Setpoints Compensation for Nonlinear Industrial Processes with Disturbances Based on Fuzzy Logic Control

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***Abstract*—This paper focuses on the performance tracking issue of complex industrial processes in double layer architecture. First, the nonlinear plants in the device layer are modeled by using Takagi-Sugeno (T-S) fuzzy technique, and are controlled by local proportional integral (PI) controller with the $\mathscr{H}_\infty$ performance guaranteed. Then, the outputs and inputs of local plants are sampled and transited to the operation layer to form the economic performance index (EPI), which is used to represent the performance of the tracking of economic objective. Furthermore, the setpoints, which are dynamically changing, are calculated via a compensator based on the error between the objective and the EPI at each step of the operation layer. Finally, the effectiveness of the proposed method is demonstrated by a nonlinear continuous stirred tank reactor (CSTR) model.**

I. Introduction

One of the main purposes of process control is to direct the system to achieve certain performance objective; even other goals, like to stabilize the states and to regulate the local plants to track given setpoints, can be regarded as the component parts [1]. For the performance objective, the economy relevant target is usually of high interest and the predictive way to calculate it through the processes can be called economic performance index (EPI). In order to achieve this goal, many methods have been proposed, which can be classified into to two main types. The first kind of ideas focus on the single-layer control, i.e. only the local plants are taken into consideration. There exist many kinds of ways to design local controllers, e.g. in [2] and [3], a predictive controller is designed according to a realtime optimization approach. The others utilize two-layer control scheme, which is also the one this paper inspects. For two-layer control architecture, the process in the device layer will be regulated by local controllers, the setpoints of which are given by the upper layer via considering a nonlinear complex model. These works have been reported in [4] and [5].

In practice, open-loop optimization, e.g. traditional model predictive control (MPC) or manual decomposition, is widely used in the upper layer (operation layer). As is presented in Figure 1, the economic objective of the overall system will be decomposed to a series of setpoints for the local plants. However, because of the disadvantages of the open-loop structure, the setpoints are not either accurate or dynamically changing. Thus, the economic performance index (EPI) can seldom track the desired target and a novel feedback architecture given in Figure 2 is necessary. In this new scheme, the information of the local plants, e.g. the outputs and control inputs, will be utilized to compute the EPI. The error between the value of EPI and the performance objective will then be used for the sepoints compensation. With the two-layer feedback structure described above, the overall performance objective can be achieved.

Moreover, processes are complex and hard to model [6], or even we can, the nonlinearity of the model and the disturbance of the system are almost inevitable. Many papers to deal with the industrial nonlinear systems, e.g. [7] and [8], have been reported. Among all the approaches, fuzzy control, which is a kind of intelligent control methodology, has been widely used in industrial processes, see [9] and [10]. It has been shown in [11] and [12] that any smooth functions can be approximated in any compact set via the fuzzy dynamic models, which will help to tackle the nonlinearity issue. $\mathscr{H}_\infty$ control is popular in restraining the disturbance and it can be introduced to the control of plants. In this paper, a fuzzy logic control strategy with $\mathscr{H}_\infty$ performance index is imported to the local control to solve the problem of model nonlinearity and the influence of disturbances. Some of existing papers may have different solutions, however, their common disadvantage is that the device layer controller has to be redesigned based on the decision of operation layer, through which, the control performance could be improved; but may even lead to more cost because of the alteration of former regulator. Thus, a compensator to adjust the setpoints will be proposed with former control structure unchanged.

As is reported, PI controllers are popular in process control [13]. However, the PI controllers are usually designed based on nominal models, i.e. the disturbance is not taken into consideration, which is not good for the tracking of the overall performance objective of the system. In this paper, the disturbance problem will be considered in the device layer by a classical PI controller with $\mathscr{H}_\infty$ performance guaranteed.

Notations: (η, θ), the combination of η and θ means $[\eta^T, \theta^T]^T$. "$*$" in a block matrix denotes the symmetric terms. $\mathbb{R}^p$ represents the set of real numbers with p dimensions.

