State estimation and state feedback control for continuous fluidized bed dryers

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Abstract

Moisture content is one of the key controlled variables in drying processes. However, this variable cannot or is difficult to be measured directly and it is often an inferred quantity based on experience. Therefore, there is a need to design a state observer to estimate the moisture content on-line for the purpose of direct control of drying product quality.

A linear state space dynamic model is used to describe drying in continuous fluidized bed dryers. The estimation technique based on Kalman filter design is used to provide state estimates for an optimal state feedback control system. The filter shows acceptable performance in reducing the noises present in the system and in converging to the actual states from incorrect initial states. Also, state feedback controller shows acceptable performance in tracking set points changes when using either actual states or estimated ones.

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1. Introduction

Control of drying processes is concerned mainly with maintenance of desired product moisture content by manipulation of heating rate and solids feed flow rate despite disturbances in the drying operation such as changes in ambient air temperature, ambient air humidity, and variations in feed supply and composition. The lack of direct, on-line and reliable methods for sensing product moisture content constitutes a major problem in developing new control strategies for drying processes. Direct control and on-line measurement of solids moisture content would enable significant improvement in dryer control.

In general, it is difficult to make on-line measurement of product moisture content. Moreover, suitable moisture sensors are not available widely in drying plants because they are expensive and/or have low reliability and often produce significant amounts of measurement noise (Jumah, Mujumdar, & Raghavan, 1995). Therefore, the moisture content of the dried product is often be inferred from the temperature and humidity of the exhaust gas. Several experimental investigations have produced empirical correlations for the determination of material moisture content (e.g., Agness & Isaacs, 1966; Alden, Torkington, & Strutt, 1988; Harber, 1974). These methods are based on deriving correlations, which relate solid moisture content to both exit air humidity and temperature (wet and dry bulb temperatures) in order to infer the value of particles moisture content. Nevertheless, the resulted correlations are limited to the studied cases and it is not applicable to other systems. In addition, due to the weak correlation between the exit air
temperature and the product moisture content, using indirect (inferential) control usually results in poor control of the drying process (Jumah et al., 1995).

Advanced control design methods such as state feedback controls, which use the states in their control laws are designed under the assumption that all state variables are accessible for measurement. In practical applications, this assumption may not be realistic since it is either impossible or too expensive to measure all the state variables. In such cases, a state observer is needed to provide “acceptable” estimates of the process states, using only the available measurements, for use in implementation of a control law feedback.

Motivated by the above considerations, the state estimation techniques should be used to estimate the unmeasured state variables and to reduce the effect of measurement and/or process noises. The most popular estimation technique for a continuous process is Kalman filter. Since the publication of the classic papers by Kalman (1960) and Kalman & Bucy (1961), the application of the Kalman filter and its extended design to chemical processes has received extensive study (e.g., Dimtratos, Georgakis, El-Aasser, & Klein, 1988; Hamilton, Seborg, & Fisher, 1973; Seinfeld, 1970; Quinterno-Marmol, Luyben, & Georgakis, 1991; Tatiraju & Soroush, 1997). In drying control, reliable sensors have been constructed for direct measurement of material moisture content, but they are restricted to a small class of agricultural products such as grains (e.g. Eltitani & Bakker-Arkema, 1987). Recently, Kiranoudis, Dimitratos, Maroulis, & Marinos-Kouris (1993) designed hybrid extended Kalman filter that employs both continuous and discrete time rules to estimate moisture content in a batch dryer of food materials. A quite satisfactory result for moisture estimation was obtained from the comparison of filter estimation and a laboratory dryer.

The problem of designing control system for continuous fluidized bed dryers was considered earlier by Abdel-Jabar, Jumah, & Al-Haj Ali (2002a). A continuous transfer functions model and equivalent state space models were derived via system identification. The derived models were used to develop model-based control algorithms such as Internal Model Control (IMC) and Model Predictive Control (MPC). Performance and robustness properties of these controllers were analyzed. However, it was assumed that the moisture content is available as an output measured variable. Also, it was assumed that process noises and measurement noises are not present. As such, these assumptions limit the applicability of the state feedback control algorithms on real drying processes.

Therefore, the purpose of this study is to present a methodology for on-line estimation of material moisture content based on Kalman filter design. In addition, closed-loop simulations are performed in order to evaluate the effectiveness of the state estimator in state feedback control.

