

V. CONCLUSION

Based on the extended nonquadratic Lyapunov function and the non-PDC law, three new stabilization results for T–S fuzzy systems are obtained by rehandling the slack matrices and the collection matrices. Each of the new results is either less conservative and computationally less expensive, or less conservative, than some of the existing results. To further improve the results, the technique given in [9] can be utilized; however, this is not the aim of this paper, while Ding and Huang [3] have utilized the technique given in [9].

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Dynamic Output Feedback Fault Tolerant Controller Design for Takagi–Sugeno Fuzzy Systems With Actuator Faults

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Abstract—This paper addresses the problem of robust fault estimation and fault tolerant control (FTC) for Takagi–Sugeno (T–S) fuzzy systems. A fuzzy-augmented fault estimation observer (AFEEO) design is proposed to achieve fault estimation of T–S models with actuator faults. Furthermore, based on the information of online fault estimation, an observer-based dynamic output feedback-fault tolerant controller (DOFFTC) is designed to compensate for the effect of faults by stabilizing the closed-loop system. Sufficient conditions for the existence of both AFEEO and DOFFTC are given in terms of linear matrix inequalities. Simulation results of an inverted pendulum system are presented to illustrate the effectiveness of the proposed method.

Index Terms—Dynamic output feedback, fault estimation, fault-tolerant control (FTC), Takagi–Sugeno (T–S) fuzzy models.

I. INTRODUCTION

Fault detection and isolation (FDI) and fault-tolerant control (FTC) have been the subjects of intensive investigations over the past two decades [1], [2]. However, since most real systems are nonlinear in nature, FDI/FTC applications to industrial and commercial processes require nonlinear models to be specifically taken into account. Takagi–Sugeno (T–S) fuzzy models are based on a set of IF–THEN rules, which give a local linear representation of an underlying nonlinear system, and it is well known that such models can describe or approximate a wide class of nonlinear systems. This is why they have

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attracted considerable attention and given rise to many important results [3]–[8]. However, most address stability analysis and feedback stabilization, and very few address the issue of fault estimation and FTC.

Based on passive FTC idea in [9] and [10], a reliable fault-tolerant controller using T–S fuzzy models was developed against actuator faults, while issues of fault detection and estimation were not involved. Robust fault detection for T–S fuzzy systems was studied in [11]–[13], but the issue of fault estimation was not included. The problem of robust fault estimation for time-delay T–S fuzzy models was dealt with in [14], but under a restrictive assumption on the faults, i.e., $f(t) \in L_2[0, \infty)$. A sliding-mode observer (SMO) and an adaptive observer (AO) were proposed to achieve fault estimation in [6] and [15], but their design needed very restrictive conditions to be satisfied.

Based on the aforementioned works, this paper further investigates the issue of robust fault estimation and FTC for T–S fuzzy systems. Restrictive constraints are relaxed through a general observer-based dynamic output feedback–fault tolerant controller (DOFFTC) design for a class of T–S fuzzy systems under actuator faults. First, a multiobjective fuzzy augmented fault-estimation observer (AFEO), including a regional pole placement and a H_∞ performance level, is proposed, not only to guarantee the convergence speed of fault estimation but to restrict the influence of disturbances as much as possible as well. Then, using the online fault estimate, a fuzzy DOFFTC is designed to guarantee the system stability in the presence of actuator faults. In the design process, the AFEO and the DOFFTC are independently designed, and their performances are considered simultaneously, which is convenient for calculating the design parameters and can avoid the coupling problem generated by the observer-based state feedback control.

II. SYSTEM DESCRIPTION

The T–S fuzzy model is described by fuzzy IF–THEN rules, whose collection represent the approximation of the nonlinear system. The i th rule of the T–S fuzzy model is of the following form.

Plant Rule i :

IF $z_1(t)$ is μ_{i1} and, \dots , and $z_s(t)$ is μ_{is} , THEN

$$\dot{x}(t) = A_i x(t) + B_i (u(t) + f(t)) + D_{1i} \omega(t) \quad (1)$$

$$y(t) = C_i x(t) + D_{2i} \omega(t) \quad (2)$$

where $x(t) \in R^n$ is the state, $u(t) \in R^m$ is the input, $y(t) \in R^p$ is the output, $f(t) \in R^m$ represents the additive actuator fault, and $\omega(t) \in R^d$ is the disturbance, which is assumed to belong to $L_2[0, \infty)$. A_i , B_i , C_i , D_{1i} , and D_{2i} are constant real matrices of appropriate dimensions. It is supposed that matrices B_i are of full column rank, i.e., $\text{rank}(B_i) = m$, the pairs (A_i, B_i) are controllable, and the pairs (A_i, C_i) are observable. $z_j(t)$ ($j = 1, \dots, s$) are the premise variables, μ_{ij} ($i = 1, \dots, q$; $j = 1, \dots, s$) are the fuzzy sets that are characterized by membership function, q is the number of IF–THEN rules, and s is the number of the premise variables. The overall fuzzy model achieved by fuzzy-blending of each individual plant rule (local model) is given by

$$\dot{x}(t) = \sum_{i=1}^q h_i(z(t)) [A_i x(t) + B_i (u(t) + f(t)) + D_{1i} \omega(t)] \quad (3)$$

$$y(t) = \sum_{i=1}^q h_i(z(t)) [C_i x(t) + D_{2i} \omega(t)] \quad (4)$$

where $z(t) = [z_1(t), \dots, z_s(t)]$

$$h_i(z(t)) = \frac{\sigma_i(z(t))}{\sum_{i=1}^q \sigma_i(z(t))}, \quad \sigma_i(z(t)) = \prod_{j=1}^s \mu_{ij}(z_j(t))$$

and $\mu_{ij}(\cdot)$ is the grade of the membership function of μ_{ij} . We assume

$$\sigma_i(z(t)) \geq 0, \quad i = 1, \dots, q, \quad \sum_{i=1}^q \sigma_i(z(t)) > 0 \quad (5)$$

for any $z(t)$. Hence, $h_i(z(t))$ satisfies

$$h_i(z(t)) \geq 0, \quad i = 1, \dots, q, \quad \sum_{i=1}^q h_i(z(t)) = 1 \quad (6)$$

for any $z(t)$.

