CONTROL OF INVERTED PENDULUM CART SYSTEM BY USE OF PID CONTROLLER

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ABSTRACT; *In this paper, we have tried to control the nonlinear inverted pendulum-cart dynamic system by the use of Proportional-Integral-Derivative (PID) controller.*
We know that the inverted pendulum-cart dynamic system is an inherently unstable and nonlinear system when we want to use PID controller for it, we should change the system to a linear system, then we can apply a PID controller on it, to this end we will use Linear Quadratic Regulator (LQR). LQR is an optimal control technique. Duration the paper simulation results are presented to verify the effectiveness of the proposed control.

Keywords; Inverted Pendulum; nonlinear system; inherently unstable; PID; control; LQR.

INTRODUCTION

The control of the inverted pendulum and, which was proposed in [1], has been studied by many authors, [2, 3, 4] and it is a common example in Control Theory. The main characteristics that make it an interesting case of study are that it is a strongly nonlinear, unstable, non minimum-phase, which affects stability margins and robustness, and under actuated system (there is only one actuator for more degrees of freedom) and, therefore, a difficult and complex system to be controlled that has been used as a benchmark to compare different control strategies. Nearly all works on pendulum control concentrate on two problems: pendulums swing up control design and stabilization of the inverted pendulums. In this paper, optimal nonlinear stabilization problem is addressed: stabilize IPC minimizing an accumulative cost functional quadratic in states and controls. For linear systems, this leads to linear feedback control, which is found by solving a Riccati equation [5], and thus referred to as linear quadratic regulator (LQR).

The inverted pendulum has two equilibrium points, one of which is stable while the other is unstable. On the one hand, the stable equilibrium point corresponds to a state in which the pendulum is pointing downwards. In the absence of any control force, the system will naturally return to this state. On the other hand, the unstable equilibrium point is upwards. The different control strategies try to reach and maintain this unstable equilibrium position.

IPC, however, is a highly nonlinear system, and its linearization is far from adequate for control design purposes. Nonetheless, the LQR will be considered as a baseline controller for our work.

In general, the control problem consists of obtaining dynamic models of systems, and using these models to determine control laws or strategies to achieve the desired system response and performance. The simplicity of control algorithm as well as to guarantee the stability and robustness in the closed-loop system is challenging task in real situations. Most of the dynamical systems such as power systems, missile systems, robotic systems, inverted pendulum, industrial processes, chaotic circuits etc. are highly nonlinear in nature. The control of such systems is a challenging task.

The control of the inverted pendulum and, which was The Proportional-Integral-Derivative (PID) control gives the simplest and yet the most efficient solution to various real-world control problems. Both the transient and steady state responses are taken care of with its three-term (i.e. P, I, and D) functionality. Since its invention the popularity of PID control has grown tremendously. The advances in digital technology have made the control system automatic. The automatic control system offers a wide spectrum of choices for control schemes, even though, more than 90% of industrial controllers are still implemented based around the PID algorithms, particularly at the lowest levels, as no other controllers match with the simplicity, clear functionality, applicability, and ease of use offered by the PID controller.

The performance of the dynamical systems being controlled is desired to be optimal. There are many optimization & optimal control techniques which are present in the literatures for linear & nonlinear dynamical systems[5-7]. The recent development in the area of artificial intelligence (AI), such as artificial neural network (ANN), fuzzy logic theory (FL), and evolutionary computational techniques such as genetic algorithm (GA), and particle swarm optimization (PSO) etc., commonly all these are known as intelligent computational techniques which have given novel solutions to the various control system problems.

The intelligent optimal control has emerged as viable recent approach by the application of these intelligent computational techniques [8-18]. There are many examples in literature in which the inverted pendulum-cart dynamical system has been applied in implementing the various control schemes [16-21]. Linear quadratic regulator (LQR), an optimal control method, and PID control which is generally used for control of the linear dynamical systems have been used in this paper to control the nonlinear inverted pendulum-cart dynamical system. In recent trends even the various advance control approaches are developing and being tried for many dynamical systems control, the proposed control method is simple, effective, and robust.

