Robust Multivariable PI Control: Applications to Process Control

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Abstract: We consider the application of a recently developed "conditional integrator" technique for the output regulation of a class of nonlinear systems to process control systems. For a special choice of the controller parameters, our controller reduces to a finely tuned saturated PI/PID type controller with an anti-windup structure. This is of particular significance because while the application of nonlinear control strategies such as feedback linearization, adaptive/neural control and nonlinear model predictive control has reached considerable maturity for process systems, industrial practice has traditionally relied on PI/PID controllers. With our design, we provide both global regulation results (under state-feedback and when the control is not required to be constrained), and regional/semi-global results with saturated control and under output feedback. When applied to different process control problems, our simulation results show that good tracking performance is achieved, in spite of partial knowledge of the plant parameters.

Keywords: Process Control; Output Regulation; Integral Control; Sliding-mode Control; Output feedback.

1. INTRODUCTION

In the past decade and a half, the analysis and application of nonlinear control strategies for process systems has reached considerable maturity. Control of chemical reactors under varying assumptions on the class of systems, available measurements, and control objectives has been widely studied, and a variety of techniques, including feedback linearization, sliding mode control, adaptive/neural control and nonlinear model predictive control; see, for example, Alvarez-Ramirez and Morales [2000], Alvarez-Ramirez et al. [1998], Antonelli and A.Astolfi [2003], Daoutidis and Kravaris [1992], Daoutidis et al. [1990], Fradkov et al. [1997], Jadot [1996], Jadot et al. [1999], Kosanovich et al. [1995], Kurtz et al. [2000], Maner et al. [1996], Valluri et al. [1998], Veil and Bastin [1997], Veil et al. [1997] and the references therein. An early survey/review of nonlinear control of chemical processes can be found in Bequette [1991], and a more recent/comprehensive overview in the research monograph Henson and Seborg [1997].

Some of the issues that have been treated in detail in the above cited literature include (i) uncertainties in the process kinetics, (ii) presence of input constraints, i.e., control saturation, and (iii) the available measurements (for example, it is well-known that in industrial practice, it is easy to measure temperatures, but usually not concentrations Antonelli and A.Astolfi [2003], Kurtz et al. [2000]). In spite of the wide variety of methodologies, in virtually all present day industrial applications the problem is efficiently solved using PI controllers. The stabilization of chemical reactors by output feedback with PI-type controllers has been reviewed and treated in detail in the Ph.D. thesis of Jadot Jadot [1996]. A robust control scheme in the face of uncertain kinematics for a class of CSTRs has been proposed by Alvarez-Ramirez *et al.* Alvarez-Ramirez et al. [1998], where it has been shown that the proposed controller has the structure of PI control. A more recent result, which goes beyond just the analysis of closed-loop stability, and focuses on transient performance, has been discussed in Alvarez-Ramirez *et al* Alvarez-Ramirez and Morales [2000].

In a recent work Seshagiri and Khalil [2005], we developed a new "conditional integrator" approach to the design of robust output regulation for multi-input multi-output (MIMO) minimum phase nonlinear systems transformable into the normal form, uniformly in a set of constant disturbances and uncertain parameters. Analytical results for regional as well as semi-global/global stability results of the design, as well as performance improvement over conventional integral control techniques were reported, both in the state-feedback and output-feedback cases. It was also shown that for a special choice of the parameters, the control is a "PID$^{\rho-1}$ controller" (ρ being the relative degree, so that for $\rho = 1, 2$, we have a PI/PID controller respectively), with a conditional (anti-windup) integrator, followed by saturation. In light of the observation on the ubiquitous use of PI/PID type controllers in industrial practice, and on account of the below mentioned advantages

- simple structure, i.e., computational simplicity,
- constraint handling capability,
- robustness to parameter uncertainty and certain classes of external disturbances, and
- non-requirement of the full state (i.e., output-feedback control design)

we believe our method is especially suited to actual implementation. To that end, we consider the application of our technique in Seshagiri and Khalil [2005] to the control of continuous-stirred tank reactors (CSTRs), and show, via simulation, the performance of the proposed method.

The rest of this paper is organized as follows. While the theoretical part of our work was presented in Seshagiri and Khalil [2005], for the sake of readability and completeness, we abstract our work in Seshagiri and Khalil [2005] in Section 2, where we briefly describe the system under consideration, assumptions and control objective, and present our control design and main analytical results. The application to a single-input single-output (SISO) relative degree two CSTR example is presented in Section 3 [1]. Section 4 discusses our results for a MIMO example, and finally, our conclusions are presented in Section 5.