For simplicity, we introduce the following notations:

$$h_i = h_i(z(t)), \quad A(h) = \sum_{i=1}^q h_i A_i, \quad B(h) = \sum_{i=1}^q h_i B_i$$

$$C(h) = \sum_{i=1}^q h_i C_i, \quad D_1(h) = \sum_{i=1}^q h_i D_{1i}, \quad \text{and} \quad D_2(h) = \sum_{i=1}^q h_i D_{2i}.$$

Then, the T–S fuzzy model (3) and (4) can be rewritten as

$$\dot{x}(t) = A(h)x(t) + B(h)(u(t) + f(t)) + D_1(h)\omega(t) \quad (7)$$

$$y(t) = C(h)x(t) + D_2(h)\omega(t). \quad (8)$$

Before ending this section, a lemma related to quadratic d -stabilizability is presented [16].

Lemma 1: For a given matrix $\mathcal{A} \in R^{n \times n}$, the eigenvalues of \mathcal{A} belong to the circular region $D(\alpha, r)$ with center $\alpha + j0$ and radius r , if and only if there exists a symmetric positive definite matrix $\mathcal{P} \in R^{n \times n}$ such that

$$\begin{bmatrix} -\mathcal{P} & \mathcal{P}(\mathcal{A} - \alpha I_n) \\ * & -r^2 \mathcal{P} \end{bmatrix} < 0 \quad (9)$$

where here and everywhere in the sequel, $*$ denotes the symmetric elements in a symmetric matrix.

III. MAIN RESULTS

A. Fuzzy AFEO Design

Now, we are ready to express our main results. In order to detect and estimate faults, the following fault estimation observer is constructed:

$$\dot{\hat{x}}(t) = A(h)\hat{x}(t) + B(h)(u(t) + \hat{f}(t)) - L(h)(\hat{y}(t) - y(t)) \quad (10)$$

$$\hat{y}(t) = C(h)\hat{x}(t) \quad (11)$$

$$\dot{\hat{f}}(t) = -F(h)(\hat{y}(t) - y(t)) \quad (12)$$

where $\hat{x}(t) \in R^n$ is the observer state, $\hat{y}(t) \in R^p$ is the observer output, and $\hat{f}(t) \in R^r$ is an estimate of the fault $f(t)$. $L(h) \in R^{n \times p}$ and $F(h) \in R^{r \times p}$ are the gain matrices to be designed $L(h) = \sum_{i=1}^q h_i L_i$, $F(h) = \sum_{i=1}^q h_i F_i$.

Denote $e_x(t) = \hat{x}(t) - x(t)$, $e_y(t) = \hat{y}(t) - y(t)$, and $e_f(t) = \hat{f}(t) - f(t)$; then, the error dynamics is given by

$$\dot{\bar{e}}(t) = (\bar{A}(h) - \bar{L}(h)\bar{C}(h))\bar{e}(t) + (\bar{L}(h)\bar{D}_2(h) - \bar{D}_1(h))\nu(t) \quad (13)$$

$$e_y(t) = \bar{C}(h)\bar{e}(t) - \bar{D}_2(h)\nu(t) \quad (14)$$

where

$$\begin{aligned}\bar{e}(t) &= \begin{bmatrix} e_x(t) \\ e_f(t) \end{bmatrix}, \quad \nu(t) = \begin{bmatrix} \omega(t) \\ \dot{f}(t) \end{bmatrix} \\ \bar{A}(h) &= \begin{bmatrix} A(h) & B(h) \\ 0 & 0 \end{bmatrix}, \quad \bar{L}(h) = \begin{bmatrix} L(h) \\ F(h) \end{bmatrix} \\ \bar{C}(h) &= [C(h) \ 0], \quad \bar{D}_1(h) = \begin{bmatrix} D_1(h) & 0 \\ 0 & I_m \end{bmatrix} \\ \bar{D}_2(h) &= [D_2(h) \ 0].\end{aligned}$$

Assumption 1: $\dot{f}(t)$ belongs to $L_2[0, \infty)$.

Remark 1: In [14] and [17], the fault estimation filter is designed under the assumption $f(t) \in L_2[0, \infty)$. In general, SMO-based fault estimation requires the preliminary knowledge of the upper bound of $f(t)$ [6], [18]. However, in many practical systems, there is a transient period during which the fault establishes itself, after which, it remains more or less constant, meaning that the derivatives of the faults are energy-bounded, i.e., $\dot{f}(t) \in L_2[0, \infty)$. This is stated by Assumption 1, which is more general than those used in the aforementioned design methods.

Remark 2: From the augmented systems (13) and (14), it can be seen that $\bar{A}(h)$, $\bar{C}(h)$, $\bar{D}_1(h)$, and $\bar{D}_2(h)$ are known matrices, while $\bar{L}(h)$ contains the two gains $L(h)$ and $F(h)$ to be designed. Therefore, we can see that a necessary condition for the existence of fuzzy AFEO is that the pairs (\bar{A}_i, \bar{C}_i) are observable, and globally robust stability of fuzzy AFEO can be guaranteed by the following Theorem 1.

Next, a multiobjective AFEO design method under regional pole placement and H_∞ performance specifications is proposed to achieve robust fault estimation.

Theorem 1: For two given positive scalars α and r , the eigenvalues of $(\bar{A}(h) - \bar{L}(h)\bar{C}(h))$ belong to $D(\alpha, r)$, and the error dynamics (13) satisfy the H_∞ performance index $\|e_f(t)\|_2 < \gamma\|\nu(t)\|_2$, if there exists a symmetric positive definite matrix $\bar{P} \in R^{(n+m) \times (n+m)}$ and matrices $\bar{Y}_i \in R^{(n+m) \times p}$ such that

min γ subject to

$$\Psi_{ii} < 0, \quad i = 1, \dots, q \quad (15)$$

$$\Psi_{ij} + \Psi_{ji} < 0, \quad 1 \leq i < j \leq q \quad (16)$$

$$\Phi_{ii} < 0, \quad i = 1, \dots, q \quad (17)$$

$$\Phi_{ij} + \Phi_{ji} < 0, \quad 1 \leq i < j \leq q \quad (18)$$

where

$$\begin{aligned}\Psi_{ij} &= \begin{bmatrix} -\bar{P} & \bar{P}\bar{A}_i - \bar{Y}_i\bar{C}_j - \alpha\bar{P} \\ * & -r^2\bar{P} \end{bmatrix}, \quad \bar{I}_m = \begin{bmatrix} 0 \\ I_m \end{bmatrix} \\ \Phi_{ij} &= \begin{bmatrix} \phi_{11} & \bar{Y}_i\bar{D}_{2j} - \bar{P}\bar{D}_{1i} & \bar{I}_m \\ * & -\gamma I_{d+m} & 0 \\ * & * & -\gamma I_m \end{bmatrix}, \quad Y_i = PL_i \\ \phi_{11} &= \bar{P}\bar{A}_i + \bar{A}_i^T\bar{P} - \bar{Y}_i\bar{C}_j - \bar{C}_j^T\bar{Y}_i^T.\end{aligned}$$

