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Dynamic Feedback Controller of Euler Angles and Wind parameters estimation for a Quadrotor Unmanned Aerial Vehicle

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***Abstract*— A nonlinear dynamic model for a quadrotor unmanned aerial vehicle is presented with a new vision of state parameter control which is based on Euler angles and open loop positions state observer. This method emphasizes on the control of roll, pitch and yaw angle rather than the translational motions of the UAV. For this reason the system has been presented into two cascade partial parts, the first one relates the rotational motion whose the control law will be applied in a closed loop form and the other one reflects the translational motion. A dynamic feedback controller is developped to transform the closed loop part of the system into linear, controllable and decoupled subsystem. The wind parameters estimation of the quadrotor is used to avoid more sensors. Hence an estimator of resulting aerodynamic moments via Lyapunov function is developed. Performance and robustness of the proposed controller are tested in simulation.**

***Index Terms*— dynamic feedback control, Unmanned Aerial Vehicle, nonlinear estimation**

I. Introduction

Unmanned aerial vehicles are being used more and more in civilian applications such as monitoring of traffic, recognition and surveillance vehicles, search and rescue operations [1], [2]. They are highly capable to flown without an on-board pilot. These robotic aircraft are often computerised and fully autonomous. UAVs have unmatched qualities that make them the only effective solution in specialised tasks where risks to pilots are high, where beyond normal human endurance is required, or where human presence is not necessary.

In 1907, the Breguet Brothers built their first human carrying helicopter, they called it the Breguet-Richet Gyroplane $N^\circ 1$, which was a quad-rotor. However, there was no means of control provided to the pilot other than a throttle for the engine to change the rotor speed, and the stability of the machine was found to be very poor. The machine was subsequently tethered so that it could move only vertically upward.

In 1922 Jerome de Bothezat built one of the largest quadrotor helicopters of the time which flow successfully at low altitudes and forward speeds. However because of insufficient performance the project was cancelled.

In 1920 Etienne Oemichen of france built a quadrotor machine in similar style to that of de Bothezat's but with a number of additionnal rotors for control and propulsion. The initial design was underpowered and it had to have a hydrogen balloon attached to provide additional lift and stability [3]. Although these vehicles have remained relatively simple since they were built by De Bothezat [4], recent technological developments in guidance system, and miniaturization of sensor now allow relatively cheap UAV's to operate in a wide variety of roles. They can be cost effective when properly used and the risk to on board human is eliminated.

The characteristics of the UAV system components are determined by some key operational requirements like endurance, radius of action, altitude and take on/off landing. Their successful application depends on their level of controllability and flying qualities [5]. Because of a fully autonomous operation and faster dynamics which lead to a difficult control design to obtain a satisfactory level of performance, a systematic study of the UAV dynamics representation seems to be necessary. Hence Current UAV related research activities include the following:

- Wind-tunnel and flight based experimental research in aerodynamics and flight performance.
- Modelling of engine/propeller performance and aircraft stability characteristics.
- High fidelity aircraft model development for simulation based control system validation.
- Trajectory optimisation and autonomous guidance for unmanned aircraft.
- Sensor fusion strategies for state estimation using multiple redundant sensors, including Global Positioning Systems (GPS).
- Using GPS for aircraft attitude determination.
- System Identification methods and neural networks for fault detection and reconfiguration.
- Robustness analysis of control laws in the presence of uncertain dynamics and wind gusts.
- Robust nonlinear high-performance manoeuvre tracking for autonomous aircraft.
- Autonomous launch and recovery of the UAV.
- Real-time fight control software synthesis; and design and fabrication of airframe components using advanced composite materials.

Omid Shakernia *et. al* [1] use computer vision as a feedback sensor in a control loop for landing an unmanned air vehicle

(UAV) on a landing pad. They study together the discrete and differential versions of the motion estimation, in order to obtain both position and velocity relative to the landing pad and the only auxiliary sensor are accelerometers for measuring acceleration. They present a performance evaluation of the motion estimation algorithms under varying levels of image measurement noise, altitudes of the camera above the landing pad. Using geometric nonlinear control theory, their dynamics are decoupled into an inner system and outer system. For the overall closed-loop system, conditions are provided under which exponential stability can be guaranteed.

Erdinç Altug *et. al* [6] present two methods of control of the quadrotor, the first one using feedback linearizing controllers, and the other using back-stepping-like control law. Both approaches have been used to control translational motion (x, y, z) and yaw angle (ψ), which needs higher order lie derivatives (first method) and huge calculation (second method).

Vincent Mistler *et. al* [7] developped the dynamic model in non linear state space representation, and used an exact linearization and non-interacting control for the global system to evaluate translational motion and yaw angle outputs. A delay of control inputs here seems to be necessary to avoid singularity and make the problem solvable.