Description mathematical modelling

Inverted pendulum system equations

The free body diagram of an inverted pendulum mounted on a motor driven cart is shown in Fig. 1 [1-4]. The system equations of this nonlinear dynamic system can be derived as follows. It is assumed here that the pendulum rod is mass-less, and the hinge is frictionless. In such assumption, the whole pendulum mass is concentrated in the centre of gravity (COG) located at the center of the pendulum ball. The cart mass and the ball point mass at the upper end of the inverted pendulum are denoted as M and m, respectively. There is an externally x-directed force on the cart, u(t) , and a gravity force acts on the point mass at all times. The coordinate system considered is shown in Fig. 1, where x(t) represents the cart position, and $\dot{x}$ (t) is the tilt angle referenced to the vertically upward direction. A force balance on the system in the x-direction can be written as

$$M\frac{d^2}{dt^2}x + m\frac{d^2}{dt^2}x_G = u \qquad (1)$$

Where, the time-dependent centre of gravity (COG) of the point mass is given by the coordinates, (x_G ,y_G).

For the point mass assumed here, the location of the center of gravity of the pendulum mass is simply

$X_G = X + l\sin\theta$ and $y_G = l\cos\theta$ (2)

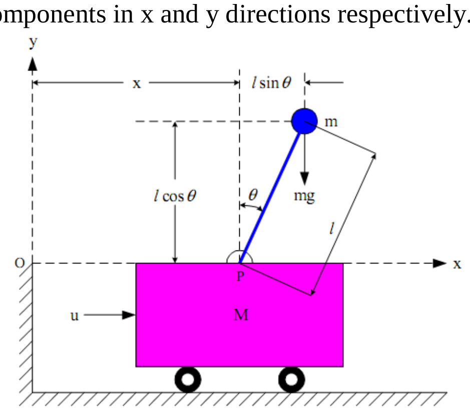
Where l is the pendulum rod length. Substituting (2) into (1) it is written as:

$$(M+m)\ddot{x} - ml\sin\theta\,\ddot{\theta} + ml\cos\theta\,\ddot{\theta} = u \qquad (3)$$

In a similar way, a torque balance on the system is performed. Fig. 2 shows the force components acting on the system. The resultant torque balance can be written as:

$(F_x \cos\theta)l - (F_y \sin\theta)l = (mg \sin\theta)l$ (4)

Where, $F_x = m\frac{d^2}{dt^2}x_G$ and $F_y = m\frac{d^2}{dt^2}y_G$ are the force components in x and y directions respectively.



Inverted Pendulum - Cart System

Fig. 1 Motor driven inverted pendulum-cart system

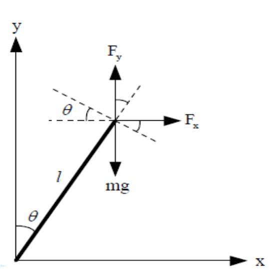


Fig. 2 Vector diagram for force components in torque balance

After manipulation (4) is written as:

$$m\ddot{x}\cos\theta + ml\ddot{\theta} = mg\sin\theta \qquad (5)$$

Equations (3) and (5) are the defining equations for this system. These two equations are manipulated algebraically to have only a single second derivative term in each equation. Finally, we may derive the system equations describing the cart position dynamics and the pendulum angle dynamics respectively. Thus we have:

$$\ddot{x} = \frac{u + ml(\sin\theta)\theta^2 - mg\cos\theta\sin\theta}{M + m - m\cos^2\theta} \qquad (6)$$

$$\ddot{\theta} = \frac{u\cos\theta - (M+m)g\sin\theta + ml(\cos\theta\sin\theta)\dot{\theta}}{ml\cos^2\theta - (M+m)l} \qquad (7)$$

Equations (6) and (7) represent a nonlinear system which is relatively complicated from a mathematical viewpoint. Following subsection presents the standard state space form of these two nonlinear equations.