2. Drying model

A rigorous nonlinear dynamic model for a continuous fluidized bed dryer (FBD) has been developed earlier by Abdel-Jabar, Jumah, & Al-Haj Ali (2002b). The model was based on combining the drying kinetics for diffusion-controlled systems and residence time density function with no adjustable parameters. Following that, the model was utilized in performing step testing to generate input–output dynamic data.

Then, the input–output data were used to derive reduced-order linear models for FBD via system identification (Abdel-Jabar et al., 2002a). The derived continuous transfer function model, which relates the
product moisture content \((M)\), and product temperature \((T)\) as controlled output variables to the solid flow rate \((G_s)\) and the inlet gas temperature \((T_g)\) as the manipulated input variables, and to the inlet moisture content \((M_i)\) and inlet air humidity \((Y_i)\) as disturbance (load) variables, is given as

\[
\begin{bmatrix}
M \\
T
\end{bmatrix} = \begin{bmatrix}
0.934e^{-50} & -1.88 \times 10^{-3} \\
8.22 \times 10^3 + 1 & 2.31 \times 10^3 + 1 \\
-4.62 \times 10^2 & 7.52 \times 10^{-1} \\
1.38 \times 10^5 + 1 & 2.06 \times 10^3 + 1 \\
\end{bmatrix} \begin{bmatrix}
G_s \\
T_g
\end{bmatrix}
\begin{bmatrix}
1.87 \times 10^{-2} \\
8.7 \times 10^3 + 1 \\
-90 \\
2.71 \times 10^5 + 1
\end{bmatrix} \begin{bmatrix}
M_i \\
Y_i
\end{bmatrix}
\]

(1)

The above model has shown to be adequately representing the process dynamics. An equivalent state space model was then obtained from the above continuous transfer function model using the state space realization routine available in MATLAB.

3. Observer design

The state estimator (observer) is based on Kalman filter design. In this filter, the process and measurement noise are assumed to be white Gaussian with zero mean. Also, this filter uses continuous-time models of dynamics and observation. The mathematical formulation of this estimator is well-known and therefore only a summary will be presented here. Consider a linear, time-invariant, state space process model of the form

\[
\begin{align*}
\dot{x} &= Ax + Bu + \phi d + \zeta w \\
y &= Cx + \eta
\end{align*}
\]

(2)

here \(x\) is \(n \times 1\) state vector, \(u\) is \(m \times 1\) manipulated inputs vector, \(d\) is \(d \times 1\) disturbances vector, \(y\) is \(p \times 1\) outputs vector, \(w\) is the \(nv \times 1\) process noise vector and \(\eta\) is the \(p \times 1\) measurements noise vector. \(A, B, C, \phi, \text{ and } \zeta\) are constant matrices of appropriate dimensions. It is assumed that \(w\) and \(\eta\) are white Gaussian noises with zero means and have covariance matrices of \(Q (nv \times nv)\) and \(R (p \times p)\), respectively. In practice, these covariance matrices are taken to be diagonal positive definite, both for convenience and for lack of information regarding covariance.

Before proceeding in observer design, a very important issue must be considered, which is the observability. To be able to design an observer for a system, the system has to be observable. A linear system is observable if the observability matrix

\[
\Xi = \begin{bmatrix}
C & CA & CA^2 & \cdots & CA^{n-1}
\end{bmatrix}^T
\]

be of rank \(n\). If the observability condition is not satisfied, this means that unobservable states are present. To overcome this obstacle, state space realization can be employed. Realization is defined as the removal of unnecessary states without affecting the input/output relation.

After checking system observability, Kalman filter can be designed. This observer has the form of linear observers (Åström & Wittenmark, 1990)

\[
\dot{x} = Ax + Bu + \phi d + L(y - Cx)
\]

(4)

where \(\dot{x}\) is the estimated state vector (\(n\)-vector).

The problem of designing this observer becomes merely that of determining the observer gain matrix \(L\), such that the observer error dynamics defined by

\[
\dot{e} = (A - LC)e + \zeta w - L\eta
\]

(5)

where \(e = x - \hat{x}\) is the \(n \times 1\) error vector, is asymptotically stable with sufficient speed of response (Ogata, 1997). The asymptotic stability and speed of response of the error dynamics are determined by the eigenvalues \((\lambda)\) of

\[
A_c = A - LC
\]

(6)

such that they lie to the left of the imaginary axis. The gain matrix can be selected in various ways. If the system is observable and linear, it is theoretically possible to place the eigenvalues of the matrix in Eq. (6) at any desired location. This method is referred to as a pole placement. Although this approach is a convenient way of selecting the control gains, it may not always be satisfactory, for several reasons. One reason is that a good set of pole locations is not always obvious. Also, in any system with more than one input, pole placement does not yield a unique solution for the gain matrix. Moreover, the design that results when the poles are placed at arbitrary locations may not be robust with respect to variation of the loop gain.