II. Preliminaries

We only consider the case that single plant or single loop exists in the device layer for brevity. First, a general industrial

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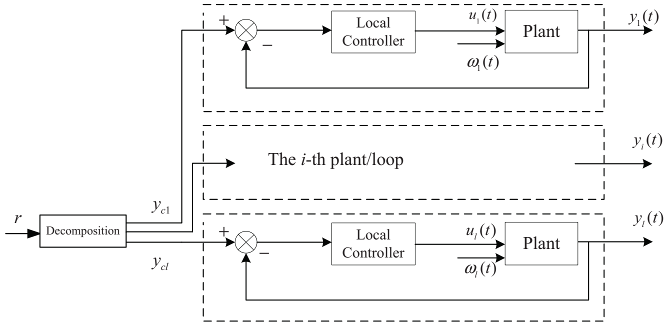

Fig. 1. Conventional scheme for industrial process control

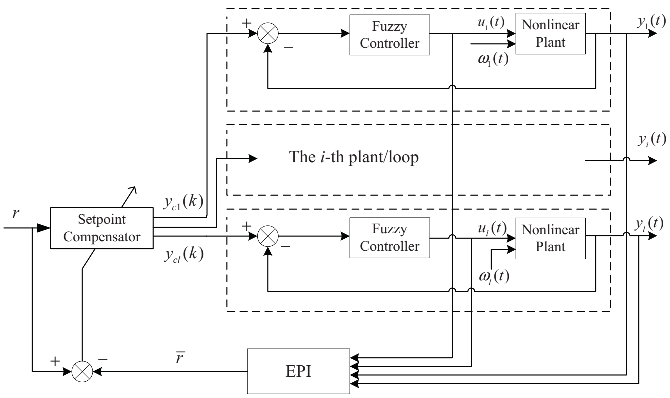

Fig. 2. Setpoints compensation scheme for industrial process control

process is described by a nonlinear model:

$$\begin{cases} \dot{x}(t) = f(x(t)) + g(x(t))u(t) + d(t) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^{n_x}$, $u(t) \in \mathbb{R}^{n_u}$, $d(t) \in \mathbb{R}^{n_d}$ and $y(t) \in \mathbb{R}^{n_y}$ are the state vector, input vector, external influence vector and output vector, respectively. $f(x(t))$ and $g(x(t))$ defined on a domain $D \in \mathbb{R}^n$ are assumed to be sufficiently smooth. From [14], the nonlinear system (1) can be described as T-S model,

Plant Rule $\mathscr{R}^i$:

IF $z_1(t)$ is M_{i1} and $z_2(t)$ is M_{i2} and $\cdots$ and z_q is M_{iq}, THEN

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) + d(t) \\ y(t) = Cx(t), i = 1, 2, \cdots, l. \end{cases} \tag{2}$$

where $\mathscr{R}^i$ means the lth fuzzy inference rule with l as the total number. For $i = 1, 2, \cdots, p; j = 1, 2, \cdots, q$, $z_i(t)$ and M_{ij} denote the premise variables and the fuzzy sets corresponding to $z_i(t)$, respectively. Parameter matrices A_i, B_i and C_i (can be written as C) are known with appropriate dimensions. The fuzzy system can also be written as

$$\begin{cases} \dot{x}(t) = \sum\limits_{i=1}^{l} h_i(z(t))[A_i x(t) + B_i u(t) + d(t)] \\ y(t) = \sum\limits_{i=1}^{l} h_i(z(t)) C_i x(t) \end{cases} \tag{3}$$

where

$$h_i(z(t)) = \frac{\mu_i(z(t))}{\sum_{i=1}^{l} \mu_i(z(t))},$$
$$\mu_i(z(t)) = \prod_{j=1}^{g} M_{ij} z_j(t),$$
$$z(t) = [z_1(t), z_2(t), \cdots, z_q(t)],$$

where $M_{ij}(z_j(t))$ is the degree of membership function of $z_j(t)$ in M_{ij}, $\mu_i(z(t)) \geq 0$, $i = 1, 2, \cdots, l$, $\sum_{i=1}^{l} \mu_i(z(t)) > 0$, $\forall t \geq t_0$. Define the integral error $E(t)$ as