2. SYSTEM DESCRIPTION AND CONTROL DESIGN

Consider a MIMO nonlinear system with uniform vector relative degree $\rho = \{\rho_1, \rho_2, \ldots, \rho_m\}$, transformable to the following "error normal form"

$$\left.\begin{aligned} \dot{z} &= \phi(z, e+\nu, d) \\ \dot{e}^i &= A_i e^i + B_i\, [b_i(z, e+\nu, d) - r_i^{(\rho_i)} \\ &\quad + \sum_{j=1}^{m} a_{ij}(z, e+\nu, d)(u_j + \delta_j(z, e+\nu, d, \tilde{w}))] \end{aligned}\right\} \quad (1)$$

where $e^i \in R^{\rho_i}$ is vector of the error in the output y_i and its derivatives up to order $\rho_i - 1$, z the "internal dynamics", $r_i^{(\rho_i)}(t)$ is the ρ_ith derivative of the ith component of the reference, $d = (r_{ss}, \theta, w_{ss})$, $r_{ss} \in R^m$ is the steady-state vector of reference values, θ a vector of unknown constant parameters that belongs to a compact set Θ, w_{ss} is the steady-state exogenous signal, $\nu(t)$ and $\tilde{w}(t)$ are deviations of the reference and exogenous signal respectively from their steady-state values, and the pair (A_i, B_i) is a controllable canonical form that represents a chain of ρ_i integrators.

We design the control u to regulate the error e to zero and then rely on a minimum-phase-like assumption ([Seshagiri and Khalil, 2005, Assumption 4]) to guarantee boundedness of z. Our design is based on combining integral action with a continuous version of sliding mode control (SMC). In the state-feedback case, we define the ith "sliding surface" as

$$s_i = k_0^i \sigma_i + \sum_{j=1}^{\rho_i - 1} k_j^i e_j^i + e_{\rho_i}^i \quad (2)$$

where σ_i is the output of

$$\dot{\sigma}_i = -k_0^i \sigma_i + \mu_i \text{ sat}\left(\frac{s_i}{\mu_i}\right), \ \sigma_i(0) \in [-\mu_i/k_0^i, \mu_i/k_0^i] \quad (3)$$

where $k_0^i > 0$, and the positive constants $k_1^i, \cdots, k_{\rho_i - 1}^i$ are chosen such that the polynomial

$$\lambda^{\rho_i - 1} + k_{\rho_i - 1}^i \lambda^{\rho_i - 2} + \cdots + k_1^i$$

is Hurwitz, and μ_i a small positive parameter, which denotes the width of the ith boundary layer. The relation of (3) to integral control is explained in Seshagiri and Khalil [2005].

[1] Preliminary results for a relative degree one example were presented in Seshagiri and Khalil [2001].

The control is taken as

$$\left.\begin{aligned} u &= \hat{A}^{-1}(e, \nu)[-\hat{F}(e, \nu, \varpi) + v], \\ v_i &= -\beta_i(e, \nu, \varpi) \text{ sat}(s_i/\mu_i) \end{aligned}\right. \quad (4)$$

where $\hat{A}$ is a known nonsingular matrix such that

$$A(z, e+\nu, d) = \{a_{ij}(\cdot)\} = \Gamma(z, e+\nu, d)\hat{A}(e, \nu)$$

and $\Gamma = \text{diag}[\gamma_1, \cdots, \gamma_m]$, with $\gamma_i(\cdot) \geq \gamma_0 > 0, \ 1 \leq i \leq m$, for some positive constant γ_0, $\hat{F}(e, \nu, \varpi)$ is chosen to cancel any known nominal terms in $\dot{s}$, and v to bound the remaining terms in $\dot{s}$. The choice of the functions β_i is specified in [Seshagiri and Khalil, 2005, Section 4.1].

The control (4) can be extended to the output-feedback case by replacing e_j^i, the $(j-1)^{th}$ derivative of e_1^i, by its estimate $\hat{e}_j^i$, obtained using the high-gain observers (HGOs)

$$\left.\begin{aligned} \dot{\hat{e}}_j^i &= \hat{e}_{j+1}^i + \alpha_j^i (e_1^i - \hat{e}_1^i)/(\epsilon_i)^j, \ 1 \leq j \leq \rho_i - 1 \\ \dot{\hat{e}}_{\rho_i}^i &= \alpha_{\rho_i}^i (e_1^i - \hat{e}_1^i)/(\epsilon_i)^{\rho_i} \end{aligned}\right\} \quad (5)$$

where $\epsilon_i > 0$, and the positive constants α_j^i are chosen such that the roots of $\lambda^{\rho_i} + \alpha_1^i \lambda^{\rho_i - 1} + \cdots + \alpha_{\rho_i - 1}^i \lambda + \alpha_{\rho_i}^i = 0$ have negative real parts.