To alleviate these deficiencies, an alternative approach based on quadratic optimization can be used. In this approach, the dynamics of the observer is optimized in a statistical sense to result in the well-known Kalman filter. Referring to the above error dynamics Eq. (5), let \(e_w(t)\) be the estimation error induced by a unit impulse process noise \((w(t) = \delta(t))\), and let \(e_d(t)\) be the estimation error induced by a unit impulse measurement noise \((\eta(t) = \delta(t))\). Then a reasonable optimization criterion would be a weighted sum of the “sizes” of \(e_w\) and \(e_d\). For the above process model, the following quadratic performance index is selected (Kravaris, 1995):

\[
J = \int_0^{\infty} (e_w^T Q e_w + e_d^T R e_d) dt
\]

(7)

The solution of this optimization problem is given by

\[
L = PC^T R^{-1}
\]

(8)
where $P$ is a positive definite matrix that satisfies the algebraic Riccati equation

$$\mathbf{AP} + \mathbf{PA}^T - \mathbf{PC}^T\mathbf{R}^{-1}\mathbf{CP} + \mathbf{Q}\zeta^2 = 0 \quad (9)$$

It should be noted that a general dynamic matrix Riccati equation of the form

$$\frac{d\mathbf{P}}{dt} = \mathbf{AP} + \mathbf{PA}^T - \mathbf{PC}^T\mathbf{R}^{-1}\mathbf{CP} + \mathbf{Q}\zeta^2$$

can be obtained if the above optimization problem is solved over a finite time instead of infinite time as given in Eq. (7). However, we are interested here in the infinite time horizon solution.

For the observer design, diagonal $Q$ and $R$ matrices were assumed with each matrix having equal diagonal elements such that (Hamilton et al., 1973);

$$Q = qI_{nw} \quad \text{and} \quad R = rI_p \quad (11)$$

where $I_{nw}$ and $I_p$ are identity matrices of dimensions $nw \times nw$ and $p \times p$, respectively. The matrices $Q$ and $R$ must be chosen in order to achieve good filter performance and tracking capabilities.

From the above, it is clear that for linear systems, the gain $L$ does not depend on the state estimates, and therefore, it can be computed off-line before the filter is implemented and stored in the filter’s memory. Here, we will consider a full-order state observer in which the order of the state observer is the same as that of the system, i.e., the state estimator observes all state variables of the system.

4. State feedback control design

The Kalman filter designed here generates estimates for the unknown states and outputs. Thus, different advanced control strategies can be designed and implemented with Kalman filter. One possible strategy is to use state feedback controller with the following law (Kravaris, 1995):

$$\mathbf{u} = \mathbf{K}_{sp}\mathbf{y}_{sp} - \mathbf{K}_{FB}\mathbf{x} \quad (12)$$

where $\mathbf{K}_{sp}$ is chosen for unity steady-state gain between $\mathbf{y}_{sp}$ and $\mathbf{y}$ as follows:

$$\mathbf{K}_{sp} = \frac{-1}{\mathbf{C}(\mathbf{A} - \mathbf{BK}_{FB})^{-1}\mathbf{B}} \quad (13)$$

here, $\mathbf{K}_{FB}$ is a constant (or time-varying) gain matrix that can be obtained by either pole placement or the optimization of certain cost function. For the optimization approach, the control is sought which gives the best trade-off between performance and cost of control. A standard form of the optimal control for state feedback control design is to seek the control which minimizes the value of a performance index $J_{FB}$ of the form (Kravaris, 1995)

$$J_{FB} = 0.5 \int_{0}^{\infty} (\mathbf{x}^T\mathbf{Q}_{FB}\mathbf{x} + \mathbf{u}^T\mathbf{R}_{FB}\mathbf{u})dt \quad (14)$$