$$E(t) = \int_0^t e(\tau) d\tau = \int_0^t (y_c - y(\tau)) d\tau. \tag{4}$$

Therefore, we have the augmented system as follows,

$$\begin{bmatrix} \dot{E}(t) \\ \dot{x}(t) \end{bmatrix} = \sum_{i=1}^{l} h_i(z) \left\{ \begin{bmatrix} 0 & -C_i \\ 0 & A_i \end{bmatrix} \begin{bmatrix} E(t) \\ x(t) \end{bmatrix} + \begin{bmatrix} 0 \\ B_i \end{bmatrix} u(t) + \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} y_c \\ d(t) \end{bmatrix} \right\}. \tag{5}$$

Letting $\eta(t) = (E(t), x(t))$ and $\omega(t) = (y_c, d(t))$ the nominal system derived from the augmented system (5) can be expressed as

$$\dot{\eta}(t) = \sum_{i=1}^{l} h_i(z) \left[\mathcal{A}_i \eta(t) + \mathcal{B}_i u(t) + \omega(t) \right] \tag{6}$$

where

$$\mathcal{A}_i = \begin{bmatrix} 0 & -C_i \\ 0 & A_i \end{bmatrix}, \quad \mathcal{B}_i = \begin{bmatrix} 0 \\ B_i \end{bmatrix}.$$

All the following paper will be based on the system (6) and we are on the way of designing the local regulator and upper layer compensator such that the local system can track given setpoints and the upper layer can provide good reference signals for the device layer.

III. Local regulator design

The goal of the controllers in the device layer is forcing the output of the plants to track setpoints provided by the operation layer. For certain setpoint y_c^T, the tracking requirement can be described as

$$\lim_{t \to \infty} e(t) = 0. \tag{7}$$

As the disturbance is considered in the device layer, a PI controller is needed such that the following $\mathscr{H}_\infty$ tracking index for $\omega(t)$ is satisfied,

$$\int_0^t e^T(\tau) e(\tau) \mathrm{d}\tau \leq \gamma^2 \int_0^t \omega^T(\tau) \omega(\tau) \mathrm{d}\tau, \tag{8}$$

in which the scale γ is positive. The state feedback fuzzy controller, whose premise parts are the same with the system (1), is described as

Controller Rule $\mathscr{R}^i$: IF $z_1(t)$ is M_{i1} and $z_2(t)$ is M_{i2} and $\cdots$ and z_q is M_{iq}, THEN

$$\tilde{u}(t) = K_i \eta(t), i = 1, 2, \cdots, l. \tag{9}$$

where $\tilde{u}(t)$ is the control input under certain rule. Then the overall controller is

$$u(t) = \sum_{i=1}^{l} h_i(z) K_i \eta(t). \tag{10}$$

Substituting (10) into (6) and denoting $h_i(z) = h_i$, one can obtain the following augmented system

$$\dot{\eta}(t) = \sum_{i=1}^{l}\sum_{j=1}^{l} h_i h_j \left\{ \left[\mathcal{A}_i + \mathcal{B}_i K_j\right] \eta(t) + \omega(t) \right\}. \tag{11}$$

And the following theorem can be obtained.

Theorem 1. *For a given constant $\gamma > 0$, the closed-loop fuzzy system in (11) is asymptotically stable with the $\mathscr{H}_\infty$ performance in (8), if there exist symmetric matrices $X > 0$ and a series of matrices Y_j, ($j = 1, 2, \ldots, l$), such that the following linear matrix inequalities (LMIs) hold:*

$$\mathcal{M}_{ii} < 0, \quad i = 1, 2, \ldots, l \tag{12}$$

$$\frac{1}{l-1}\mathcal{M}_{ii} + \frac{1}{2}(\mathcal{M}_{ij} + \mathcal{M}_{ji}) < 0, \quad 1 \le i \ne j \le l \tag{13}$$

where

$$\begin{aligned} \mathcal{M} &= \begin{bmatrix} \Phi_{ij} & \gamma I & X \\ \gamma I & -I & 0 \\ X & 0 & -I \end{bmatrix}, \\ \Phi_{ij} &= (\mathcal{A}_i X + Y_j) + (\mathcal{A}_i X + Y_j)^T. \end{aligned} \tag{14}$$

Hence, the state feedback regulator is described in (10) with the parameter $K_j = Y_j X^{-1}$.