Proof: Constraints (15) and (16): Setting $(\bar{A}(h) - \bar{L}(h)\bar{C}(h)) \rightarrow \mathcal{A}$ and $\bar{P} \rightarrow \mathcal{P}$ in Lemma 1, one gets

$$\Psi := \begin{bmatrix} -\bar{P} & \bar{P}(\bar{A}(h) - \bar{L}(h)\bar{C}(h)) - \alpha\bar{P} \\ * & -r^2\bar{P} \end{bmatrix} < 0 \quad (19)$$

which can be rewritten as

$$\Psi = \sum_{i=1}^q h_i^2 \Psi_{ii} + \sum_{i=1}^q \sum_{i < j}^q h_i h_j (\Psi_{ij} + \Psi_{ji}) < 0. \quad (20)$$

Therefore, if (15) and (16) hold, then the eigenvalues of $(\bar{A}(h) - \bar{L}(h)\bar{C}(h))$ belong to $D(\alpha, r)$.

For constraints (17) and (18), consider the following Lyapunov function:

$$V(t) = \bar{e}^T(t)\bar{P}\bar{e}(t). \quad (21)$$

Its derivative with respect to time is

$$\begin{aligned}\dot{V}(t) &= \bar{e}^T(t) \left(\bar{P}(\bar{A}(h) - \bar{L}(h)\bar{C}(h)) \right. \\ &\quad \left. + (\bar{A}(h) - \bar{L}(h)\bar{C}(h))^T \bar{P} \right) \bar{e}(t) \\ &\quad + 2\bar{e}^T(t)\bar{P}(\bar{L}(h)\bar{D}_2(h) - \bar{D}_1(h))\nu(t).\end{aligned} \quad (22)$$

Now, let us define

$$\begin{aligned}J &= \int_{t_f}^{\infty} \left[\frac{1}{\gamma} \bar{e}_f^T(t) e_f(t) - \gamma \nu^T(t) \nu(t) \right] dt \\ &= \int_{t_f}^{\infty} \left[\frac{1}{\gamma} \bar{e}^T(t) \bar{I}_m \bar{I}_m^T \bar{e}(t) - \gamma \nu^T(t) \nu(t) \right] dt\end{aligned} \quad (23)$$

where t_f denotes the fault occurrence time. Then, under zero initial condition at time t_f , it can be shown that

$$J \leq \int_{t_f}^{\infty} \left[\dot{V}(t) + \frac{1}{\gamma} \bar{e}^T(t) \bar{I}_m \bar{I}_m^T \bar{e}(t) - \gamma \nu^T(t) \nu(t) \right] dt. \quad (24)$$

Substituting (22) into (24), one gets

$$\begin{aligned}\dot{V}(t) &+ \frac{1}{\gamma} \bar{e}^T(t) \bar{I}_m \bar{I}_m^T \bar{e}(t) - \gamma \nu^T(t) \nu(t) \\ &= \bar{e}^T(t) (\bar{P}(\bar{A}(h) - \bar{L}(h)\bar{C}(h)) \\ &\quad + (\bar{A}(h) - \bar{L}(h)\bar{C}(h))^T \bar{P}) \bar{e}(t) \\ &\quad + 2\bar{e}^T(t) \bar{P}(\bar{L}(h)\bar{D}_2(h) - \bar{D}_1(h))\nu(t) \\ &\quad + \frac{1}{\gamma} \bar{e}^T(t) \bar{I}_m \bar{I}_m^T \bar{e}(t) - \gamma \nu^T(t) \nu(t) \\ &= \zeta^T(t) \Omega \zeta(t) \\ &= \sum_{i=1}^q \sum_{j=1}^q h_i h_j \zeta^T(t) \Omega_{ij} \zeta(t) \\ &= \sum_{i=1}^q h_i^2 \zeta^T(t) \Omega_{ii} \zeta(t) + \sum_{i=1}^q \sum_{i < j}^q h_i h_j \zeta^T(t) (\Omega_{ij} + \Omega_{ji}) \zeta(t)\end{aligned} \quad (25)$$

where

$$\begin{aligned}\zeta(t) &= \begin{bmatrix} \bar{e}(t) \\ \nu(t) \end{bmatrix}, \quad \Omega = \begin{bmatrix} \eta_{11} & \bar{P}(\bar{L}(h)\bar{D}_2(h) - \bar{D}_1(h)) \\ * & -\gamma I_{d+m} \end{bmatrix} \\ \Omega_{ij} &= \begin{bmatrix} \phi_{11} + \frac{1}{\gamma} \bar{I}_m \bar{I}_m^T & \bar{Y}_i \bar{D}_{2j} - \bar{P} \bar{D}_{1i} \\ * & -\gamma I_{d+m} \end{bmatrix} \\ \eta_{11} &= \bar{P}(\bar{A}(h) - \bar{L}(h)\bar{C}(h)) \\ &\quad + (\bar{A}(h) - \bar{L}(h)\bar{C}(h))^T \bar{P} + \frac{1}{\gamma} (\bar{I}_m \bar{I}_m^T).\end{aligned}$$

By the Schur complement, (17) and (18) are equivalent to $\Omega_{ii} < 0$ and $\Omega_{ij} + \Omega_{ji} < 0$. It follows that the error dynamics (13) are robustly

stable with a H_∞ performance index $\|e_f(t)\|_2 < \gamma\|\nu(t)\|_2$, provided (17) and (18) hold true. \square

Remark 3: In Theorem 1, the purpose of introducing the disk constraints (15) and (16) is to improve the transient performance of fault estimation. Note that, other pole-placement constraints, such as α -stability, vertical strips, sectors, and the intersection thereof can also be considered [19], [20].