In the present work a feedback linearization has been used to control a partial dynamic system based on rotational motion (tilt angles) rather than translational motion. This technique has been adopted for different reasons:

1) To encounter the translational motion measure which is rather difficult and instrumentation in this case (GPS, camera,...) are not accurate enough.
2) Tilt angles velocity and acceleration can be measured by inertial sensors accurately.
3) Dynamic equation which relates yaw, pitch, roll angles to external torques represent a full rank system so singularity is avoided directly and there is no need to delay control inputs.
   An observer is used to construct positions (x, y) through the controlled tilt angles output and thrust power input. The altitude z is also controlled since this last is affected directly by thrust power and roll-pitch angles, and its construction through an observer is not evident.

The paper is organized as follows: the equation of motion is given in section 2, the control law is given in section 3, wind parameters estimation is given in section 4 and simulation results with comparison is given in section 5.

II. UAV BEHAVIOR DESCRIPTION

An anmanned aerial vehicle can be defined as any flying machine using rotating wing (ie. rotors) to provide lift, propulsion, and control forces that enable the vehicule to hover relative to the ground without forward flight speed. The thrust on the rotors is generated by the aerodynamic lift forces. The rotor is the primary source of control and propulsion for the UAV. The euler angle orientation to the flow provides the forces and moments to control the altitude and position of the system.

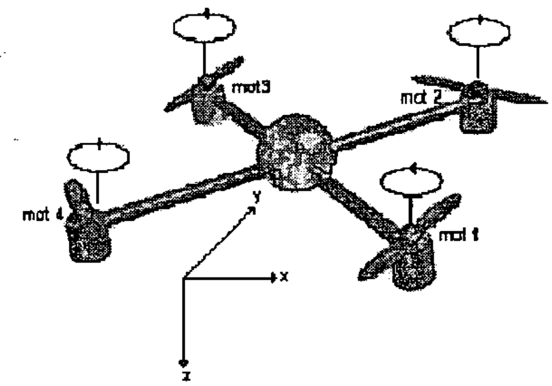

Fig. 1. The miniature four rotors helicopter.

The quadrotor helicopter is shown in 1. Two diagonal motors (1 and 3) are running in the same direction (anticlockwise) whereas the two others (2 and 4) in the clockwise direction to eliminate the anti-torque. On varying the rotor speeds altogether with the same quantity the lift forces will change affecting in this case the altitude z of the system and enabling vertical take-off/on landing. Yaw angle is obtained by speeding up the clockwise motors or slowing down depending on the desired angle direction. Tilting around x (roll angle) axe allows the quadrotor to move toward y direction. The sense of direction depend on the sense of angle whether it is positive or negative. Tilting around y (pitch angle) axe allows the quadrotor to move toward x direction.

However there are some fundamental technical problems which can be identified:

- understanding the aerodynamics of vertical flight and knowing The theoretical power required to produce a fixed amount of lift.
- Keeping structural weight and engine weight down so the machine could lift a payload.
- Conquering the problem of vibrations. This was a source of many mechanical failure of the rotors and airframe.
  The reasons behind the complexity of control design for underactuated systems is that they are not fully feedback linearizable. Moreover, many recent and traditional methods of nonlinear control design including backstepping [8] [11], forwarding [10] [13] [14] [15], high-gain/low-gain designs [12] [13], and sliding mode control [9] are not directly applicable to underactuated systems with the exception of a few special cases.

A. UAV Dynamics

The dynamic model describing the UAV position and attitude is obtained using Newton equations. The considered UAV is a miniature four rotors helicopter (Figure 1). Each rotor consists of an electric DC motor, a drive gear and a rotor blade. Forward motion is accomplished by increasing the speed of the rear rotor while simultaneously reducing the forward rotor by the same amount. Back, left and right motion work in the same way. Yaw command is accomplished by accelerating the two clockwise turning rotors while decelerating the counter-clockwise turning rotors.

The equations describing the attitude and position of an UAV are basically those of a rotating rigid body with six degrees of freedom [16] [17]. They may be separated into *kinematic* equations and *dynamic* equations [18].

The kinematic equations may be represented as follows. The *absolute position* of the UAV is described by the three coordinates (x_0, y_0, z_0) of its center of mass with respect to an earth fixed inertial reference frame and its *attitude* by the three Euler's angles (ψ,θ,ϕ). These three angles are respectively called yaw angle $(-\pi \leq \psi < \pi)$, pitch angle $(-\frac{\pi}{2} < \theta < \frac{\pi}{2})$ and roll angle $(-\frac{\pi}{2} < \phi < \frac{\pi}{2})$.

The derivatives with respect to time of the angles (ψ,θ,ϕ) can be expressed in the form

$$col(\dot{\psi}, \dot{\theta}, \dot{\phi}) = N(\psi, \theta, \phi)\omega \quad (1)$$

in which $\omega = col(p, q, r)$ is the angular velocity expressed with respect to a body reference frame and $N(\psi, \theta, \phi)$ is the 3x3 matrix given by

$$N(\psi,\theta,\phi) = \begin{bmatrix} 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \\ 0 & \cos\phi & -\sin\phi \\ 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \end{bmatrix}$$

This matrix, as shown, depends only on (ψ, θ, ϕ) and is invertible if the above conditions on (ψ, θ, ϕ) hold.