Proof: Following the proof of Lemma 1 in [15], Theorem 1 can be proved. ■

IV. Setpoint compensator design

The industrial process in the device layer with disturbance can track the given setpoints under the control of the regulator solved by Theorem 1. However, to provide good setpoints such that performance of the overall system can achieve the desired economic objective, a setpoints compensator needs to be designed in the operation layer.

Here we introduce the linear economic objective index (EPI) as

$$\bar{r}(k) = M y(k) + N u(k), \tag{15}$$

where, $\bar{r}(k) \in \mathbb{R}^{n_r}$, $y(k)$ and $u(k)$ are the value of EPI, output and input signals of the device layer at sampling instant k, respectively. $M \in \mathbb{R}^{n_r \times n_y}$ and $N \in \mathbb{R}^{n_u \times n_y}$ are the benefit and consumption coefficient matrices, respectively.

Hence, the compensator will work at each sampling instant as

$$\Delta y_c(k) = F \Delta r(k), \tag{16}$$

where $\Delta r(k) = r(k) - \bar{r}(k)$ and the setpoints sent to the local plants are updated through the following approach,

$$y_c(k+1) = y_c(k) + \Delta y_c(k). \tag{17}$$

At the very beginning, $y_c(0) = y_{c0}$ serves as the initial value of (17), where y_{c0} can be obtained via more upper layer optimization as well as human experience.

Since a sampled-data control is utilized in the operation layer, the model of device layer in (6) should be described in discrete-time form. Along with the methodology proposed in Theorem 1, the influence of the disturbance can be well restrained by the regulator in device layer; so $\omega(t)$ will not be considered in operation layer. Thus, we can obtain,

$$\eta(k+1) = \sum_{i=1}^{l} h_i \left[\bar{\mathcal{A}}_i \eta(k) + \bar{\mathcal{B}}_i u(k) + \mathcal{I} y_c(k)\right], \tag{18}$$

where $\bar{\mathcal{A}}_i = \mathrm{e}^{A_i T}$ and $\bar{\mathcal{B}}_i = \int_0^T \mathrm{e}^{A_i(T-\tau)} B_i \mathrm{d}\tau$. Substituting $u(k) = \sum_{j=1}^{l} h_j K_j \eta(k)$ into the discrete-time model in (18), one can obtain the closed-loop model of the device layer at each sampling instant of the operation layer as follows,

$$\eta(k+1) = \sum_{i=1}^{l}\sum_{j=1}^{l} h_i h_j \left[\tilde{\mathcal{A}}_{ij} \eta(k) + \mathcal{I} y_c(k)\right], \tag{19}$$

where $\tilde{\mathcal{A}}_{ij} = \bar{\mathcal{A}}_i + \bar{\mathcal{B}}_i K_j$.

On the other hand, according to (15)-(17), along with the expression of the control $u(k)$, we have,

$$\begin{aligned} y_c(k+1) &= \sum_{j=1}^{l} h_j \left[y_c(k) + Fr \right. \\ &\quad \left. -(FM\bar{C} + FNK_j)\eta(k)\right], \end{aligned} \tag{20}$$

where, $\bar{C} = [0, C]$. Then denoting $\zeta(k) = (\eta(k), y_c(k))$, along with (19) and (20), we can obtain the overall system as,

$$\zeta(k+1) = \sum_{i=1}^{l}\sum_{j=1}^{l} h_i h_j \left[\mathscr{A}_{ij} \zeta(k) + \mathcal{I} y_c(k)\right], \tag{21}$$

where,

$$\mathscr{A}_{ij} = \begin{bmatrix} \bar{\mathcal{A}}_{ij} & \tilde{I} \\ -FM\bar{C} - FNK_j & I \end{bmatrix}.$$

And we are on our way to present the main result of this paper.