The parameters μ_i result from replacing an ideal SMC with its continuous approximation, and hence should be chosen "sufficiently small" to recover the performance of the ideal SMC. Similarly, in order for the output-feedback controller to recover the performance under state-feedback, the high-gain observer parameters ϵ_i should also be chosen "sufficiently small". Therefore, one might view μ_i and ϵ_i as tuning parameters and first reduce μ_i gradually until the transient response of the partial state feedback control (4) is close enough to ideal SMC that does not contain an integrator, and then reduce ϵ_i gradually until the transient response under output feedback is close enough to that under state feedback. The asymptotic results of Seshagiri and Khalil [2005] guarantee that this tuning procedure will work. Both regional as well as semi-global results for error convergence under output-feedback are given in [Seshagiri and Khalil, 2005, Theorem 1], while analytical results showing the "closeness of trajectories"' of the output-feedback continuous SMC to a state-feedback ideal (discontinuous) SMC (without integral control) are provided in [Seshagiri and Khalil, 2005, Theorem 2] [2].

For SISO systems, the flexibility that is available in the choice of the functions $\hat{F}$ and β can be exploited to simplify the controller (3) to

$$u = -k \text{ sat}\left(\frac{k_0\sigma + k_1 e_1 + k_2 e_2 + \cdots + e_\rho}{\mu}\right) \quad (6)$$

This particular design, while having a simple structure, is also natural if the control is required to be bounded. Since, from (6), derivatives of the error up to order $\rho - 1$ appear in the control, the controller is a "PID$^{\rho-1}$ controller" with anti-reset windup, and followed by saturation (see [Seshagiri and Khalil, 2005, Section 6]). In the case of relative degree $\rho = 1$ and $\rho = 2$, the controller (6) is simply a specially tuned saturated PI/PID controller with anti-windup.

[2] Global results for error convergence and closeness to ideal SMC in the state-feedback case were given in Seshagiri and Khalil [2002].

3. SISO CSTR

Our first example involves the following multicomponent isothermal liquid-phase kinetic sequence carried out in a CSTR [Scaratt et al., 2000, Henson and Seborg, 1997, Chapter 3]:

$$A \rightleftharpoons B \rightarrow C$$

The desired product concentration is component C, and the manipulated input is the feedflow rate of the component B. The dimensionless mass balances for A, B and C are given by the following third-order nonlinear differential equation

$$\begin{aligned} \dot{x}_1 &= 1 - (1 + D_{a1})x_1 + D_{a2}x_2^2 \\ \dot{x}_2 &= -x_2 + D_{a1}x_1 - D_{a2}x_2^2 - D_{a3}x_2^2 + u \\ \dot{x}_3 &= -x_3 + D_{a3}x_2^2 \\ y &= x_3 \end{aligned} \tag{7}$$

where

- x_1: normalized concentration $\frac{C_A}{C_{AF}}$ of species A
- x_2: normalized concentration $\frac{C_B}{C_{AF}}$ of species B
- x_3: normalized concentration $\frac{C_C}{C_{AF}}$ of species C
- C_{AF}: feed concentration of species A ($\mathrm{mol} \cdot m^-1$)
- u: ratio of the per-unit volumetric molar feed rate of species B, denoted by N_{BF} and the feed concentration C_{AF}
- F: volumetric feed rate ($m^3 s^{-1}$)
- V: volume of the reactor (m^3)
- k_i: first order rate constants (s^{-1})

and the D_{ai} terms are the respective Damkőhler terms for the reactions, defined by $D_{a1} = k_1 V/F$, $D_{a2} = k_2 V C_{AF}/F$, and $D_{a3} = k_3 V C_{AF}/F$. The operating region is the orthant $D_x = \{x \in R^3 | x_i > 0\}$, and it is easily verified that the system has relative degree $\rho = 2$, uniformly in D_x, and that for each constant desired $y = \bar{y}$, the system has a unique equilibrium point $x = \bar{x}$, and an equilibrium input $u = \bar{u}$, at which $y = \bar{y}$.

