{Q}$  and  $\mathbf{R}$  by Genetic Algorithm. Simulation results show that by this strategy we have optimal system characteristics (settling time, overshoot amplitude).

According to the simulation results the latest controller has

the better performance encountering with noise and disturbance and parameter variation.

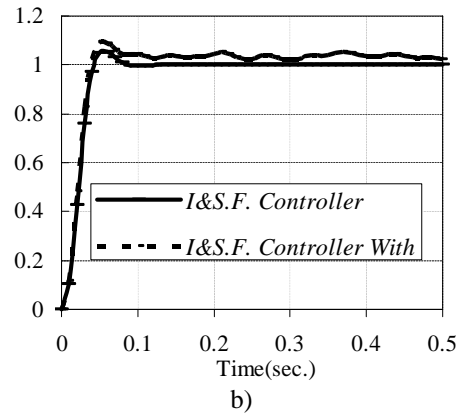
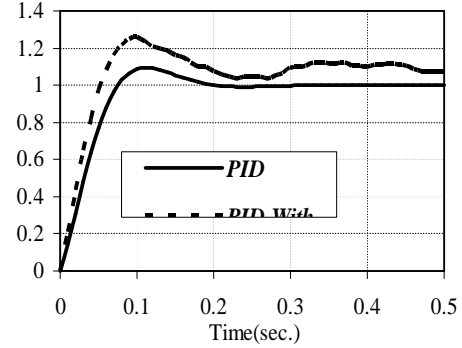


Fig. 4 Step Responses for Second Controller

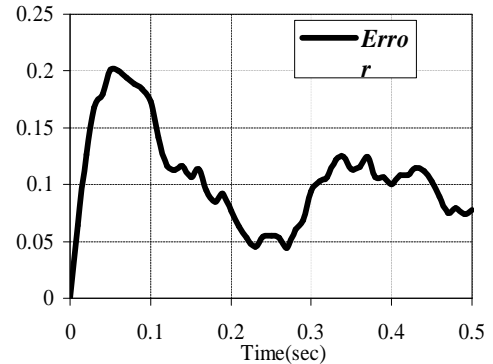


Fig. 5 Errors for PID Controller Applying Noise & 20% Parameter Variation

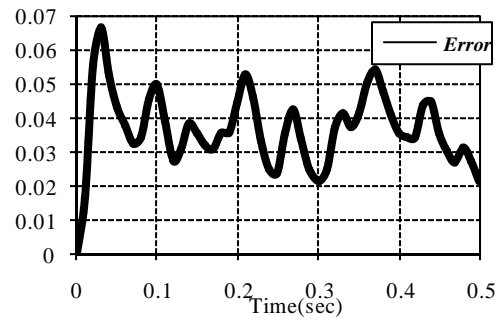


Fig.6 Errors for Second Controller Applying Noise & 20% Parameter Variation

TABLE III: SPEED CONTROLLER PERFORMANCE

	PID Controller	I&S.F. Controller
Settling Time	0.176 (sec)	0.0748 (sec)
Over Shoot	9.47%	5.16%

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