Theorem 2. *Given matrix $\bar{G}$, the closed-loop system in (21)is asymptotically stable, if there exist symmetric matrices Q_1, Q_3 and matrix $\bar{L}$, such that the following LMIs hold,*

$$\Xi_{ii} < 0, \quad i = 1, 2, \ldots, l \tag{22}$$

$$\frac{1}{l-1}\Xi_{ii} + \frac{1}{2}(\Xi_{ij} + \Xi_{ji}) < 0, \quad 1 \le i \ne j \le l \tag{23}$$

$$Q = \begin{bmatrix} Q_1 & \bar{G} Q_3 \\ * & Q_3 \end{bmatrix} < 0, \tag{24}$$

where,

$$\Xi_{ij} = \begin{bmatrix} -Q & \Psi_{ij}^T \\ * & -Q \end{bmatrix},$$

$$\Psi_{ij} = \begin{bmatrix} Q_1 \mathscr{A}_{ij} - \bar{G}\bar{L}M\bar{C} - \bar{G}\bar{L}NK_j & Q_1 \mathcal{I} + Q_2 \\ Q_2 \mathscr{A}_{ij} - \bar{L}M\bar{C} - \bar{L}NK_j & Q_2 \mathcal{I} + Q_3 \end{bmatrix}. \tag{25}$$

Moreover, the compensator parameter is calculated as $F = Q_3^{-1}\bar{L}$.

Proof: Let us consider $V(k) = \zeta^T(k)Q\zeta(k)$ as the Lyapunov function. Considering the case $r = 0$, we can obtain the increment,

$$\Delta V(k) = \sum_{i=1}^{l}\sum_{j=1}^{l} h_i h_j \left[\zeta^T(k)\left(\mathscr{A}_{ij}^T Q \mathscr{A}_{ij} - Q\right)\zeta(k)\right]. \quad (26)$$

Set $\Delta V(k) < 0$ and we can have the result for all $0 \le i, j \le l$ in detail after a congruent transformation by blkdiag$\{I, Q\}$ as follows,

$$\begin{bmatrix} -Q_1 & -Q_2 & \psi_1^T & \psi_2^T \\ * & -Q_3 & \mathcal{I}^T Q_1 + Q_2^T & \mathcal{I}^T Q_2 + Q_3 \\ * & * & -Q_1 & -Q_2 \\ * & * & * & -Q_3 \end{bmatrix} < 0, \quad (27)$$

where,

$$\begin{aligned} \psi_1 &= Q_1\mathscr{A}_{ij} - Q_2FM\bar{C} - Q_2FNK_j \\ \psi_2 &= Q_2\mathscr{A}_{ij} - Q_3FM\bar{C} - Q_3FNK_j. \end{aligned} \quad (28)$$

However, Q_2F and Q_3F are bilinear items and cannot be easily solved. The nonlinear items can be eliminated by denoting $\bar{L} = Q_3F$ and $Q_2 = \bar{G}Q_3$, in which $\bar{G}$ is a given constant matrix. By Schur complement and using the approach in [16], (27) is equivalent to (22) and (23). And obviously, the compensator matrix is obtained by $F = Q_3^{-1}\bar{L}$. ■

V. APPLICATION TO CSTR

Consider a single first-order, irreversible reaction in isothermal CSTR [17]

$$A \rightarrow B \quad r = kc_A$$

where k is the rate constant, which is equal to 1.2 L/(mol·min) in this case. The dynamic model based on material balances are:

$$\begin{aligned} \frac{\mathrm{d}c_A(t)}{\mathrm{d}t} &= \frac{Q(t)}{V}(c_{Af} - c_A(t)) - kc_A \\ \frac{\mathrm{d}c_B(t)}{\mathrm{d}t} &= \frac{Q(t)}{V}(c_{Bf} - c_B(t)) + kc_A \end{aligned} \quad (29)$$

in which, for reactor A, $c_A(t)$ and $c_{Af} = 1$ mol/L are the concentration and the given feed concentration, respectively; And the same go for $c_B(t)$ and $c_{Bf} = 0$ for the product B. And $Q(t)$ is the flow to be controlled through the reactor, whose volume is $V = 10$ L. We choose $x(t) = (c_A(t),\ c_B(t))$ as the states of the plant; $u(t) = Q(t)$, the available manipulated variable, as the input; and $y(t) = c_B(t)$, the concentration of the product as the output. Thus the differential equation in (29) equals to:

$$\begin{aligned} \dot{x}_1(t) &= -1.2x_1(t) + 0.1(1 - x_1(t))u(t) \\ \dot{x}_2(t) &= 1.2x_1(t) - 0.1x_2(t)u(t) \end{aligned} \quad (30)$$