As previously stated, the control objective is to regulate y at a desired constant value by manipulating the normalized feedrate u. Similar to Scaratt et al. [2000], we assume that the Damkőhler coefficients are unknown, and that they constitute the unknown parameter vector θ in (**??**). Note that our system formulation also allows for matched uncertainties that are possibly dependent on the state, the parameter and even time-varying exogenous disturbances.

The parameter dependent change of variables

$$e_1 = x_3 - \bar{y},\ e_2 = \dot{e}_1 = -x_3 + D_{a3}x_2^2,\ z = x_1 - \bar{x}_1$$

transforms the system into the error normal form (1), and it is trivial to verify that the zero dynamics are ISS with e as the driving input, and that furthermore, with $e \equiv 0$, the zero dynamics are simply $z = -(1 + D_{a1})z$, which are exponentially stable.

Equation(6) then is simply the saturated PID controller

$$\begin{aligned} \dot{\sigma} &= -k_0\sigma + \mu \ \mathrm{sat}\left(\frac{k_0\sigma + k_1 e_1 + e_2}{\mu}\right) \\ u &= -k \ \mathrm{sat}\left(\frac{k_0\sigma + k_1 e_1 + e_2}{\mu}\right) \end{aligned} \tag{8}$$

where k_0, and $k_1 > 0$. Also, since e_2 is typically not measured (note that even when x_2 is measured, $e_2 = \dot{e}_1 = -y + D_{a3}x_2^2$ is dependent on the unknown parameter D_{a3}), we replace $e_2 = \dot{e}_1$ with its estimate $\hat{e}_2$, obtained using the high-gain observer

$$\left.\begin{aligned} \dot{\hat{e}}_1 &= \hat{e}_2 + \alpha_1(e_1 - \hat{e}_1)/\epsilon \\ \dot{\hat{e}}_2 &= \alpha_2(e_1 - \hat{e}_1)/\epsilon^2 \end{aligned}\right\}$$

where $\epsilon > 0$, and the positive constants α_1, α_2 are chosen such that the roots of $\lambda^2 + \alpha_1\lambda + \alpha_2 = 0$ have negative real parts. Note that the controller we have is *much simpler* than the ones in Scaratt et al. [2000] [3] that are based on adaptive backstepping. Furthermore, our controller is simply the industrial workhorse PID controller, with the derivative replaced by an estimated derivative, and integrator anti-windup.

For the purpose of simulation and to facilitate comparison with the results of Scaratt et al. [2000], we use the following numerical values specified in that reference: $D_{a1} = 3.0$, $D_{a2} = 0.5$, and $D_{a3} = 1.0$, and consider stabilizing the system at the desired output value $\bar{y} = 0.7753$, for which choices, we have the equilibrium values $\bar{x} = (0.3467, 0.8796, 0.7753)$, $\bar{u} = 1$. We also choose $x(0) = (0.5, 0.5, 0.5)$, $\sigma(0) = 0$, $\hat{e}_1(0) = \hat{e}_2(0) = 0$, $k_0 = 1$, $k_1 = 5$, $k = 2$, $\mu = 0.1$, $\epsilon = 0.1$, $\alpha_1 = 15$, and $\alpha_2 = 50$. Figure 1 shows the error e_1 and the input u for the above numerical values. It is clear from the figure that we achieve regulation in spite of not knowing (or at least not specifically using the knowledge of) the plant parameter values. However, as seen from the figure, there is some kind of "chattering" in the control u, which is a result of the small value of μ. Such a chattering was also observed in the adaptive backstepping sliding mode control (DAB-SMC) controller of Scaratt et al. [2000].

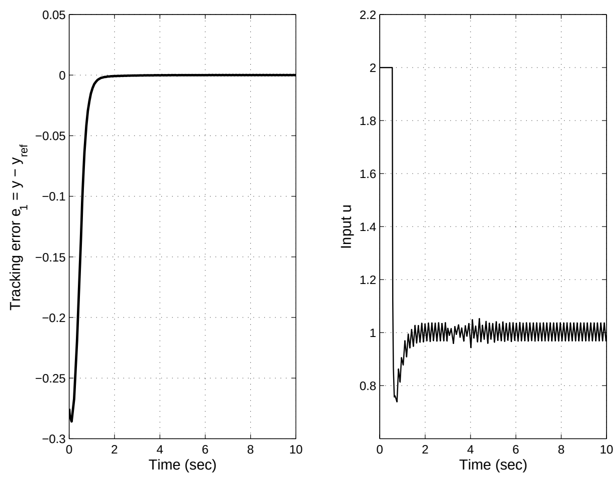

Fig. 1. Output-feedback control with PID controller : Tracking error $e_1 = y - \bar{y}$ and input u