Nonlinear system state space equations of inverted pendulum

For numerical simulation of the nonlinear model for the inverted pendulum-cart dynamic system, it is required to represent the nonlinear equations (6) and (7) into standard state space form:

$$\frac{d}{dt}x = f(x,u,t) \qquad (8)$$

Considering the state variables as following:

$$x_1 = \theta \;,\quad x_2 = \dot{\theta} = \dot{x}_1 \;,\quad x_3 = x \;,\quad x_4 = \dot{x} = \dot{x}_3 \qquad (9)$$

Then, the final state space equation for the inverted pendulum system may be written as:

$$\frac{d}{dt}x = \frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \frac{d}{dt}\begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix}$$

Where

$$f_1 = x_2 \qquad (11)$$

$$f_2 = \frac{u\cos x_1 - (M+m)g\sin x_1 + ml(\cos x_1\sin x_1)x_2^2}{ml\cos^2 x_1 - (M+m)l} \qquad (12)$$

$$f_3 = x_4 \qquad (13)$$

$$f_4 = \frac{u + ml(\sin x_1)x_2^2 - mg\cos x_1\sin x_1}{M + m - m\cos^2 x_1} \qquad (14)$$

If both the pendulum angle θ and the cart position x are the variables of interest, then the output equation may be written as:

$$y = cx \quad or \quad y = \begin{bmatrix} \theta \\ x \end{bmatrix} = cx = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix} \qquad (15)$$

Equations (10) and (15) give a complete state space representation of the nonlinear inverted pendulum-cart dynamic system.

Linear system state space equations of inverted pendulum

Since the goal of this particular system is to keep the inverted pendulum in upright position around θ=0, the linearization might be considered about this upright equilibrium point. The linear model for the system around the upright stationary point is derived by simply linearization of the nonlinear system given in (10). Since the usual A and B matrices are zero for this case; and so every term is put into the nonlinear vector function, f (x,u, t) , then the linearized form for the system becomes:

$$\frac{d}{dt} = \delta x = J_x(x_0, u_0)\delta x + J_u(x_0, u_0)\delta u \qquad (16)$$

Where, the reference state is defined with the pendulum stationary and upright with no input force. Under these conditions, X0=0 , and u0=0. Since the nonlinear vector function is rather complicated, the components of the Jacobian matrices are determined systemically, term by term. The elements of the first second, third, and fourth Columns of $J_x(x_0, u_0)$ are given by $\frac{\partial f_1}{\partial x_1}$, respectively. Thus Combining all these separate terms gives:

$$J_x(x_0, u_0) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{(M+m)g}{Ml} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{mg}{M} & 0 & 0 & 0 \end{bmatrix} \qquad (17)$$

For the derivative of the nonlinear terms with respect to u , we have

$$J_u(x_0, u_0) = \begin{bmatrix} \delta f_1 / \delta u \\ \delta f_2 / \delta u \\ \delta f_3 / \delta u \\ \delta f_4 / \delta u \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{-1}{M} \\ 0 \\ \frac{1}{M} \end{bmatrix} \qquad (18)$$

Finally, after all these manipulations (16) may be written explicitly as

$$\frac{d}{dt}\delta x = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{(M+m)g}{Ml} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{mg}{M} & 0 & 0 & 0 \end{bmatrix} \delta x + \begin{bmatrix} 0 \\ \frac{-1}{Ml} \\ 0 \\ \frac{1}{M} \end{bmatrix} \delta u \qquad (19)$$

T his is the open loop linearized model for the inverted pendulum with a cart force, δu(t) , (written in perturbation form). Thus, LTI system is in standard state space form. The equation (19) may be written in general as:

$$\frac{d}{dt}\delta x = A\delta x + B\delta u \qquad (20)$$

Equation (20) along with the output equation (15) represents the final linear model of the inverted pendulum-cart system. This is the simplified model which is used to study the system behavior in general and to design LQR.

Inverted pendulum system equations with disturbance input

The system equations of this nonlinear dynamic system with disturbance input can be derived as follows. Consider a disturbance input due to wind effects, acting on the inverted pendulum in addition to force on the cart, u(t). Let Fw represent the horizontal wind force on the pendulum point mass. With this additional force component, the force balance equation (1) becomes:

$$M\frac{d^2}{dt^2}x + m\frac{d^2}{dt^2}x_G = u + F_w \qquad (21)$$

which can be manipulated as to give:

$$(M+m)\ddot{x} - ml\sin\theta\,\dot{\theta} + ml\cos\theta\,\ddot{\theta} = u + F_w \qquad (22)$$

Similarly, the torque in the clockwise direction caused by the horizontal wind disturbance is (Fx cosθ)l.