Remark 4: Achieving the estimation of actuator faults by the SMO-based and AO-based approaches proposed in [6] and [15] would require that the constraints $\text{rank}(C_i B_i) = \text{rank}(B_i)$ and $C_i(sI - A_i)^{-1} B_i$ be minimum phase to be satisfied. Details on the conservativeness of these constraints can be found in [21] and [18]. It is seen that no equality constraint is used in the design process of the proposed fuzzy AFEO, which resumes in finding matrices $\tilde{L}(h)$ such that (13) is robustly stable.

Remark 5: Note that a fault-estimation filter followed from the design idea in [14] and [17] is constructed as

$$\dot{x}_f(t) = A_f(h)x_f(t) + B_f(h)y(t) \quad (26)$$

$$\hat{f}(t) = C_f(h)x_f(t) + D_f(h)y(t) \quad (27)$$

where $x_f(t) \in R^{n_f}$ is the filter state, and $\hat{f}(t) \in R^m$ is an estimate of the fault $f(t)$. $A_f(h)$, $B_f(h)$, $C_f(h)$, and $D_f(h)$ are the filter gain matrices of appropriate dimensions to be designed. Then, substituting (7) and (8) into (26) and (27), we obtain

$$\dot{\tilde{x}}_f(t) = \tilde{A}_f(h)\tilde{x}_f(t) + \tilde{B}_f(h)\rho(t) \quad (28)$$

$$e_f(t) = \tilde{C}_f(h)\tilde{x}_f(t) + \tilde{D}_f(h)\rho(t) \quad (29)$$

where

$$\tilde{x}_f(t) = \begin{bmatrix} x(t) \\ x_f(t) \end{bmatrix}, \quad \rho(t) = \begin{bmatrix} u(t) \\ f(t) \\ \omega(t) \end{bmatrix}$$

$$\tilde{A}_f(h) = \begin{bmatrix} A(h) & 0 \\ B_f(h)C(h) & A_f(h) \end{bmatrix}$$

$$\tilde{B}_f(h) = \begin{bmatrix} B(h) & B(h) & D_1(h) \\ 0 & 0 & B_f(h)D_2(h) \end{bmatrix}$$

$$\tilde{C}_f(h) = [D_f(h)C(h) \quad C_f(h)]$$

$$\tilde{D}_f(h) = [0 \quad -I_m \quad D_f(h)D_2(h)].$$

Under the H_∞ performance $\|e_f(t)\|_2 < \gamma\|\rho(t)\|_2$, the fault-estimation filter design requires the assumption $f(t) \in L_2[0, \infty)$. From (28) to (29), it is shown that, since the occurred fault is viewed as a ‘‘disturbance,’’ it is impossible to realize asymptotical estimation of the constant fault (except for $f(t) = 0$ that is of no interest). Meanwhile, from system matrices $\tilde{A}_f(h)$, it can be seen that the filter design is only suitable for open-loop stable systems. However, in practical situations, most systems are open-loop unstable, and therefore, such a constraint limits its application scopes.

B. Fuzzy DOFFTC Design

On the basis of the obtained online fault-estimation information, we design a fault-tolerant controller to guarantee stability in the presence of faults. Since the state $x(t)$ is unmeasurable, we use the fuzzy dynamical output feedback-controller scheme [8], [22] to construct the DOFFTC for T-S fuzzy models as

$$\dot{\xi}(t) = A_K(h, h)\xi(t) + B_K(h)y(t) \quad (30)$$

$$u(t) = C_K(h)\xi(t) + D_K y(t) - \hat{f}(t) \quad (31)$$

where $\xi(t)$ is the state, $A_K(h, h) \in R^{n \times n}$, $B_K(h) \in R^{n \times p}$, $C_K(h) \in R^{m \times n}$, and $D_K \in R^{m \times p}$ are the designed DOFFTC matrices, and $A_K(h, h) = \sum_{i=1}^q \sum_{j=1}^q h_i h_j A_{Kij}$, $B_K(h) = \sum_{i=1}^q h_i B_{Ki}$, $C_K(h) = \sum_{i=1}^q h_i C_{Ki}$.

Substituting (8) into (30) and (31), one obtains

$$\dot{\xi}(t) = A_K(h, h)\xi(t) + B_K(h)C(h)x(t) + B_K(h)D_2(h)\omega(t) \quad (32)$$

$$u(t) = C_K(h)\xi(t) + D_K C(h)x(t) + D_K D_2(h)\omega(t) - \hat{f}(t). \quad (33)$$

Then, one gets

$$\dot{\tilde{x}}(t) = \tilde{A}(h, h)\tilde{x}(t) + \tilde{D}_1(h, h)\mu(t) \quad (34)$$

$$y(t) = \tilde{C}(h)\tilde{x}(t) + \tilde{D}_2(h)\mu(t). \quad (35)$$

where

$$\tilde{x}(t) = \begin{bmatrix} x(t) \\ \xi(t) \end{bmatrix}, \quad \mu(t) = \begin{bmatrix} \omega(t) \\ e_f(t) \end{bmatrix}$$

$$\tilde{A}(h, h) = \begin{bmatrix} A(h) + B(h)D_K C(h) & B(h)C_K(h) \\ B_K(h)C(h) & A_K(h, h) \end{bmatrix}$$

$$\tilde{D}_1(h, h) = \begin{bmatrix} D_1(h) + B(h)D_K D_2(h) & -B(h) \\ B_K(h)D_2(h) & 0 \end{bmatrix}$$

$$\tilde{C}(h) = \tilde{C}(h) = [C(h) \quad 0]$$

$$\tilde{D}_2(h) = \tilde{D}_2(h) = [D_2(h) \quad 0].$$

Theorem 2: Assume a fault $f(t)$ occurs at some unknown time t_f , and let α and r be two positive scalars. The eigenvalues of $\tilde{A}(h, h)$ belong to $D(\alpha, r)$, and the system dynamics (34) and (35) satisfy the H_∞ performance index $\|y(t)\|_2 < \gamma\|\mu(t)\|_2$, if there exist two symmetric positive definite matrices $X, Y \in R^{n \times n}$ and matrices $\hat{A}_{ij} \in R^{n \times n}$, $\hat{B}_i \in R^{n \times p}$, $\hat{C}_i \in R^{m \times n}$, and $\hat{D} \in R^{m \times p}$ such that

