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Fuzzy model-based predictive control of dissolved oxygen in activated sludge processes *



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ABSTRACT

The paper is concerned with the design of a fuzzy model-based predictive controller for activated sludge wastewater treatment processes. The control purpose is to maintain the dissolved oxygen concentration in an aerobic reactor of the wastewater treatment plant at the set-point. The fuzzy model of the activated sludge processes is derived based on the Activated Sludge Model No. 1 (ASM1), including the structure of the fuzzy rules. The required fuzzy space of input variables is partitioned by fuzzy c-means cluster algorithm and the consequent parameters are identified using the method of least squares. Compared with both traditional PID control and dynamic matrix control schemes, the proposed fuzzy model-based predictive control paradigm achieves satisfactory benefits in terms of both transient and steady performances.

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1. Introduction

The activated sludge treatment approach, which uses the bacteria and other microorganisms to remove contaminants by assimilating them, has been widely adopted in most wastewater treatment plants (WWTPs). Modeling and control of the activate sludge processes (ASPs) play an important role for improving the effectiveness of this approach. So far, some models have been proposed, such as Activated Sludge Models (ASMs) of International Water Association (IWA) including ASM1, ASM2, ASM2d and ASM3. It has been well recognized in the area that the ASM1 is the most successful one used to represent the processes dynamics [11,12,14,15]. However, due to the complexity of the model, e.g., high-dimensional with many nonlinear terms and parameters that are hard to identify, it is quite limited to apply ASM1 directly for controller design of WWTPs. To

overcome this difficulty, big efforts have been made towards proposing more efficient models such as the modified ASM models, the intelligent models and hybrid models (see [2,5–7]). Fuzzy modeling approach [17,26,33,34], which is commonly adopted in approximating a broad class of nonlinear systems, becomes more and more popular to be used in modeling ASP. A large number of results have been available in the literature demonstrating that fuzzy models can adequately reflect the dynamics of the ASPs [9,10,18,20,31]. An efficient method of identifying the structure of fuzzy model is proposed in [37] and this modeling approach has been successfully applied to predict chemical oxygen demand of the ASPs.

In general, the complexity in controlling wastewater treatment processes are mainly caused by seriously high nonlinearity and various uncertainties due to, for instance, the time-varying influent parameters, the intricacy of structure and the huge number of coefficients of the model. Model predictive control, capable of dealing with multi-variable systems and constraints, has been extensively applied to ASPs (see [23,24,29,30], for example). In the existing results, there are many manipulated variables which are frequently employed, such as dissolved oxygen concentration [3,13,16,28], ammonia concentration [32], residual substrate [22], internal recycle flow rate and external carbon dosing rate. Effective control of dissolved oxygen can not only guarantee the common behavior and activity of the microorganisms living in the activated

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Nomenclature		K_{NH} S_S	immonium half-saturation coefficient for autotrophs eadily biodegradable substrate	
S_{l} X_{l} X_{BH} S_{NH} X_{P} S_{O} S_{ND} Y_{H} η_{g} b_{H} μ_{H} K_{S} K_{OA}	soluble inert organic matter particulate inert organic matter active heterotrophic biomass ammonium and ammonia nitrogen particulate products arising from biomass decay dissolved oxygen soluble biodegradable organic nitrogen heterotrophic yield correction factor for anoxic growth of heterotrophs decay rate for heterotrophs maximum heterotrophic specific growth rate half-saturation coefficient for heterotrophs oxygen half-saturation coefficient for autotrophs	X_S X_{BA} S_{NO} f_p S_{ALK} X_{ND} Y_A η_h b_A μ_A K_{OH}	slowly biodegradable substrate active autotrophic biomass nitrate and nitrite nitrogen fraction of biomass yielding decay products alkalinity particulate biodegradable organic nitrogen autotrophic yield correction factor for anoxic hydrolysis decay rate for autotrophs maximum autotrophic specific growth rate oxygen half-saturation coefficient for heterotrophs nitrate half-saturation coefficient for heterotrophs	

sludge, but also significantly reduce the operational costs of the wastewater treatment. It is worth mentioning that most of the research results of model predictive control are focused on neural network model [3,13], linear state-space model [16], bilinear model [8] and reduced ASM1 [32], the fuzzy model-based predictive control of ASPs has not been sufficiently investigated. In [19], the hierarchical fuzzy predictive control for nitrogen removal in biological wastewater treatment processes has been investigated. However, the parameters of the obtained fuzzy model lack definite physical meaning.