Adding the torque contribution of this term the torque balance equation (4) becomes:

(Fx cosθ)l – (Fy sinθ)l = (mg sinθ)l+(Fw cosθ)l (23)

Which can be modified to give:

$$m\ddot{x}\cos\theta + ml\ddot{\theta} = mg\sin\theta + F_w\cos\theta \qquad (24)$$

Equations (22) and (24) are the defining equations for this system with a disturbance input.

The state space equation for inverted pendulum system with disturbance input is derived as same of equation (10) with following modification:

$$\ddot{f_2} = \frac{u\cos x_1 - (M+m)g\sin x_1 + ml(\cos x_1 \sin x_1)x_2^2 - \frac{M}{m}F_W\cos x_1}{ml\cos^2 x_1 - (M+m)l} \qquad (25)$$

$$f_4 = \frac{u + ml(\sin x_1)x_2^2 - mg\cos x_1 \sin x_1 + F_W \sin^2 x_1}{M + m - m\cos^2 x_1} \qquad (26)$$

The output equation of the nonlinear inverted pendulum system with disturbance input remains same as equation (15).

The linearized model can also be developed as following:

$$\frac{d}{dt}\delta x=\begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{(M+m)g}{Ml} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{mg}{M} & 0 & 0 & 0 \end{bmatrix}\delta x+\begin{bmatrix} 0 \\ \frac{-1}{Ml} \\ 0 \\ \frac{1}{M} \end{bmatrix}\delta u+\begin{bmatrix} 0 \\ \frac{-1}{ml} \\ 0 \\ 0 \end{bmatrix}\delta F_w \qquad (27)$$

This is the open loop linearized model for the inverted pendulum with a cart force, $\delta u(t)$ and a horizontal wind disturbance, $\delta F_w(t)$. The two inputs have been separated for convenience, thus the LTI system can be written as:

$$\frac{d}{dt}\delta x = A\delta x + b_1\delta u + b_2\delta F_w \qquad (28)$$

CONTROL METHODS

The following control methods are presented here to control the nonlinear inverted pendulum-cart dynamic system [3, 4].

3.1 PID control

To stabilize the inverted pendulum in upright position and to control the cart at desired position using PID control approach two PID controllersangle PID controller, and cart PID controller have been designed for the two control loops of the system. The equations of PID control are given as

$$u_p = k_{pp}e_\theta(t) + k_{ip}\int e_\theta(t)dt + k_{dp}\frac{de_\theta(t)}{dt} \qquad (29)$$

following:

$$u_c = k_{pc}e_x(t) + k_{ic}\int e_x(t)dt + k_{dc}\frac{de_x(t)}{dt} \qquad (30)$$

Where, $e_\theta(t)$ and $e_x(t)$ are angle error and cart position error. Since the pendulum angle dynamics and cart position dynamics are coupled to each other so the change in any controller parameters affects both the pendulum angle and cart position which makes the tuning tedious. The tuning of controller parameters is done using trial & error method and observing the responses of SIMULINK model to be optimal.

Optimal Control using LQR

Optimal control refers to a class of methods that can be used to synthesize a control policy which results in best possible behavior with respect to the prescribed criterion (i.e. control policy which leads to maximization of performance). The main objective of optimal control is to determine control signals that will cause a process (plant) to satisfy some physical constraints and at the same time extremize (maximize or minimize) a chosen performance criterion (performance index (PI) or cost function). The optimal control problem is to find a control which causes the dynamical system to reach a target or follow a state variable (or trajectory) and at the same time extremize a PI which may take several forms [1-7].

Linear quadratic regulator (LQR) is one of the optimal control techniques, which takes into account the states of the dynamical system and control input to make the optimal control decisions. This is simple as well as robust [1-7].