In the absence of integral control, we will need to make μ small if we want small steady-state errors, as the order of

[3] We can also use the more general form of the controller in (4), with the "equivalent control" component $\hat{F}$ chosen to cancel any known/nominal terms, and the "switching gain" $\beta(\cdot)$ chosen possibly as a function of the measures states, exogenous signals and time. As previously mentioned, the choice (8) that we make above is not only simpler, but also in some sense intuitively natural if the control u is bounded in magnitude, in which case we simply choose $\beta(\cdot) = k = u_{max}$, where u_{max} is the maximum available control magnitude.

the regulation order will in general be $O(\mu)$. A smaller μ will thus result in a smaller steady-state error, but at the expense of control chattering. As mentioned in Seshagiri and Khalil [2005], as a consequence of using integral control, we will not require μ to be small in order to reduce the steady-state error, but only small enough to stabilize the disturbance-dependent equilibrium point. To illustrate this, we repeat the previous simulation with $\mu = 1$. Figure 2 shows the result of the simulation, and we see that the steady-state error is still zero on account of integral control, but that now there is no chattering in the control [4]. For comparison, we have also shown the error and input for a continuous SMC without integral action, i.e., $u = -k \text{ sat}\left(\frac{k_1 e_1 + \hat{e}_2}{\mu}\right)$, with the numerical values for k, k_1 and μ the same. Note that there is no chattering in the control (because μ is "not small"), but now the steady-state error is also non-zero. Without integral control, reducing μ will make the steady-state error smaller, but will lead to chattering again for small enough μ. Chattering was also removed in Scaratt et al. [2000] using a second-order sliding mode control(DAB-SOSMC). The results reported above are comparable, i.e., the transient responses are at least as good (for the chosen values), with the ones in Scaratt et al. [2000], where the controllers are "a combination of dynamical adaptive backstepping and sliding mode control of first and second order order", and are considerably more complex than our design. In fact, we claim that the transient response in our design is better than the one in Scaratt et al. [2000] in that the output in our design exhibits no "overshoot"', and moreover, while our control is constrained with $k = 2$, the maximum control value in Scaratt et al. [2000] is roughly four times this value.

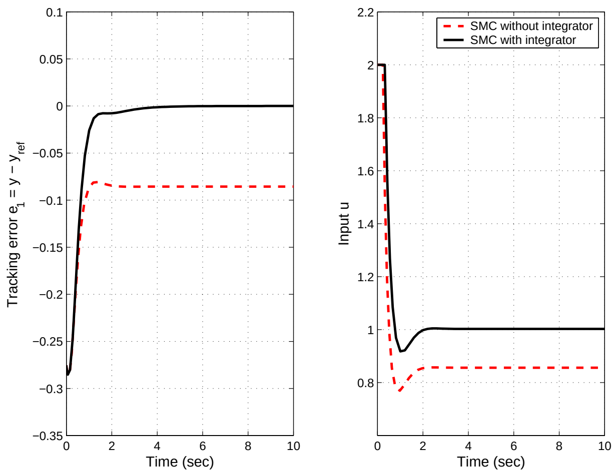

Fig. 2. Effect of increasing μ, no chattering in control but longer "settling time"; non-zero steady-state error without integral control

Finally, in order to illustrate the robustness of our controller to both parameter uncertainties and to matched disturbances, we repeat the previous simulations, but with the numerical values of the Damkőhler coefficients changed to $D_{a1} = 3.5$, $D_{a2} = 0.2$, and $D_{a3} = 1.5$. We also assume that there is an input additive disturbance $\delta(t) = 1.5\sqcap(t - 5)$. All other values are retained from the previous simulation, except $\mu = 0.2$. The results of the simulation are shown in Figure 3, and is clear that good regulation is achieved with the output-feedback controller, in spite of parameter uncertainties and disturbances.