Similarly, the derivative with respect to time of the position (x_0, y_0, z_0) is given by

$$col(\dot{x}_0, \dot{y}_0, \dot{z}_0) = V_0 \quad (2)$$

where $V_0 = col(u_0, v_0, w_0)$ is the absolute velocity of the UAV expressed with respect to an earth fixed inertial reference frame. Let $V = col(u, v, w)$ be the absolute velocity of the UAV expressed in a body fixed reference frame. V and V_0 are related by

$$V_0 = R(\psi, \theta, \phi)V$$

where $R(\psi, \theta, \phi)$ is the rotation matrix given by[1]

$$R = \begin{bmatrix} C\theta C\psi & C\psi S\theta S\phi - C\phi S\psi & C\phi C\psi S\theta + S\phi S\psi \\ C\theta S\psi & S\theta S\phi S\psi + C\phi C\psi & C\phi S\theta S\psi - C\psi S\phi \\ -S\theta & C\theta S\phi & C\theta C\phi \end{bmatrix}$$

(1) and (2) are the kinematic equations. The dynamic equations are now expressed. Using the Newton's laws about the center of mass, one obtains the dynamic equations for the miniature four rotors helicopter[2]

$$\begin{aligned} m\dot{V}_0 &= \sum F_{ext} \\ J\dot{\omega} &= -\omega \times J\omega + \sum T_{ext} \end{aligned} \quad (3)$$

m is the mass, J is the inertia matrix given by

$$J = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix}$$

and $\sum F_{ext}$, $\sum T_{ext}$ represent the vector of external forces and external torques respectively. They contain the helicopter's weight, the aerodynamic forces vector, the thrust and the torque developed by the four rotors. Some calculations yield the following form for theses two vectors

$$\begin{aligned} \sum F_{ext} &= \begin{bmatrix} A_x - (C\phi C\psi S\theta + S\phi S\psi)u_1 \\ A_y - (C\phi S\theta S\psi - C\psi S\phi)u_1 \\ A_z + mg - (C\theta C\phi)u_1 \end{bmatrix} \quad (4) \\ \sum T_{ext} &= \begin{bmatrix} A_p + u_2 d \\ A_q + u_3 d \\ A_r + u_4 \end{bmatrix} \end{aligned}$$

in which

- $col(A_x, A_y, A_z)$ and $col(A_p, A_q, A_r)$ are the resulting aerodynamic forces and moments acting on the UAV and are computed from the aerodynamic coefficients C_i as $A_i = \frac{1}{2}\rho_{air}C_iW^2$ (ρ_{air} is the air density, W is the velocity of the UAV with respect to the air) [17].
- g is the gravity constant ($g = 9.81ms^{-2}$);
- d is the distance from the center of mass to the rotors;
- u_1 is the resulting thrust of the four rotors;
- u_2 is the difference of thrust power between the left rotor and the right rotor (y direction);
- u_3 is the difference of thrust power between the front rotor and the back rotor (x direction);
- u_4 is the difference of torque between the two clockwise turning rotors and the two counter-clockwise turning rotors.

Each rotor undergoes thrust and torque and of course leaves a wake behind as it moves. If the velocity induced by the wake is omitted, it can be shown that they are proportional to the square of the angular speed of the rotor shaft [19] . Assuming that the electric motors are velocity controlled, then (u_1, u_2, u_3, u_4) may be considered directly as control inputs.

B. *Partial decomposition*

With the state vector $x_1 = col(\psi, \theta, \phi)$; and the state vector $x_2 = col(x_0, y_0, z_0)$ and from equations (1),(3) and (4) , we may write the dynamic equation of the system as:

$$M(x_1)\ddot{x}_1 + K(x_1, \dot{x}_1) = \sum T_{ext} \quad (5)$$

$$m\ddot{x}_2 = \sum F_{ext} \quad (6)$$

with

$$M(x_1) = \begin{bmatrix} -I_x sin(\theta) & 0 & I_x \\ I_y cos(\theta)sin(\phi) & I_y cos(\phi) & 0 \\ I_z cos(\theta)cos(\phi) & -I_z sin(\phi) & 0 \end{bmatrix}$$

and

$$K(x_1, \dot{x}_1) = \begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix}$$

[1] $C\theta$, $S\theta$ and $T\theta$ denote respectively $\cos(\theta)$, $\sin(\theta)$ and $\tan(\theta)$