After linearization of nonlinear system equations about the upright (unstable) equilibrium position having initial conditions as $x_0=[0,0,0,0]^T$ the linear state-space equation is obtained as

$$\dot{x} = Ax + Bu \qquad (31)$$

Where $x = \left[\theta, \dot{\theta}, x, \dot{x}\right]^T$ The state feedback control u=-Kx leads to

$$\dot{x} = (A - BK)x \qquad (32)$$

where, K is derived from minimization of the cost function:

$$J = \int \left(x^T Qx + u^T Ru\right)dt \qquad (33)$$

where, Q and R are positive semi-definite and positive definite symmetric constant matrices respectively. The LQR gain vector K is given by:

$$K = R^{-1}B^T P \qquad (34)$$

Where, P is a positive definite symmetric constant matrix obtained from the solution of matrix algebraic reccatti equation (ARE):

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \qquad (35)$$

In the optimal control of nonlinear inverted pendulum dynamical system using PID controller & LQR approach, all the instantaneous states of the nonlinear system, pendulum. Angle θ, angular velocity $\dot{\theta}$ cart position x , and cart velocity $\dot{x}$ have been considered available for measurement which are directly fed to the LQR. The LQR is designed using the linear state-space model of the system. The optimal control value of LQR is added negatively with PID control value to have a resultant optimal control. The tuning of the PID controllers which are used here either as PID control method or PID+LQR control methods is done by trial & error method and observing the responses achieved to be optimal.

SIMULATION & RESULTS

The MATLAB-SIMULINK models for the simulation of modelling, analysis, and control of nonlinear inverted pendulum-cart dynamical system have been developed. The typical parameters of inverted pendulum-cart system setup are selected as [16,20]: mass of the cart (M): 2.4 kg, mass of the pendulum (m): 0.23 kg, length of the pendulum (l): 0.36 m, length of the cart track (L): ± 0.5 m, friction coefficient of the cart & pole rotation is assumed negligible. After linearization the system matrices used to design LQR are computed as below:

TABLE I. PID CONTROLLER PARAMETERS

Control Schemes	Angle PID Control			Cart PID Control		
	K_{pp}	K_{ip}	K_{dp}	K_{pc}	K_{ic}	Kdc
PID	-40	0	-8	-1	0	-3
2 PID+LQR	1	1	1	1.5	-7.5	5
1 PID+LQR	---	---	---	1.5	-7.5	5

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 29.8615 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -0.9401 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ -1.1574 \\ 0 \\ 0.4167 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad and \quad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

With the choice of

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 500 & 0 \\ 0 & 0 & 0 & 250 \end{bmatrix} and \; R = 1$$

we obtain LQR gain vector as following:

$$K = [-137.7896 \quad -25.9783 \quad -22.3607 \quad -27.5768]$$

Here three control schemes have been implemented for optimal control of nonlinear inverted pendulum-cart dynamical system: 1. PID control method having two PIDs i.e. angle PID & cart PID, 2. Two PIDs (i.e. angle PID & cart PID) with LQR control method, 3. One PID (i.e. cart PID) with LQR control method. Both alternatives of PID+LQR control method are similar in all respect of control techniques but they differ only in number of PID controllers used. The SIMULINK models for these control schemes are shown in Figs. 2, 4, and 6 respectively. The corresponding simulation results are shown in Figs. 3, 5, and 7 respectively.

The reference angle has been set to 0 (rad), and reference cart position is set to 0.1 (m). The tuned PID controller parameters of these control schemes are given as in table