$\mathbf{Q}_{FB}$ and $\mathbf{R}_{FB}$ are weighting matrices and usually diagonal with unity elements. The solution of this optimization problem is given by

$$\mathbf{K}_{FB} = \mathbf{R}_{FB}^{-1}\mathbf{B}^T\mathbf{p} \quad (15)$$

again $\mathbf{P}_{FB}$ is a positive definite matrix that satisfies the algebraic Riccati equation

$$\mathbf{A}^T\mathbf{P}_{FB} + \mathbf{P}_{FB}\mathbf{A} - \mathbf{P}_{FB}\mathbf{BR}_{FB}^{-1}\mathbf{B}^T\mathbf{P}_{FB} + \mathbf{Q}_{FB} = 0 \quad (16)$$

5. Observer-state feedback design

A block diagram representation of a general observer-state feedback is depicted in Fig. 1. The dynamics of the observed-state feedback control system equations (2), (5), (6) and (12) are combined to give the overall closed-loop system

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{e}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{BK}_{FB} & \mathbf{BK}_{FB} \\ 0 & \mathbf{A} - \mathbf{LC} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{e} \end{bmatrix} + \begin{bmatrix} \mathbf{BK}_{sp} & \phi & \zeta & 0 \\ 0 & 0 & \zeta & -\mathbf{L} \end{bmatrix} \begin{bmatrix} \mathbf{y}_{sp} \\ \mathbf{d} \\ \mathbf{w} \\ \mathbf{\eta} \end{bmatrix}$$

$$\mathbf{y} = \begin{bmatrix} \mathbf{C} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{e} \end{bmatrix} + \mathbf{\eta}$$

The above combined regulator–observer design is based on the separation principle (Friedland, 1996), by which control system is designed in two independent phases, which are

1. Design a full state feedback for the subsystem $(\mathbf{A} - \mathbf{BK}_{FB})$ and
2. Design an observer for the subsystem $(\mathbf{A} - \mathbf{LC})$.

That is to design two separate subsystems and then put them together such that the poles (eigenvalues) of the

![Fig. 1. A block diagram representation of a general observer-state feedback.](image-url)
the closed-loop system comprise the poles of the observer and the poles that would be present if full state feedback were implemented. In other word, the stability of the resulting closed-loop system will be guaranteed by designing stable state feedback and observer dynamics. However, this separation property holds only when the model of the plant used in implementing the observer is a faithful model of the physical plant (Friedland, 1996).

6. Results and discussion

The state space model which was obtained from Eq. (1) via state space realization comprises of fifteen state variables \( (x) \), four inputs (manipulated and disturbances) \( (u \) lumped with \( d) \), and two outputs \( (y) \). It should be noticed here that these states are not actual states for the drying system. Rather, these states are just artificial ones and their output mapping matrix \( C \) gives the actual output variables \( (M \) and \( T) \). To ensure the observability of the system (Kalman, 1960), the minimal realization function in MATLAB was employed to remove excess states. Here, it was found that three states must be removed. As such, the drying state space model has 12 states, 4 inputs, and 2 output variables with a state matrix \( A \) of dimension \( (12 \times 12) \), an input matrix \( B \) lumped with \( d \) of size \( (12 \times 4) \), and an output map matrix \( C \) of size \( (2 \times 12) \).

The systems simulated in this study are divided into three types. The first system consists of Kalman filter and the dryer model. The second one consists of state feedback controller and the dryer model. The last one consists of Kalman filter, state feedback controller and the dryer model.

In this work, a full-order observer design is considered. The Kalman filter gain matrix \( L \) is uniquely determined by the solution of Eqs. (7)–(9), for the specified values of \( Q \) and \( R \) matrices. Gaussian noise sequences with zero means and standard deviations of 0.3 were used in all simulation runs. For all runs, \( q = r = 0.09 \) is used in Eq. (11). Table 1 lists the observer gain matrix with the associated observer poles (eigenvalues). It is noticed that all computed observer eigenvalues are negative, and hence, the error dynamics of the observer is asymptotically stable.

Simulation of the observer dynamics is performed in order to evaluate the effectiveness of the observer with respect to reducing the effect of noises in the system and its ability to accurately estimate solid moisture content with incorrect initial state estimates. The effectiveness of the Kalman filter in providing state estimates of the moisture content from noisy measurements is illustrated by Fig. 2. The plot indicates that the noise level can be dramatically reduced. In addition, the state estimates from the observer are very close to the actual states and represent a considerable improvement over the unfiltered values.