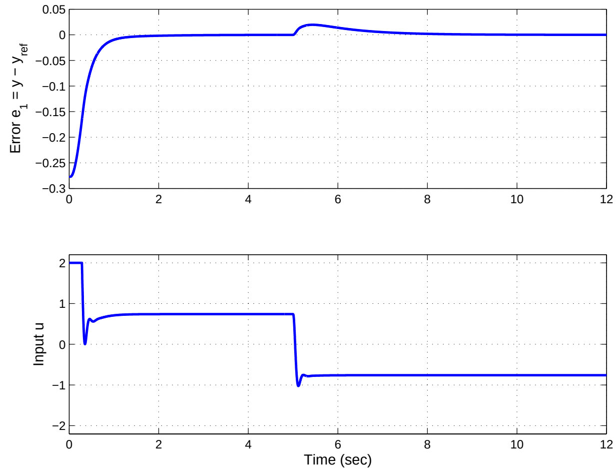

Fig. 3. Effect of uncertainties (parametric and external disturbances) on the response of the output-feedback controller

To emphasize the contribution of our work, we mention again that our controller is simply a saturated PID controller with anti-windup with a special choice of the controller gains, and the simulations above show the robustness to parameter uncertainties and disturbances, with constrained inputs, and only using output feedback.

4. MIMO CSTR

In the previous section, we considered the control of a SISO system. Our next example is the MIMO free radical polymerization of methyl methacrylate in a constant volume exothermic CSTR Adebekun and Schork [1989], Kurtz et al. [2000]. The solvent is ethyl acetate, while the reactor is benzoyl peroxide. As abstracted from Kurtz et al. [2000], the model equations are

$$\begin{aligned}
\dot{M} &= \frac{q}{V}(M_f - M) - k_p M P \\
\dot{T} &= \frac{q}{V}(T_f - T) + \left(\frac{-\Delta H}{\rho c_p}\right) k_p M P - \frac{h A_c}{V \rho c_p}(T - T_c) \\
\dot{I} &= \frac{q}{V}(I_f - I) - k_d I \\
\dot{S} &= \frac{q}{V}(S_f - S)
\end{aligned}$$

where M, T, I and S are respectively the monomer concentration, reactor temperature, initiator concentration and solvent concentration respectively, V is the reactor volume, M_f, I_f and S_f are the monomer, initiator and solvent feed concentrations respectively, T_f is the feed temperature, q is the feed flow rate, T_c is the coolant temperature, $P = \sqrt{\frac{2 f k_d I}{k_t}}$ is the total concentration of live radicals, f is the initiator efficiency, and k_t is the termination rate constant. The rate constants k_p, k_d follow the Arrhenius dependence on temperature, i.e.,

$$k_p = k_p' \exp\left(\frac{-E_p}{RT}\right), \quad k_d = k_d' \exp\left(\frac{-E_d}{RT}\right)$$

[4] Note that the error does take a longer time to settle to zero, which is to be expected

while the expression for k_t is computed using the Schmidt-Ray correlation for the gel-effect as

$$g_t = \frac{k_t}{k_{t0}} = \begin{cases} g_{t1}, & \text{if } V_f > \bar{V}_f(T) \\ g_{t2}, & \text{if } V_f \leq \bar{V}_f(T) \end{cases}$$

where

$$\begin{aligned} \bar{V}_f(T) &= 0.1856 - 2.965 \times 10^{-4}(T - 273.2), \\ g_{t1} &= 0.10575\exp[17.15V_f - 0.01715(T - 273.2)], \\ g_{t2} &= 2.3 \times 10^{-6}\exp[75V_f] \end{aligned}$$

where $k_{t0} = k'_{t0}\exp\left(\frac{-E_{t0}}{RT}\right)$, and $V_f(M, T, I, S)$ is the free volume calculated from the volume fractions of the monomer, polymer and solvent in the reactor (see Kurtz et al. [2000] for the functional dependence of V_f on the temperature T and concentrations, calculated under the assumption of ideal mixing). For the purposes of control design, it is more convenient to write the above equations in dimensionless form, resulting in the following state model

$$\begin{aligned} \dot{x}_1 &= x_{1f} - x_1 - Da_p W(x)E_x(x_2)x_1 \\ \dot{x}_2 &= -x_2 + BDa_p\gamma_p W(x)E_x(x_2)x_1 + \beta(x_{2c} - x_2) \\ \dot{x}_3 &= x_{3f} - x_3 - Da_d E_{xd}(x_2)x_3 \\ \dot{x}_4 &= x_{4f} - x_4 \end{aligned} \tag{9}$$

where $\dot{x}_i \overset{\text{def}}{=} \frac{dx_i}{d\tau}$, $\tau = tq/V$, and the dimensionless variables are defined as (see Adebekun and Schork [1989], Kurtz et al. [2000])