min γ subject to

$$\Pi_{ii} < 0, \quad i = 1, \dots, q \quad (36)$$

$$\Pi_{ij} + \Pi_{ji} < 0, \quad 1 \leq i < j \leq q \quad (37)$$

$$\Xi_{ii} < 0, \quad i = 1, \dots, q \quad (38)$$

$$\Xi_{ij} + \Xi_{ji} < 0, \quad 1 \leq i < j \leq q \quad (39)$$

where

$$\Pi_{ij} = \begin{bmatrix} -X & -I_n & \pi_{13} & A_i + B_i \hat{D} C_j - \alpha I_n \\ * & -Y & \hat{A}_{ij} - \alpha I_n & Y A_i + \hat{B}_j C_i - \alpha Y \\ * & * & -r^2 X & -r^2 I_n \\ * & * & * & -r^2 Y \end{bmatrix}$$

$$\text{and } \Xi_{ij} = \begin{bmatrix} \chi_{11} & \chi_{12} & D_{1i} + B_i \hat{D} D_{2j} & -B_i & X C_i^T \\ * & \chi_{22} & Y D_{1i} + \hat{B}_j D_{2i} & -Y B_i & C_i^T \\ * & * & -\gamma I_d & 0 & D_{2i}^T \\ * & * & * & -\gamma I_m & 0 \\ * & * & * & * & -\gamma I_p \end{bmatrix}$$

with

$$\pi_{13} = A_i X + B_i \hat{C}_j - \alpha X$$

$$\chi_{11} = A_i X + X A_i^T + B_i \hat{C}_j + \hat{C}_j^T B_i^T$$

$$\chi_{12} = \hat{A}_{ij}^T + A_i + B_i \hat{D} C_j$$

and $\chi_{22} = Y A_i + A_i^T Y^T + \hat{B}_j C_i + C_i^T \hat{B}_j^T$.

The parameter matrices of the DOFFTC are given by

$$\begin{aligned} D_K &= \hat{D} \\ C_{K_i} &= (\hat{C}_i - D_K C_i X) M^{-T} \\ B_{K_i} &= N^{-1} (\hat{B}_i - Y B_i D_K) \\ A_{K_{ij}} &= N^{-1} (\hat{A}_{ij} - Y (A_i + B_i D_K C_j) X) M^{-T} \\ &\quad - B_{K_j} C_i X M^{-T} - N^{-1} Y B_i C_{K_j} \end{aligned}$$

where $M, N \in R^{n \times n}$ satisfy $MN^T = I_n - XY$.

Proof: We start with the proof of (38) and (39), and (36) and (37) will be considered subsequently. For constraints (38) and (39), consider the following Lyapunov function:

$$V(t) = \tilde{x}^T(t) \tilde{P} \tilde{x}(t). \quad (40)$$

Its derivative with respect to time is

$$\begin{aligned} \dot{V}(t) &= \tilde{x}^T(t) \left(\tilde{P} \tilde{A}(h, h) + \tilde{A}^T(h, h) \tilde{P} \right) \tilde{x}(t) \\ &\quad + 2\tilde{x}^T(t) \tilde{P} \tilde{D}_1(h, h) \mu(t). \end{aligned} \quad (41)$$

Let us introduce

$$J = \int_{t_f}^{\infty} \left[\frac{1}{\gamma} y^T(t) y(t) - \gamma \mu^T(t) \mu(t) \right] dt. \quad (42)$$

It can be shown that

$$J \leq \int_{t_f}^{\infty} \left[\dot{V}(t) + \frac{1}{\gamma} y^T(t) y(t) - \gamma \mu^T(t) \mu(t) \right] dt. \quad (43)$$

Substituting (41) into (43), one obtains

$$\begin{aligned} \dot{V}(t) + \frac{1}{\gamma} y^T(t) y(t) - \gamma \mu^T(t) \mu(t) &= \tilde{x}^T(t) (\tilde{P} \tilde{A}(h, h) + \tilde{A}^T(h, h) \tilde{P}) \tilde{x}(t) + 2\tilde{x}^T(t) \tilde{P} \tilde{D}_1(h, h) \mu(t) \\ &\quad + \frac{1}{\gamma} \tilde{x}^T(t) \tilde{C}^T(h) \tilde{C}(h) \tilde{x}(t) + \frac{2}{\gamma} \tilde{x}^T(t) \tilde{C}^T(h) \tilde{D}_2(h) \mu(t) \\ &\quad + \frac{1}{\gamma} \mu^T(t) \tilde{D}_2^T(h) \tilde{D}_2(h) \mu(t) - \gamma \mu^T(t) \mu(t) = \varsigma^T(t) \Theta \varsigma(t) \end{aligned} \quad (44)$$

where

$$\varsigma(\tau) = \begin{bmatrix} \tilde{x}(\tau) \\ \mu(\tau) \end{bmatrix} \text{ and } \Theta = \begin{bmatrix} \theta_{11} & \theta_{12} \\ * & \theta_{22} \end{bmatrix}$$

with

$$\theta_{11} = \tilde{P} \tilde{A}(h, h) + \tilde{A}^T(h, h) \tilde{P} + \frac{1}{\gamma} \tilde{C}^T(h) \tilde{C}(h)$$

$$\theta_{12} = \tilde{P} \tilde{D}_1(h, h) + \frac{1}{\gamma} \tilde{C}^T(h) \tilde{D}_2(h)$$

$$\text{and } \theta_{22} = \frac{1}{\gamma} \tilde{D}_2^T(h) \tilde{D}_2(h) - \gamma I_{d+m}.$$

Using the Schur complement, $\Theta < 0$ is equivalent to

$$\begin{bmatrix} \tilde{P} \tilde{A}(h, h) + \tilde{A}^T(h, h) \tilde{P} & \tilde{P} \tilde{D}_1(h, h) & \tilde{C}^T(h) \\ * & -\gamma I_{d+m} & \tilde{D}_2^T(h) \\ * & * & -\gamma I_p \end{bmatrix} < 0. \quad (45)$$

Then, we express the symmetric positive definite matrix \tilde{P} and its inverse matrix \tilde{P}^{-1} as $\tilde{P} = \begin{bmatrix} Y & N \\ N^T & W \end{bmatrix}$ and $\tilde{P}^{-1} = \begin{bmatrix} X & M \\ M^T & Z \end{bmatrix}$.