Three equilibriums are chosen, i.e. the process is linearized around $x_2(t) = 0.5$, $\frac{2}{3}$ and 0.4. And we can have the T-S fuzzy model of the plant according to the **Plant Rule**, with the parameters

$$A_1 = \begin{bmatrix} -2.4 & 0 \\ 1.2 & -1.2 \end{bmatrix},\ B_1 = \begin{bmatrix} 0.05 \\ -0.05 \end{bmatrix};$$

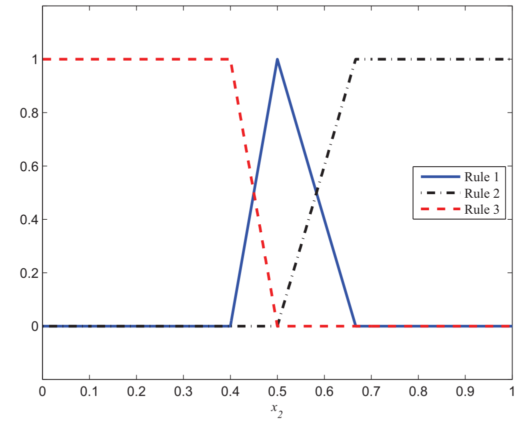

Fig. 3. Tracking of setpoints without setpoints compensation

$$A_2 = \begin{bmatrix} -1.8 & 0 \\ 1.2 & -0.6 \end{bmatrix},\ B_2 = \begin{bmatrix} 0.0667 \\ -0.0667 \end{bmatrix};$$

$$A_3 = \begin{bmatrix} -3 & 0 \\ 1.2 & -1.8 \end{bmatrix},\ B_3 = \begin{bmatrix} 0.04 \\ -0.04 \end{bmatrix};$$

The membership functions are shown in Figure 3. According to the approach in Theorem 1, we can obtain the parameters of the controller as follows,

$$\begin{aligned} K_1 &= [-248.5829,\quad 11.3565,\quad 192.8199], \\ K_2 &= [-188.4418,\quad 1.9995,\quad 151.9916], \\ K_3 &= [-317.7794,\quad 18.8900,\quad 239.8705]. \end{aligned}$$

The economic objective is set to 65 (and will also set as a variable in the last part of this simulation) in the simulation. The parameters in EPI in (15) are $M = 20$ and $N = 5$. Sampling period is $T = 10$ min. Random Gaussian noise is considered in the simulation.

The simulations are made up of three parts. First, when the fixed setpoint y_c serves as the reference signal of the plant, which is presented in Figure 4, the local plant can track the setpoint. However, as is shown in Figure 5, since the optimal setpoint cannot be easily obtained, the EPI fails to track economic objective.

Second, a feedback compensator is used in the operation layer. The tracking performance in Figure 6 is still good. Meanwhile, the economic objective can be tracked by $\bar{r}(k)$, which can be seen in Figure 7.

In the end, the economic objective rises in the 9th step from 65 to 75 in the operation layer and this may happen when the order of more upper layer changes. As is presented in Figure 8, the EPI can still track the economic objective, which shows the robustness of the compensator.

VI. CONCLUSION

In this paper, a combined setpoints compensation and fuzzy logic control scheme has been proposed for a class of nonlinear industrial processes with disturbances. At device layer, the nonlinear plant was described by T-S fuzzy model, and then the $\mathscr{H}_\infty$ performance is guaranteed by constructing PI controllers.