[2] $\times$ denotes the usual "vector" product

$$
\begin{aligned}
K_1 = & -I_x cos(\theta)\dot{\theta}\dot{\psi} + cos(\theta)^2 sin(\phi)\dot{\psi}^2 I_z cos(\phi) - \\
& cos(\theta)sin(\phi)^2\dot{\psi}I_z\dot{\theta} + cos(\phi)^2\dot{\theta}I_z cos(\theta)\dot{\psi} - \\
& cos(\phi)\dot{\theta}^2 I_z sin(\phi) - cos(\theta)^2 cos(\phi)\dot{\psi}^2 I_y sin(\phi) - \\
& cos(\theta)cos(\phi)^2\dot{\psi}I_y\dot{\theta} + sin(\phi)^2\dot{\theta}I_y cos(\theta)\dot{\psi} + \\
& sin(\phi)\dot{\theta}^2 I_y cos(\phi)
\end{aligned}
$$

$$
\begin{aligned}
K_2 = & -I_y\dot{\psi}sin(\theta)sin(\phi)\dot{\theta} + I_y\dot{\psi}cos(\theta)cos(\phi)\dot{\phi} - \\
& I_y sin(\phi)\dot{\phi}\dot{\theta} - cos(\theta)cos(\phi)\dot{\psi}^2 I_x sin(\theta) + \\
& cos(\theta)cos(\phi)\dot{\psi}I_x\dot{\phi} + sin(\phi)\dot{\theta}I_x sin(\theta)\dot{\psi} - \\
& sin(\phi)\dot{\theta}I_x\dot{\phi} + sin(\theta)\dot{\psi}^2 I_z cos(\theta)cos(\phi) - \\
& sin(\theta)\dot{\psi}I_z sin(\phi)\dot{\theta} - \dot{\phi}I_z cos(\theta)cos(\phi)\dot{\psi} + \\
& \dot{\phi}I_z sin(\phi)\dot{\theta}
\end{aligned}
$$

$$
\begin{aligned}
K_3 = & -I_z\dot{\psi}sin(\theta)cos(\phi)\dot{\theta} - I_z\dot{\psi}cos(\theta)sin(\phi)\dot{\phi} - \\
& I_z cos(\phi)\dot{\phi}\dot{\theta} - sin(\theta)\dot{\psi}^2 I_y cos(\theta)sin(\phi) - \\
& sin(\theta)\dot{\psi}I_y cos(\phi)\dot{\theta} + \dot{\phi}I_y cos(\theta)sin(\phi)\dot{\psi} + \\
& \dot{\phi}I_y cos(\phi)\dot{\theta} + cos(\theta)sin(\phi)\dot{\psi}^2 I_x sin(\theta) - \\
& cos(\theta)sin(\phi)\dot{\psi}I_x\dot{\phi} + cos(\phi)\dot{\theta}I_x sin(\theta)\dot{\psi} - \\
& cos(\phi)\dot{\theta}I_x\dot{\phi}
\end{aligned}
$$

To avoid complications on control aim and to not fall in an underactuated system, it is necessary to set the number of input equal to the number of output channels. Since the input signals are u_1, u_2, u_3, u_4 the output signals to be controlled are chosen as :

$$y_c = col(x_0, y_0, z_0, \psi)$$

To encounter the translational motion measure which is rather difficult and instrumentation in this case (GPS, camera,...) are not accurate enough, we have chosen to use the following measured signals for control:

$$y_m = col(\psi, \theta, \phi, z_0)$$

This choice is argued from the following:

1) Tilt angles, velocities and accelerations can be measured by inertial sensors accurately.
2) The dynamic equation which relates yaw, pitch, roll angles to external torques represent a full rank system so singularity is avoided directly and there is no need to delay control inputs.
3) An observer can be used to construct positions (x, y) through the controlled tilt angles output and thrust power input.
4) The altitude z_0 is also controlled since this last is affected directly by thrust power and roll-pitch angles, and its construction through an observer is not possible.

So the system to be controlled can be represented as follow

$$\begin{bmatrix} \frac{-m}{C\theta C\phi} & 0 \\ 0 & M(x_1) \end{bmatrix} \begin{bmatrix} \ddot{z} \\ \ddot{x}_1 \end{bmatrix} + \begin{bmatrix} \frac{m}{C\theta C\phi} g \\ K(x_1, \dot{x}_1) \end{bmatrix} = \begin{bmatrix} \frac{A_z}{C\theta C\phi} + u_1 \\ A_p + u_2 d \\ A_q + u_3 d \\ A_r + u_4 \end{bmatrix} \quad (7)$$

III. Feedback linearization control

The feedback linearization control is applied to the equation (7) with inputs u_1, u_2, u_3, u_4 and outputs ψ, θ, ϕ, z_0. Though these methods were rather successful in local analysis of nonlinear systems affine in control they usually fail to work for a global analysis and nonlinear systems that are non-affine in control [20].