Due to $\tilde{P} \tilde{P}^{-1} = I_{2n}$, one gets $\tilde{P} \begin{bmatrix} X \\ M^T \end{bmatrix} = \begin{bmatrix} I_n \\ 0_n \end{bmatrix}$ and $\tilde{P}^{-1} \begin{bmatrix} X & I_n \\ M^T & 0_n \end{bmatrix} = \begin{bmatrix} I_n & Y \\ 0_n & N^T \end{bmatrix}$, respectively.

Denote $F_1 = \begin{bmatrix} X & I_n \\ M^T & 0_n \end{bmatrix}$ and $F_2 = \begin{bmatrix} I_n & Y \\ 0_n & N^T \end{bmatrix}$; then, it follows that $\tilde{P} F_1 = F_2$. Premultiplying and postmultiplying by $\text{diag}(F_1^T, I_{d+m}, I_p)$, and its transpose in (45), and denoting

$$\begin{aligned} \hat{A}(h, h) &= Y(A(h) + B(h) D_K C(h)) X + N B_{K_i}(h) C(h) X \\ &\quad + Y B(h) C_K(h) M^T + N A_K(h, h) M^T \end{aligned}$$

$$\hat{B}(h) = Y B(h) D_K + N B_K(h)$$

$$\hat{C}(h) = D_K C(h) X + C_K(h) M^T, \text{ and } \hat{D} = D_K.$$

Then, (45) can be expressed as

$$\Xi := \begin{bmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13} & -B(h) & X C^T(h) \\ * & \varphi_{22} & \varphi_{23} & -Y B(h) & C^T(h) \\ * & * & -\gamma I_d & 0 & D_2^T(h) \\ * & * & * & -\gamma I_m & 0 \\ * & * & * & * & -\gamma I_p \end{bmatrix} < 0 \quad (46)$$

where

$$\varphi_{11} = A(h) X + X A^T(h) + B(h) \hat{C}(h) + \hat{C}^T(h) B^T(h)$$

$$\varphi_{12} = \hat{A}^T(h, h) + A(h) + B(h) \hat{D} C(h)$$

$$\varphi_{13} = D_1(h) + B(h) \hat{D} D_2(h)$$

$$\varphi_{22} = Y A(h) + A^T(h) Y^T + \hat{B}(h) C(h) + C^T(h) \hat{B}^T(h)$$

$$\varphi_{23} = Y D_1(h) + \hat{B}(h) D_2(h).$$

Therefore, $\Xi < 0$ can be rewritten as

$$\begin{aligned} \Xi &= \sum_{i=1}^q \sum_{j=1}^q h_i h_j \Xi_{ij} \\ &= \sum_{i=1}^q h_i^2 \Xi_{ii} + \sum_{i=1}^q \sum_{i < j} h_i h_j (\Xi_{ij} + \Xi_{ji}) < 0. \end{aligned} \quad (47)$$

Therefore, if (38) and (39) hold, the system dynamics (34) and (35) are robustly stable with a H_∞ performance index $\|y(t)\|_2 < \gamma \|\mu(t)\|_2$.

Constraints (36) and (37): Setting $\tilde{A}(h, h) \rightarrow \mathcal{A}$ and $\tilde{P} \rightarrow \mathcal{P}$ in Lemma 1, one has

$$\begin{bmatrix} -\tilde{P} & \tilde{P} \tilde{A}(h, h) - \alpha \tilde{P} \\ * & -r^2 \tilde{P} \end{bmatrix} < 0. \quad (48)$$

Premultiplying and postmultiplying by $\text{diag}(F_1^T, F_1^T)$ and its transpose in (48) and then using the definition $\hat{A}(h, h)$, $\hat{B}(h)$, $\hat{C}(h)$, and \hat{D} , one gets

$$\Pi := \begin{bmatrix} -X & -I_n & \varpi_{13} & \varpi_{14} \\ * & -Y & \hat{A}(h, h) - \alpha I_n & \varpi_{24} \\ * & * & -r^2 X & -r^2 I_n \\ * & * & * & -r^2 Y \end{bmatrix} < 0 \quad (49)$$

where

$$\varpi_{13} = A(h) X + B(h) \hat{C}(h) - \alpha X$$

$$\varpi_{14} = A(h) + B(h) D_K C(h) - \alpha I_n$$

$$\varpi_{24} = Y A(h) + \hat{B}(h) C(h) - \alpha Y.$$

This can be rewritten as

$$\begin{aligned} \Pi &= \sum_{i=1}^q \sum_{j=1}^q h_i h_j \Pi_{ij} \\ &= \sum_{i=1}^q h_i^2 \Pi_{ii} + \sum_{i=1}^q \sum_{i < j}^q h_i h_j (\Pi_{ij} + \Pi_{ji}) < 0. \end{aligned} \quad (50)$$

Therefore, if (36) and (37) hold, then the eigenvalues of $\tilde{A}(h, h)$ belong to $D(\alpha, r)$. \square

Remark 6: From the top left subblock $\begin{bmatrix} -X & -I_n \\ * & -Y \end{bmatrix}$ in (36), we can obtain $Y > 0$ and $X - Y^{-1} > 0$, which imply that $I_n - XY$ is nonsingular. Therefore, we can always find nonsingular matrices M and N satisfying $MN^T = I_n - XY$, and they can be calculated by the `qr` function of MATLAB toolbox.

Remark 7: In the whole design process, the AFEO and DOFFTC are designed separately, and their performances are considered simultaneously, which can avoid design difficulties caused by the coupling between the fault-diagnosis observer and the observer-based state feedback fault-tolerant controller. It is the coupling that makes the separate principle that does not hold [15], [23], [24].

Remark 8: It is noted that more fuzzy rules can approximate nonlinear systems more accurately. However, as the number of fuzzy rules increases, computation burden and design conservatism are unavoidable such that there might be no solution. Therefore, the two conditions are partially conflicting with each other, and a tradeoff between the number of fuzzy rules and design conservatism should be made.

IV. SIMULATION RESULTS

Consider the problem of balancing and swing-up of an inverted pendulum on a cart [7], [22]. The equations of the pendulum motion are given by

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= \frac{[g \sin(x_1(t)) - amlx_2^2(t) \sin(2x_1(t))/2 -]}{4l/3 - aml \cos^2(x_1(t))} u(t) \\ y(t) &= x_1(t) \end{aligned}$$

where $x_1(t)$ is the angle (rad) of the pendulum from the vertical, and $x_2(t)$ is the angular velocity (rad/s). Note that, we assume only the first state to be measured, rather than all states [7], to achieve the actuator faults estimation, thus setting a more restrictive problem; $g = 9.8$ m/s is the gravity constant, m is the pendulum mass, M is the cart mass, $2l$ is the pendulum length, and $a = 1/(m + M)$. In all simulations, $m = 2.0$ kg, $M = 2.0$ kg, and $2l = 1.0$ m.