- $x_1 = M/M_{f0}$, $x_2 = \left(\frac{T-T_f}{T_f}\right)\left(\frac{E_p}{RT_f}\right)$,
- $x_3 = I/M_{f0}$, $x_4 = S/M_{f0}$,
- $x_{1f} = M_f/M_{f0}$, $x_{2c} = \left(\frac{T_c-T_f}{T_f}\right)\left(\frac{E_p}{RT_f}\right)$,
- $x_{3f} = I_f/M_{f0}$, $x_{4f} = S_f/M_{f0}$,
- $\gamma_p = E_p/(RT_f)$, $\beta = hA_c/(\rho c_p q)$,
- $B = (-\Delta H)M_{f0}/(\rho c_p T_f)$, $W(x) = P(\cdot)/M_{f0}$,
- $Da_p = k'_p e^{-\gamma_p} M_{f0}V/q$, $D_{ad} = k'_d e^{-\gamma_d\gamma_p}V/q$,
- $E_x(x_2) = \exp\left(\frac{x_2}{1+x_2/\gamma_p}\right)$, $E_{xd}(x_2) = \exp\left(\frac{\gamma_d x_2}{1+x_2/\gamma_p}\right)$

The control objective is to regulate the monomer concentration $y_1 = x_1$ and the reactor temperature $y_2 = x_2$ by manipulating the monomer feed concentration $u_1 = x_{1f}$ and the coolant temperature $u_2 = x_{2c}$. As in Kurtz et al. [2000], we assume the availability of on-line measurements of the outputs; the initiator and solvent concentrations x_3 and x_4 respectively are assumed to be unmeasurable. Furthermore, the inputs are assumed to be constrained by 0 mol$/L \leq M_f \leq 9$ mol$/L$, $300K \leq T_c \leq 440K$, which for the nominal values of the parameters and operating point specified in Kurtz et al. [2000] translate to $0 \leq u_1 \leq 2.0535$, and $-0.42 \leq u_2 \leq 2.571$. For the sake of convenience, we transform these to symmetric saturation bounds by defining $u_{1\delta} = u_1 - \hat{u}_1$, $u_{2\delta} = u_2 - \hat{u}_2$, where $\hat{u}_1 = 1.02675$, $\hat{u}_2 = 1.0755$, so that $|u_{1\delta}| \leq 1.02675 \overset{\text{def}}{=} k_1$, and $|u_{2\delta}| \leq 1.4955 \overset{\text{def}}{=} k_2$.

Note that the system has well-defined vector relative degree $\rho = \{1, 1\}$, and that for each specified equilibrium value $\bar{y} = (\bar{y}_1, \bar{y}_2)$, there is a unique equilibrium point $\bar{x}$ and equilibrium input $\bar{u} = (\bar{u}_1, \bar{u}_2)$ at which $y = \bar{y}$. Defining

$$\begin{aligned} e_1 &= y_1 - \bar{y}_1,\ e_2 = y_2 - \bar{y}_2 \\ \eta_1 &= x_3 - \bar{x}_3,\ \eta_2 = x_4 - \bar{x}_4 \end{aligned}$$

where $\bar{x}_3 = \frac{x_{3f}}{1+Da_d E_{xd}(\bar{y}_2)}$, $\bar{x}_4 = x_{4f}$, we can rewrite (9) in the form of (1). [5] It is easy to verify that the zero dynamics are exponentially stable. The matrix $A(\cdot)$ in simply diag$\{1, \beta\}$, so that we can take $\hat{A}(\cdot)$ in (4) to be the identity. Then, the control is "decoupled" and we simple take

$$\begin{aligned} \dot{\sigma}_1 &= -k_0^1\sigma_1 + \mu_1\text{sat}\left(\frac{s_1}{\mu_1}\right) \\ s_1 &= k_0^1\sigma_1 + e_1,\ u_1 = \hat{u}_1 - k_1\text{sat}\left(\frac{s_1}{\mu_1}\right) \\ \dot{\sigma}_2 &= -k_0^2\sigma_2 + \mu_2\text{sat}\left(\frac{s_2}{\mu_2}\right) \\ s_2 &= k_0^2\sigma_2 + e_2,\ u_2 = \hat{u}_2 - k_2\text{sat}\left(\frac{s_2}{\mu_2}\right) \end{aligned} \tag{10}$$

where $k_0^1, k_0^2 > 0$, and μ_1, μ_2 are "sufficiently small" positive constants. This completes the design of the controller, which is simply a saturated PI controller (with an additive nominal component). The analysis of Seshagiri and Khalil [2005] tells that the controller (10) achieves perfect regulation (the region of attraction depends on the values of the gains k_1 and k_2 though).