A. Controller without wind disturbances

If one consider no disturbance on the system ($A_p = A_q = A_q = A_z = 0$), th control law will then be

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \left(\begin{array}{c} -\frac{m}{C(\theta)C(\phi)}(v_1 - g) \\ D^{-1}\left(M(x_1) \begin{bmatrix} v_2 \\ v_3 \\ v_4 \end{bmatrix} + K \right) \end{array} \right)$$

with

$$D = \begin{pmatrix} d & 0 & 0 \\ 0 & d & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$M(x_1)$ is a non singular matrix. Hence the system is transformed from nonlinear system into a linear and controllable one, so the closed loop system is reduced to four double integrators:

$$
\begin{aligned}
\ddot{z}_0 &= v_1 \\
\ddot{\psi} &= v_2 \\
\ddot{\theta} &= v_3 \\
\ddot{\phi} &= v_4
\end{aligned}
$$

Let the tracking error signals as:

$$
\begin{aligned}
e_z &= z_{0d} - z, \quad \dot{e}_z = \dot{z}_{0d} - \dot{z} \\
e_\psi &= \psi_d - \psi, \quad \dot{e}_\psi = \dot{\psi}_d - \dot{\psi} \\
e_\theta &= \theta_d - \theta, \quad \dot{e}_\theta = \dot{\theta}_d - \dot{\theta} \\
e_\phi &= \phi_d - \phi, \quad \dot{e}_\phi = \dot{\phi}_d - \dot{\phi}
\end{aligned}
$$

From equation of forces and taking into account only the measured signals, the relations between desired positions and desired angles (the desired roll angle $\phi_d(\ddot{x}_{0d}, \ddot{y}_{0d}, \psi_d, u_1)$ and the desired pitch angle $\theta_d(\ddot{x}_{0d}, \ddot{y}_{0d}, \psi_d, u_1)$) are derived from:

$$m \begin{bmatrix} \ddot{x}_{0d} \\ \ddot{y}_{0d} \end{bmatrix} = \begin{bmatrix} -(\cos\phi_d \cos\psi_d \sin\theta_d + \sin\phi_d \sin\psi_d) u_1 \\ -(\cos\phi_d \sin\theta_d \sin\psi_d - \cos\psi_d \sin\phi_d) u_1 \end{bmatrix}$$

The reference signals are given by:

$$
\begin{aligned}
v_1 &= \ddot{z}_{0d} + \lambda_v \dot{e}_z + \lambda_p e_z \\
v_2 &= \ddot{\psi}_d + \lambda_v \dot{e}_\psi + \lambda_p e_\psi \\
v_3 &= \ddot{\theta}_d + \lambda_v \dot{e}_\theta + \lambda_p e_\theta \\
v_4 &= \ddot{\phi}_d + \lambda_v \dot{e}_\phi + \lambda_p e_\phi
\end{aligned}
$$

The differential equation for the tracking error is then given as follows:

$$
\begin{aligned}
\ddot{e}_z + \lambda_v \dot{e}_z + \lambda_p e_z &= 0 \\
\ddot{e}_\psi + \lambda_v \dot{e}_\psi + \lambda_p e_\psi &= 0 \\
\ddot{e}_\theta + \lambda_v \dot{e}_\theta + \lambda_p e_\theta &= 0 \\
\ddot{e}_\phi + \lambda_v \dot{e}_\phi + \lambda_p e_\phi &= 0
\end{aligned}
$$

if λ_v and λ_p are chosen to assign a specific set of eigenvalues then the system will converge exponentially.

B. Wind parameters compensation

The control law with the estimating aerodynamic moments can be chosen as an adaptive controller drawn from the adaptive control theory applied for robot manipulators because it avoids acceleration measurements as shown in [22] [23]. The proposed controller is derived from equation (5) written as:

$$M(x_1)\,\ddot{x}_1 + K(x_1,\dot{x}_1) = Du + A_m \tag{8}$$

with $u = col(u_2, u_3, u_4)$ the control inputs and $A_m = col(A_p, A_q, A_r)$ is the resulting aerodynamic moments acting on the UAV.

The control law is given by:

$$u = D^{-1}\left[M(x_1)\left[\ddot{x}_{1r} - K_v s\right] + K(x_1,\dot{x}_1) - \hat{A}_m\right] \tag{9}$$

where $\hat{A}_m$ is the estimated value of A_m and the reference and composite signals are given by:

$$\ddot{x}_{1r} = \ddot{x}_{1d} - \lambda\dot{e}; \quad e = x_1 - x_{1d}$$

hence:

$$s = \dot{e} + \lambda e = \dot{x}_1 - \dot{x}_{1r}, \quad \dot{s} = \ddot{x}_1 - \ddot{x}_{1r}$$

The gain matrices K_v and λ are chosen to be positive. From the equations (8) and (9) we can deduce the following equation:

$$\dot{s} + K_v s = M^{-1}\tilde{A}_m \tag{10}$$

with $\tilde{(.)} = \hat{(.)} - (.)$.