First, we represent the system by using a two-rule T-S fuzzy model.

Rule 1: IF $x_1(t)$ is about 0, THEN

$$\dot{x}(t) = A_1 x(t) + B_1 u(t), \quad y(t) = C_1 x(t).$$

Rule 2: IF $x_1(t)$ is about $\pm \frac{\pi}{2}$ ($|x_1(t)| < \frac{\pi}{2}$), THEN

$$\dot{x}(t) = A_2 x(t) + B_2 u(t), \quad y(t) = C_2 x(t).$$

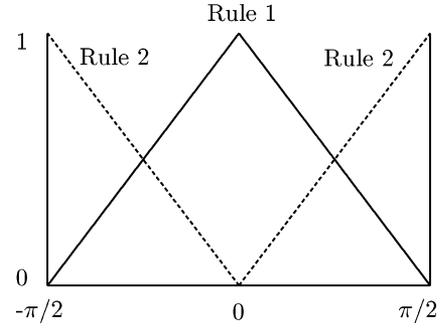


Fig. 1. Membership functions of the two-rule model.

where

$$\begin{aligned} A_1 &= \begin{bmatrix} 0 & 1 \\ \frac{g}{(4l/3 - aml)} & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ -\frac{a}{(4l/3 - aml)} \end{bmatrix} \\ A_2 &= \begin{bmatrix} 0 & 1 \\ \frac{2g}{\pi(4l/3 - aml\beta^2)} & 0 \end{bmatrix} \\ B_2 &= \begin{bmatrix} 0 \\ -\frac{a\beta}{(4l/3 - aml\beta^2)} \end{bmatrix} \\ C_1 &= [1 \ 0], \quad C_2 = [1 \ 0] \end{aligned}$$

and $\beta = \cos(88^\circ)$ (notice that when $x_1(t) = \pm\pi/2$, the system is uncontrollable). Membership functions for Rules 1 and 2 are shown in Fig. 1. Meanwhile, disturbance distribution matrices are, respectively, chosen as $D_{11} = D_{12} = [0.01 \ 0.01]^T$, and $D_{21} = D_{22} = [0.001 \ 0.001]^T$. It is easy to verify that the pairs (A_i, B_i) are controllable and that the pairs (A_i, C_i) are observable.

It can be easily shown that $\text{rank}(C_i B_i) = 0$ and (A_i, B_i, C_i) do not possess any zeros; therefore, the SMO-based and AO-based methods cannot be used for this system to achieve fault estimation [6], [15]. Meanwhile, from system matrices A_i , it is shown that the system is open-loop unstable, and therefore, the fault-estimation filter design is also not suitable for such system [14], [17].

We first design the fuzzy AFEO. By solving the conditions in Theorem 1 with the circle region $D(30, 30)$ based on the `mincx` function of MATLAB toolbox, one obtains the minimum attenuation value $\gamma = 3.6104$ with

$$\begin{aligned} L_1 &= \begin{bmatrix} 61.3775 \\ 911.1319 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 52.8509 \\ 611.4022 \end{bmatrix} \\ F_1 &= -40383.7598, \quad \text{and} \quad F_2 = -32963.7904. \end{aligned}$$

Next, we design the fuzzy DOFFTC. By solving the conditions in Theorem 2 with the circle region $D(30, 30)$, one obtains the minimum attenuation value $\gamma = 0.0388$ with

$$\begin{aligned} A_{K11} &= \begin{bmatrix} -92.0199 & 0.2014 \\ -5919.5413 & 16.3694 \end{bmatrix} \\ A_{K12} &= \begin{bmatrix} -22.8679 & -1.1238 \\ 1500.3734 & -152.4579 \end{bmatrix} \\ A_{K21} &= \begin{bmatrix} -90.1229 & 0.6926 \\ -5694.2247 & 75.2751 \end{bmatrix} \\ A_{K22} &= \begin{bmatrix} -18.7826 & -0.0260 \\ 1993.0790 & -18.9805 \end{bmatrix} \end{aligned}$$

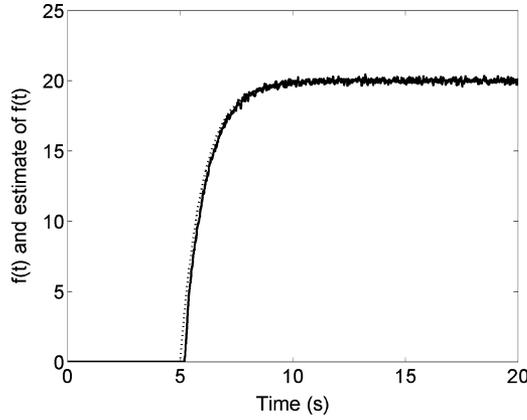


Fig. 2. Constant fault $f(t)$ (dotted) and its estimate $\hat{f}(t)$ (solid).

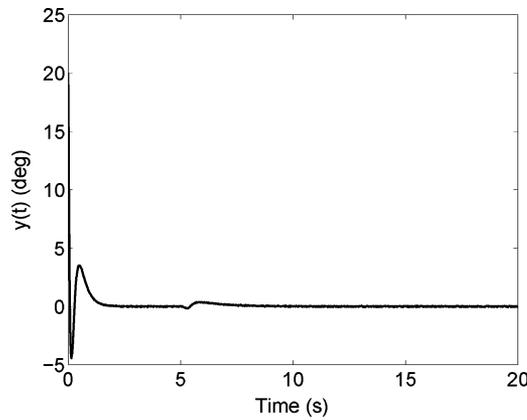


Fig. 3. For constant fault, output response under the fuzzy DOFFTC.

$$B_{K1} = \begin{bmatrix} -97.6313 \\ -6546.8816 \end{bmatrix}, \quad B_{K2} = \begin{bmatrix} -16.8077 \\ 2227.0462 \end{bmatrix}$$

$$C_{K1} = [7986.5346 \quad -24.4210]$$

$$C_{K2} = [8245.9040 \quad 47.1746], \quad D_K = 8856.8760.$$

First, it is assumed that a constant fault $f(t)$ is created as

$$f(t) = \begin{cases} 0, & 0 \leq t < 5 \\ 20(1 - e^{-(t-5)}), & 5 \leq t \leq 20. \end{cases}$$

It is supposed that $\omega(t)$ are band-limited white noise with power 0.001 and sampling time 0.01 s. Under initial value $(20\pi/180, 0)$, simulation results are displayed as follows. In the following simulation results, we apply the proposed design method to the original nonlinear system rather than T-S fuzzy models, whose purpose is to verify the robustness of the proposed method with respect to modeling errors. Fig. 2 illustrates fault-estimation simulation results. Simulation results for the system output response are shown in Fig. 3 (a 0.2-s detection delay is considered).