Table 1
Observer design parameters

<table>
<thead>
<tr>
<th>( L )</th>
<th>( \lambda(A - LC) )</th>
</tr>
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<tr>
<td>[-1.7895, -2.9682]</td>
<td>[-1.0003, -0.8944]</td>
</tr>
<tr>
<td>-0.4935, -0.7734</td>
<td>-0.4306</td>
</tr>
<tr>
<td>-0.6549, -1.0805</td>
<td>-0.0674</td>
</tr>
<tr>
<td>-0.3613, -0.5174</td>
<td>-0.0166</td>
</tr>
<tr>
<td>-0.0094, -0.0125</td>
<td>-0.013</td>
</tr>
<tr>
<td>-0.0791, -0.0907</td>
<td>-0.0049</td>
</tr>
<tr>
<td>0.00210, 0.00780</td>
<td>-0.0039</td>
</tr>
<tr>
<td>-0.3104, -0.3404</td>
<td>-0.0008</td>
</tr>
<tr>
<td>0.00190, 0.00170</td>
<td>-0.0006</td>
</tr>
<tr>
<td>0.01180, 0.17100</td>
<td>-0.0006</td>
</tr>
<tr>
<td>0.03960, 0.03570</td>
<td>-0.0006</td>
</tr>
<tr>
<td>-0.0026, -0.6399</td>
<td>-0.0006</td>
</tr>
</tbody>
</table>

Fig. 2. The effectiveness of the Kalman filter in providing state estimates of the moisture content from noisy measurements.

Fig. 3. Observer estimates for incorrect initial estimates.
Fig. 3 depicts the effect of an inaccurate initial state estimate on the performance of the observer. Clearly, the outlet solid moisture content ($M$) estimate converges to the actual values from an incorrect initial state estimates above the true values. However, it can be seen that the error dynamics response of the observer is rather slow which is due to the conservative values of the observer gain in order to ensure stability and robustness (i.e., less sensitivity to modeling errors) characteristics. A faster error dynamics response can be obtained but at the expense of less stability and more sensitivity to modeling errors.

Initially, a full state feedback controller, which utilizes the actual states, is simulated with the dryer model. The optimal feedback gain matrix $K_{FB}$ used in this work is given in Table 2. The closed-loop time responses due to setpoint step changes in the product temperature ($T$) and moisture content ($M$) are shown respectively in Figs. 4 and 5. Both responses are shown to be stable.

However, the temperature response is rather faster than the moisture one, where the later takes about 100 min to achieve steady state compared to 10 min for the temperature response.

Finally, a closed-loop simulation run is performed in order to evaluate the effectiveness of using Kalman filter in feedback control systems, that is, the combined regulator–observer design. The state estimates produced by Kalman filter are used in the multivariable control law (Eq. 12) instead of the actual values. The dashed line in Fig. 5 represents the closed-loop simulation of the coupled regulator–observer system in the presence of inaccurate initial state estimates using the gain matrix $K_{FB}$ given in Table 2. Clearly, the performance of using the observer in the state feedback control scheme (dashed line) provides satisfactory control performance for the moisture content control compared to the closed-loop response using actual states (solid line). Also, it should be noted here that the state feedback control design parameters

<table>
<thead>
<tr>
<th>$K_{FB}$</th>
<th>0 0.0011 0.0161 0 0.0240 0 0.2696 0 -0.3783 0.0002 0.4286</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 0.0001 -0.0066 0 -0.0011 0 -0.0121 0 0.0139 0.0001 -0.0234</td>
</tr>
</tbody>
</table>

\[
\lambda_{FB}(A - BK_{FB}) =
\begin{bmatrix}
-1.0003 \\
-0.8459 \\
-0.0674 \\
-0.0322 \\
-0.0167 \\
-0.0133 \\
-0.0048 \\
-0.0037 \\
-0.0005 \\
-0.0006 \\
-0.0006 \\
-0.0006
\end{bmatrix}
\]

Table 2

State feedback control design parameters

![Fig. 4. State feedback control response due to a set point change in product temperature.](image)

![Fig. 5. State feedback control response due to a set point change in product moisture content.](image)
control of drying provides smooth setpoint tracking capabilities with no overshoot compared to the IMC-PID output feedback control performance reported in Abdel-Jabar et al. (2002a).

7. Conclusions

Drying processes represent a difficult control problem, since the material moisture content can not be measured on-line using economical sensors and the available measurements contain significant amounts of random noise. To solve these problems a Kalman filter has been designed. Performance studies show that Kalman filter provides satisfactory estimates even in the presence of significant noise levels, and inaccurate initial states fed to the observer. In addition, state feedback controller is designed. Closed-loop simulation runs show that the state estimates calculated by Kalman filter can be used efficiently in controlling outlet solid moisture content.

Acknowledgments

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