The adaptation law of the adjustable parameter is given by:

$$\dot{\tilde{A}}_m = \dot{\hat{A}}_m = -\Gamma\left(M^{-1}\right)^T s \tag{11}$$

From the above equations of model and control the Lyapunov function candidate is chosen as:

$$V = \frac{1}{2}s^T s + \frac{1}{2}\tilde{A}_m^T\Gamma^{-1}\tilde{A}_m$$

$$\dot{V} = s^T\dot{s} + \tilde{A}_m^T\Gamma^{-1}\dot{\tilde{A}}_m \tag{12}$$

Using equations (10) and (11) in equation (12) it is obtained:

$$\dot{V} = -s^T K_v s < 0 \tag{13}$$

It follows from (13) that V is decreasing with along the systems's trajectory. From this it can be concluded that the tracking errors converge asymptotically to zero ($s \longrightarrow 0$ as $t \longrightarrow \infty$). Therefore, as $e(p) = H(p)s(p)$, with $H(p)$ strictly proper and exponentially stable transfer matrix, so by using Desoer lema [21] it follows that $e \longrightarrow 0$ and $\dot{e} \longrightarrow 0$ as $t \longrightarrow \infty$.

IV. Simulation results

For reason of observability the reference trajectory proposed here is chosen in a manner to avoid initial condition problem when solving equations of position (x_0, y_0, z_0). So the desired trajectory is filtered with a third order filter defined by the transfer function $H_f(p)$ to make it smooth in curve and zero initial conditions before exciting system. The chosen reference trajectories are given by :

$$\begin{aligned}
x_{0d} &= \frac{1}{2}\cos(\frac{t}{2}) \\
y_{0d} &= \frac{1}{2}\sin(\frac{t}{2}) \\
z_{0d} &= -1 - \frac{t}{10} \\
\psi_d &= \frac{\pi}{3} \\
H_f(p) &= \frac{0.125}{p^3 + 1.5p^2 + 0.7p + 0.125}
\end{aligned}$$

the constant parameters of the quadrotor are:

$$\begin{aligned}
m &= 2Kg,\, I_x = I_y = I_z = 1.2416 N/rad/s^2, \\
d &= 0.1m,\, g = 9.81 m/s^2
\end{aligned}$$

The control parameters are:

$$\lambda_v = 16;\ \lambda_p = 64$$

Simulation results for desired and observed positions and trajectories are presented:

A. Flight without perturbation

Results without perturbation are shown in Fig-4 to Fig-9

B. Flight with parametric uncertainties

For a uncertainty of -20% on m, Ix, Iy, Iz and d, parameters robustness of the quadrotor is shown bellow (Fig_10,Fig-11):

C. Compensation of wind disturbances:

Parameters Ap, Aq, and Ar have been introduced in control law to analyze the behavior of the system when taking into account perturbation in the controller. For $Ap = 0.02, Aq = 0.03, Ar = 0.04$ the following results are obtained (Fig-12 to Fig14) :

D. Discussion of results

It is seen from reference trajectories tracking without perturbation (Fig-4) that feedback linearization control on Euler angles is acceptable since the tracking of angles and positions was perfect (Fig5,6). This may be confirmed by the angle tracking errors (Fig-7) and the position tracking error (Fig-8) which vanished on time. The control signal figure (Fig_8) shows a smooth curve, this is due to the choice of optimal control gain λ_v and λ_p.

An uncertainties on inertia coefficients and mass of the quadrotor of 20% were taken to analyze the robustness of the control law. Note that all this parameters affect the angle dynamic equations which are presented in closed loop form. These variations were successfully compensated by controller. from Fig-10 representing the angle tracking error and Fig-11 representing the position tracking error it is seen that the actual output ψ, θ, ϕ, z converge perfectly to the desired output, however the actual output x, y converge with a small error less than 1%.

A study of the system with the presence of wind perturbation was presented. An estimation of the wind parameters through lyapunov function and adaptation gain was elaborated. The tracking errors for Euler angles and positions were presented in Fig-12 and Fig-13. The convergence were confirmed for angles whereas the position errors reveal a small oscillation of 1%. The estimated wind parameters for a nominal values of $A_p = 0.02; A_q = 0.03$ and $A_r = 0.04$; are shown in Fig-11 .