In order to compare results with the controller in Kurtz et al. [2000], we use the same numerical values for the parameters specified therein (see [Kurtz et al., 2000, Table II]). Other numerical parameters are $x(0) = (0.5, 0.5, 0.5)$, $\sigma_1(0) = \sigma_2(0) = 0$, $k_0^1 = k_0^2 = 1$, $\mu_1 = \mu_2 = 0.01$. As in Kurtz et al. [2000], the setpoints were chosen to be $\bar{y} = [1.2\ 0.0865]^T$, followed by a setpoint change to $\bar{y} = [0.31\ 1.06]^T$ at $\tau = 4$ s. The results are shown in Fig 4, and it clear that the controller achieves good performance, despite the fact that almost no plant parameter values are explicitly used in the control design. Our results are at least as good as the one with the MPC controller of Kurtz et al. [2000] (as compared to Fig 3 in that reference), where the design is much more involved.

As before, we mention the inherent robustness of this method to parametric uncertainties since the design does not explicitly require any knowledge of these parameter values. Robustness to disturbances is guaranteed by the use of the sliding mode technique, and the inclusion of integral control. Finally, the control designs presented are simply specially tuned versions of PI/PID controllers with an anti-windup structure.

5. CONCLUSIONS

In this paper, we presented an approach for the output feedback regulation of isothermic and exothermic chemical reactors. The method is an application of our work on the design of robust output feedback integral control for a class of minimum phase systems. For relative degree one and two systems, our controller can be designed simply as the industrially popular PI/PID controllers with anti-windup. The proposed approach offers some important advantages including

[5] Note that, even though not explicitly written, the feed solvent concentration S_f (and hence x_{4f}) depend on the feed monomer concentration u_1 (Kurtz et al. [2000]).

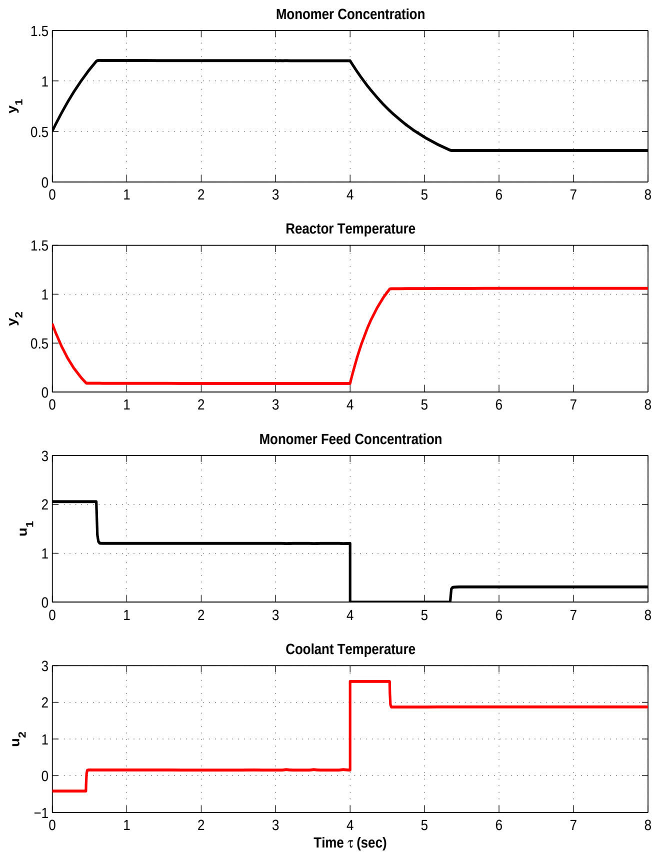

Fig. 4. Output regulation for the multivariable (MIMO) polymerization reactor

- computational simplicity,
- constraint handling capability, and
- the use of only partial state and/or output feedback.

Preliminary results were also presented in an earlier paper Seshagiri and Khalil [2001] for a SISO relative degree one system. The contribution of this paper is the application of our theory to higher relative degree and MIMO CSTRs. Some numerical simulations illustrating the theoretical results were also presented.

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