Then, a time-varying fault is simulated as

$$f(t) = \begin{cases} 0, & 0 \leq t < 5 \\ 10 \sin(t - 5), & 5 \leq t \leq 20. \end{cases}$$

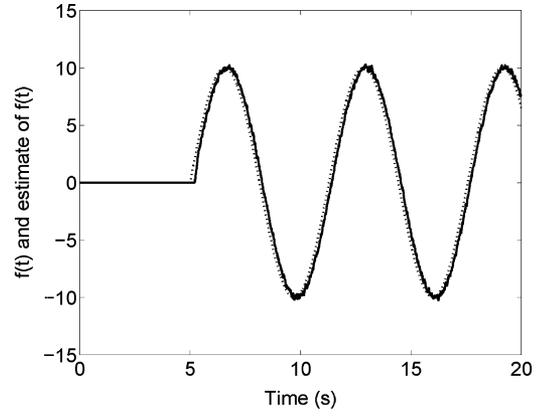


Fig. 4. (Dotted) Time-varying fault $f(t)$ and (solid) its estimate $\hat{f}(t)$.

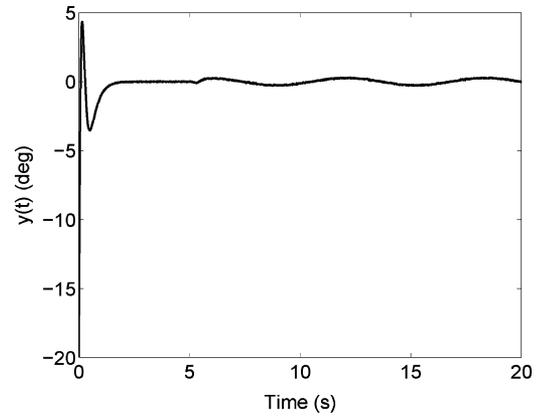


Fig. 5. For time-varying fault, output response under the fuzzy DOFFTC.

Under initial value $(-20\pi/180, 0)$, Fig. 4 illustrates the simulation result of fault estimation, while the system output response is shown in Fig. 5.

From the above simulation results, we can see that, despite the fact that $\text{rank}(C_i B_i) = \text{rank}(B_i)$ is not satisfied, and the open-loop system is unstable, the proposed design still achieves the performance under actuator faults, and the stability of the closed-loop system is guaranteed by the fuzzy DOFFTC. Note that, the proposed fuzzy AFEO design can achieve asymptotical estimation for constant fault, while for the time-varying fault, the fuzzy AFEO can almost realize accurate fault estimation.

V. CONCLUSION

In this paper, a detailed design framework for observer-based robust fault estimation and FTC is developed for a class of nonlinear systems described by a T-S fuzzy model. The framework includes a fuzzy AFEO and DOFFTC to guarantee given stability requirements, while limiting the influence of disturbances, in the presence of actuator faults. Simulation results of an inverted pendulum example are used to show the effectiveness of the obtained results. This paper focuses on fault-tolerant control design for T-S fuzzy systems with external disturbances, while approximation error of T-S fuzzy systems does exist [25]. Therefore, fault-tolerant control design for T-S fuzzy systems with approximation error and how to apply to practical nonlinear systems are meaningful and challenging issues, which will be studied in our future work.

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H_∞ -Filter Design for a Class of Networked Control Systems Via T-S Fuzzy-Model Approach

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Abstract—This paper is concerned with H_∞ -design for a class of networked control systems (NCSs) with multiple state-delays via the Takagi-Sugeno (T-S) fuzzy model. The transfer delays and packet loss that are induced by the limited bandwidth of communication networks are considered. The focus of this paper is on the analysis and design of a full-order H_∞ filter, such that the filtering-error dynamics are stochastically stable, and a prescribed H_∞ attenuation level is guaranteed. Sufficient conditions are established for the existence of the desired filter in terms of linear-matrix inequalities (LMIs). An example is given to illustrate the effectiveness and applicability of the proposed design method.

Index Terms—Filter, fuzzy systems, networked control systems (NCSs).

I. INTRODUCTION

Networked control systems (NCSs) are the control systems in which controller and plant are connected via a communication channel. The defining feature of an NCS is that information (reference input, plant output, control input, etc.) is exchanged using a network among control-system components (sensors, controller, actuators, etc.). NCSs are applicable to many fields like dc motors, advanced aircraft, spacecraft automotive, and manufacturing processes. Therefore, increasing attention has been paid to the study of networked systems (see, for example, [1]–[5] and the references therein). It should be noted that the network itself is a dynamical system including some issues, such as data dropout, limited bandwidth, time delay, and quantization. Thus, conventional control theories for point-to-point control systems must be reevaluated before they can be applied to networked systems.

On the other hand, the existence of time delays is commonly encountered in many dynamic systems, and time delay has become one of the main causes of instability and poor performance of systems [1]. Therefore, the study of NCSs in the presence of network-induced delays has attracted great attention over the past few years. The delays of NCSs are generally described by stochastic process. Such systems have gained persistent attention in the last years, for which a lot of results can be found, such as [6] and [7]. Up until now, there have been three approaches to describe the data-dropout phenomenon, i.e., the Bernoulli process [8], the Markovian jump parameter [9], and the incompleteness matrix [10]. Corresponding literature can be found about filter design for NCSs under these approaches, for example, [3], [11], and [12]. It should be pointed out that most of the considered plants on the issues are linear systems, and to the best of our knowledge, H_∞ filtering problem for NCSs has not been fully investigated, and only a few results are available in the literature, which motivates the present study.

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