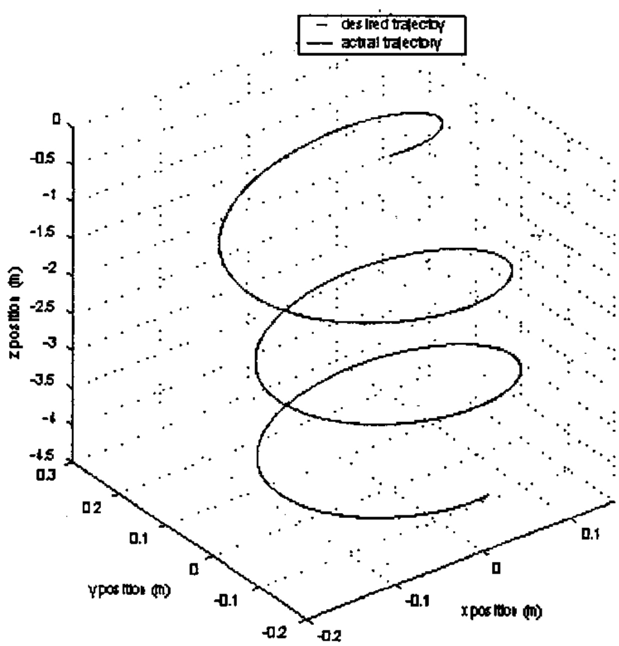

Fig. 2. Reference trajectory

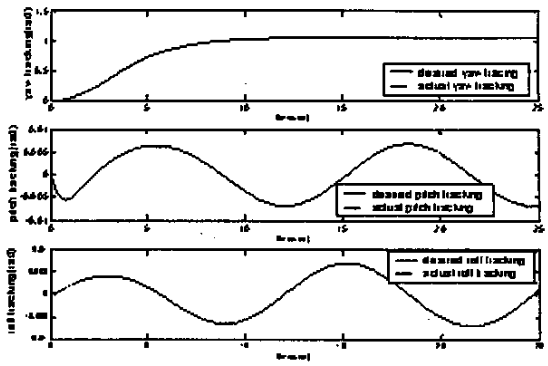

Fig. 3. Angle trajectories for ψ, θ, ϕ .

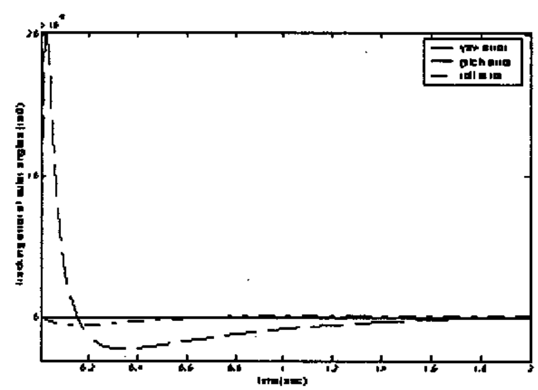

Fig. 4. Tracking error for ψ, θ, ϕ

V. Conclusion

A new vision of state parameters control has been presented. The comparison with full system representation has been done showing the same performance and stability when using observation of outputs (x_0, y_0) , without needing the use of sensors to measure them. However the system behavior toward aerodynamic forces disturbance (A_x, A_y) and moment disturbance(A_p, A_q, A_r, A_z) seems to be quite different . In the first one disturbances the system is more sensitive to variation than the other, since the variation on (A_p, A_q, A_r) can be easily compensated by control law, however (A_x, A_y) will affect the forces dynamic model of the displacement (x_0, y_0) which are represented in an open loop form and are not easy to be compensated directly by the same controller used here.

It is clearly seen that feedback linearization control is not robust enough toward wind disturbances either taking a control