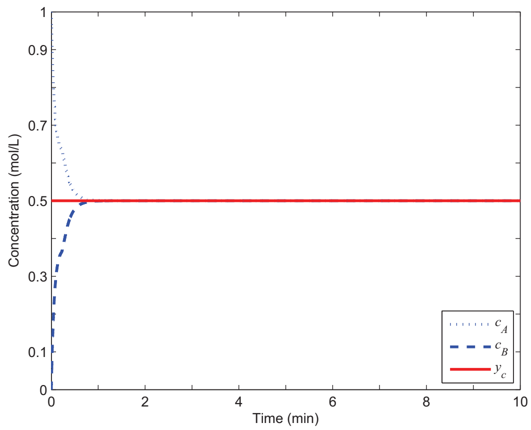

Fig. 4. Tracking of setpoints without setpoints compensation

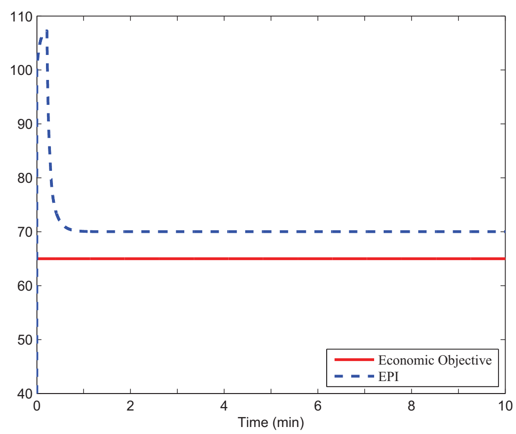

Fig. 5. Tracking of economic objective without setpoints compensation

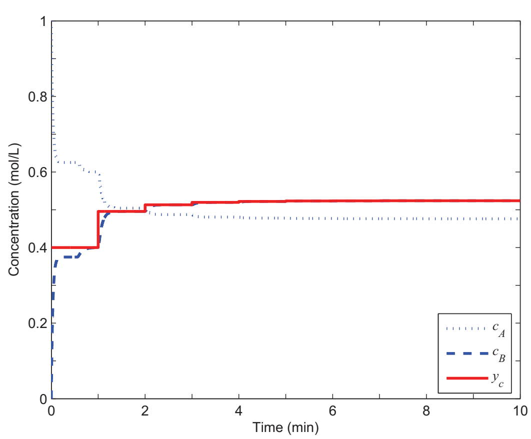

Fig. 6. Tracking of setpoints with setpoints compensation

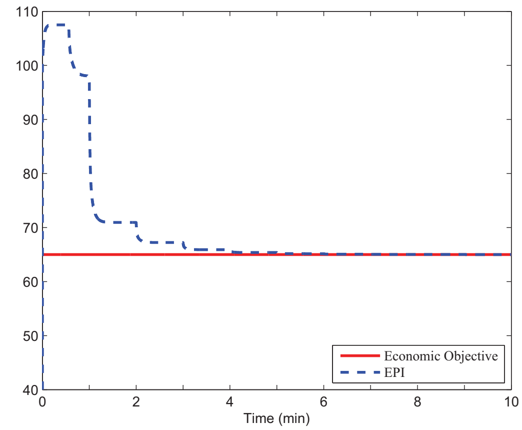

Fig. 7. Tracking of economic objective with setpoints compensation

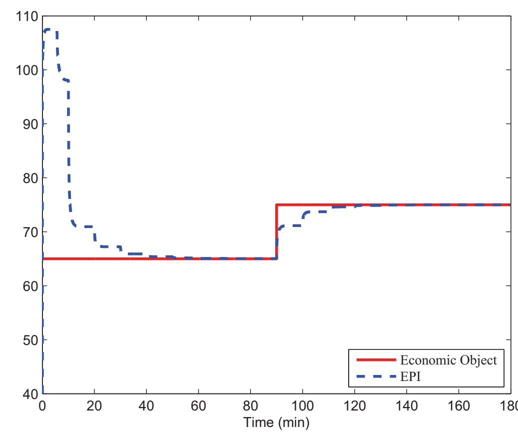

Fig. 8. Tracking of changing economic objective

Then at the operation layer, a dynamic setpoint compensator has been designed to track the desired economic objective. Furhtermore, the proposed control scheme has been verified through the simulation of nonlinear CSTR process. In our future research work, the fault detection and fault tolerance control scheme will be further considered.

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