PID control response is shown in Fig. 3. It is observed here that the pendulum stabilizes in vertically upright position after two small overshoots. The cart position x reaches the desired position of 0.1 (m) quickly & smoothly. The control input u is bounded in range [-0.1 0.1]. The response of optimal control of inverted pendulum system using two PID controllers (angle PID & cart PID) with LQR control method is shown in Fig. 5, and using one PID controller (cart PID) with LQR control method is shown in Fig. 7 respectively. Here for both control methods of PID+LQR the responses of angle θ, angular velocity $\dot{\theta}$, cart position x , cart velocity $\dot{x}$, and control u have been plotted. It is observed that in both control schemes the pendulum stabilizes in vertically upright position quickly & smoothly after two minor undershoots and a minor overshoot The angular velocity approaches 0 (rad/s) quickly. The cart position x reaches smoothly the desired position of 0.1 (m) quickly in approx. 6 seconds, and the cart velocity reaches to zero. The control input u is bounded in range [-0.1 0.1]. Comparing the results it is observed that the responses of both alternatives of PID+LQR control method are better than PID control, which are smooth & fast also. It is also observed that the responses of 2PID+LQR control and cart PID+LQR control are similar. Since 2PID+LQR method has additional degree of freedom of control added by the angle PID controller, this will have overall better response under disturbance input. But the cart PID+LQR control has structural simplicity in its credit. The performance analysis of the control schemes gives that these control schemes are effective & robust.

position x , cart velocity $\dot{x}$,and control force u of nonlinear inverted pendulum system with Angle PID , Cart PID & LQR Control.

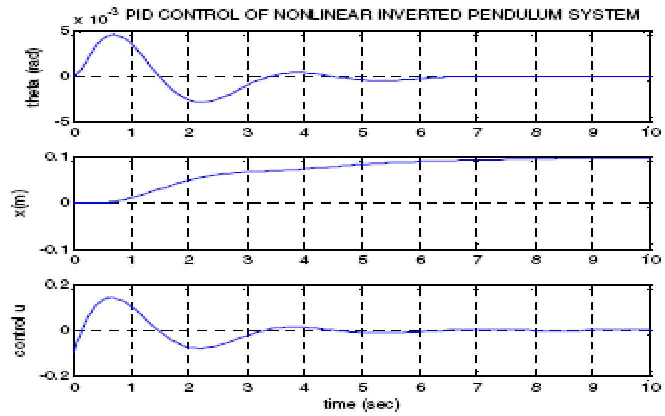


Figure 3. Responses of pendulum angle θ, cart position *x* , and control

force u of nonlinear inverted pendulum system with PID control.

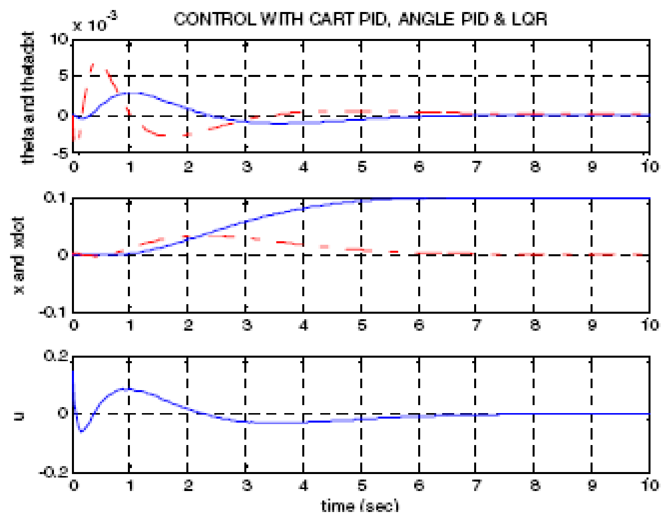


Figure 4. Responses of pendulum angle θ, angular velocity $\dot{\theta}$, cart

CONCLUSION

In this paper we have introduced the nonlinear inverted pendulum-cart dynamic system and by use of PID controller and LQR, an optimal control technique to make the optimal control decisions, have been implemented to control this system. In the optimal control of nonlinear inverted pendulum dynamical system using PID controller and LQR approach all the instantaneous states of the nonlinear system are considered available for measurement, which are directly fed to the LQR. The tuning of the PID controllers which are used here either as PID control method or PID+LQR control methods is done by trial & error method and observing the responses achieved to be optimal. The analysis of the responses of control schemes gives that the performance of proposed PID+LQR control method is better than PID control. This comparative performance investigation for this benchmark system establishes that the proposed PID+LQR control approach being simple, effective & robust control scheme for the optimal control of nonlinear dynamical systems.

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