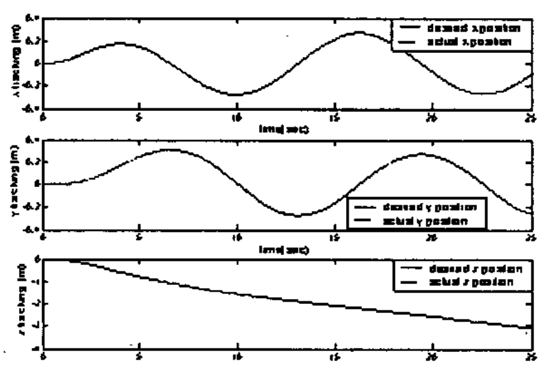

Fig. 5. Position trajectories for x, y, z

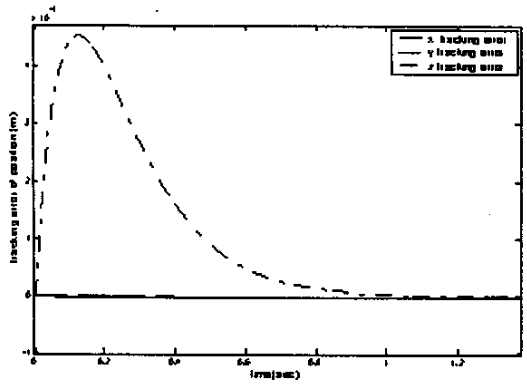

Fig. 6. Tracking errors for x, y, z

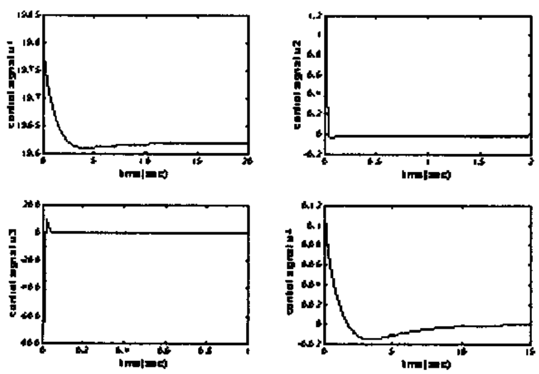

Fig. 7. Control signals u_1, u_2, u_3, u_4

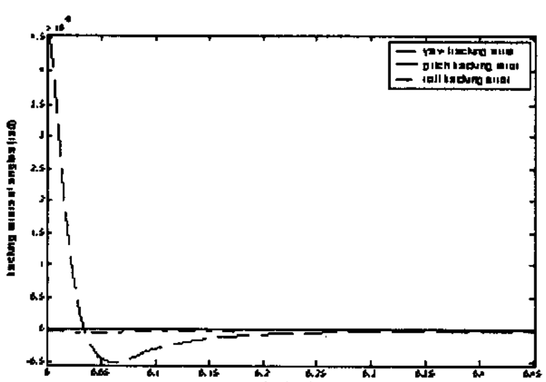

Fig. 8. Angle tracking errors for incertainties of 20% on m, I_x, I_y, I_z

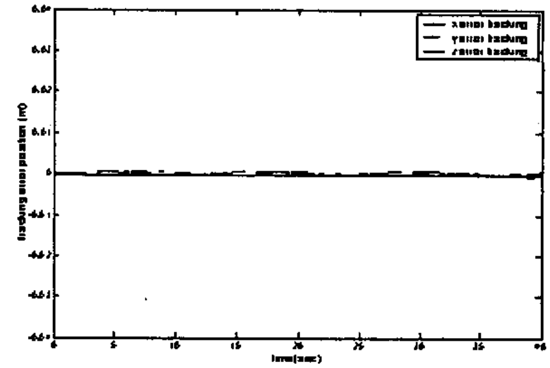

Fig. 9. Position tracking errors for incertainties of 20% on m, I_x, I_y, I_z

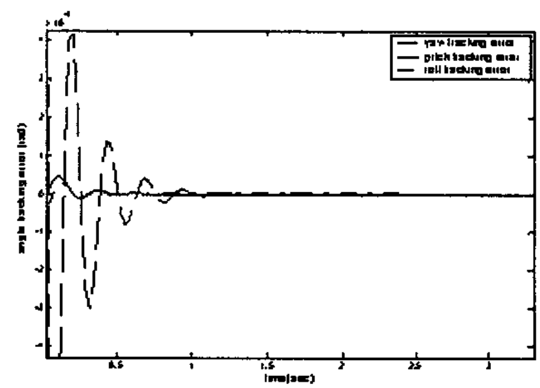

Fig. 10. Angle tracking errors with with disturbance for $Ap = 0.02; Aq = 0.03; Ar = 0.04$

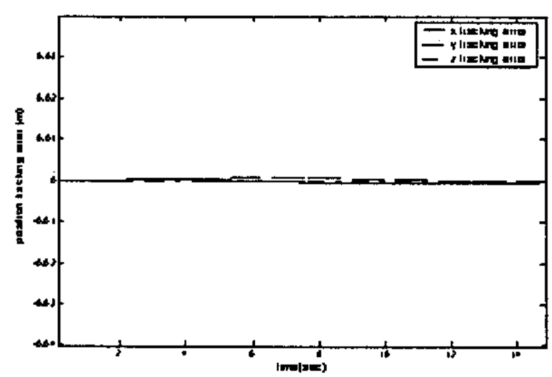

Fig. 11. Position tracking errors with with disturbance for $Ap = 0.02; Aq = 0.03; Ar = 0.04$

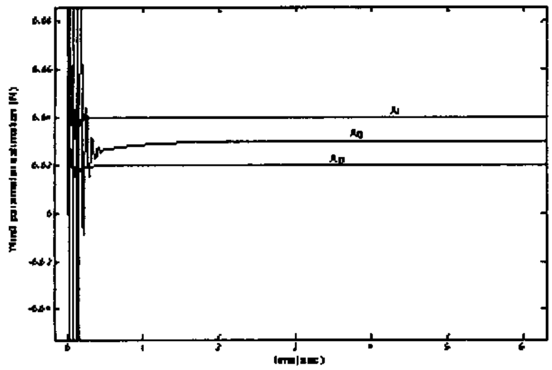

Fig. 12. Estimation of wind parameters for nominal values $Ap = 0.02; Aq = 0.03; Ar = 0.04$

law for (x_0, y_0, z_0, ψ) or $(\psi, \theta, \phi, z_0)$ outputs. This is because of feedback linearization method which does not really reflect the nonlinear system since it transforms the system into a set of cascade integrals (Brunovski form). However full system controller or partial system controller with observer give satisfactory desired trajectory tracking without extern disturbances. Further investigation will be based on robust feedback linearization to allow robust linear control like H_∞, GH_∞, μ synthesis and so on to be implemented.

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