Motivated by the aforementioned observations, in this paper, the problem of fuzzy model-based predictive control of dissolved oxygen in ASPs is considered. The control goal is to maintain the concentration of dissolved oxygen in an aerobic reactor of the WWTP at the set-point. In the considered fuzzy modeling processes, the fuzzy space of required input variables is partitioned by the fuzzy c-means cluster algorithm and the consequent parameters of the fuzzy rules are identified using the method of least squares. Moreover, in contrast with recent studies on structure identification of the fuzzy rules, the premise variables and consequent structure in our approach can be obtained through ASM1 directly. By comparing performance with PID and dynamic matrix control (DMC) strategies, it can be seen that the fuzzy modelbased predictive controller can efficiently control the dissolved oxygen with smaller overshoot and shorter settling time. The remainder of this paper is organized as follows. Section 2 briefly introduces ASM1 and the underlying WWTP. The actual modeling procedure and the model testification results are presented in Sections 3.1 and 3.2. Section 3.3 gives the controller design method and the related comparison results are given in Section **3.4.** The last section of the paper presents some conclusions.

Notation: The notation used throughout the paper is fairly standard. The superscript "T" stands for matrix transposition; \mathbb{R}^n denotes the n-dimensional Euclidean space; the notation P>0 (≥ 0) means that P is real symmetric and positive (semipositive) definite and A>B ($\geq B$) means A-B>0 (≥ 0). I and 0 represent identity matrix and zero matrix, respectively. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. The notation $\|\cdot\|_Q$ stands for the weighted norm, defined by $\|x\|_Q^2 = x^T Qx$ for all $x \in \mathbb{R}^n$, where Q is a positive-definite symmetric matrix.

2. Preliminaries

In order to make the results easier to understand, in this section, the relevant aspects of Activated Sludge Model No. 1 (ASM1) will be

briefly introduced. Then, the underlying wastewater treatment plant (WWTP) which is designed based on the so-called Benchmark Simulation Model No. 1 (BSM1) is further given.

2.1. ASM1

As commonly considered, a well-known characteristic of ASM1 is that the matrix form is used to present the activated sludge processes (ASPs). The matrix is constructed with 13 components and these components are generally described by the following mass balance equation (see [14] for more details):

$$\frac{d\xi}{dt} = R(\xi) + \frac{Q}{V}(\xi_{in} - \xi) \tag{1}$$

where

$$\xi \triangleq [S_I \ S_S \ X_I \ X_S \ X_{BH} \ X_{BA} \ X_P \ S_O \ S_{NO} \ S_{NH} \ S_{ND} \ X_{ND} \ S_{ALK}]^T$$

is a vector gathering concentrations of the 13 components,

 $\xi_{in} \triangleq$

$$[S_{I,in} \ S_{S,in} \ X_{I,in} \ X_{S,in} \ X_{BH,in} \ X_{BA,in} \ X_{P,in} \ S_{O,in} \ S_{NO,in} \ S_{NH,in} \ S_{ND,in} \ X_{ND,in} \ S_{ALK,in}]^T$$

stands for the concentrations of the process components in the influent water, Q is the influent flow rate, V is the reactor volume. $R(\xi)$ is the reaction rate modeled by the product of a reaction rate vector ρ and a stoichiometric matrix $\mathcal S$ where ρ and $\mathcal S$ are given in (2) and (3), respectively. For simplicity, in this paper, the notation with respect to time t or k will be dropped, e.g., ξ instead of $\xi(t)$ or $\xi(k)$ will be used, if it will not lead to ambiguity. Nevertheless, it should be kept in mind that the concentrations of the components are related to time:

$$\rho \triangleq \begin{bmatrix}
\mu_{H} \frac{S_{S}}{K_{S} + S_{S}} \frac{S_{O}}{K_{OH} + S_{O}} X_{BH} \\
\mu_{H} \frac{S_{S}}{K_{S} + S_{S}} \frac{S_{O}}{K_{OH} + S_{O}} X_{BH} \\
\mu_{A} \frac{S_{S}}{K_{S} + S_{S}} \frac{S_{O}}{K_{OH} + S_{O}} X_{BO} \eta_{g} X_{BH} \\
\mu_{A} \frac{S_{NH}}{K_{NH} + S_{NH}} \frac{S_{O}}{K_{OA} + S_{O}} X_{BA} \\
b_{H} X_{BH} \\
b_{A} X_{BA} \\
k_{a} S_{ND} X_{BH} \\
k_{h} \frac{X_{S}/X_{BH}}{K_{X} + X_{S}/X_{BH}} \left(\frac{S_{O}}{K_{OH} + S_{O}} + \eta_{h} \frac{K_{OH}}{K_{OH} + S_{O}} \frac{S_{NO}}{K_{NO} + S_{NO}} \right) X_{BH} \\
\rho_{7} \left(\frac{X_{ND}}{X_{S}} \right)
\end{bmatrix}$$
(2)

Remark 1. Note that in (2), some "switching functions" are used to describe the environmental conditions change via affecting the reaction rates. For example, the output of the switching function $S_O/(K_{OH}+S_O)$, where K_{OH} is a sufficiently small constant, can be adjusted under the aerobic and anoxic conditions since S_O is different for aerobic and anoxic conditions. As a result, the reaction rates, e.g., ρ_1 , ρ_2 , ρ_7 , ρ_8 , will change correspondingly. On the contrary, $S_O/(K_{OH}+S_O)$ can be seen as a constant if there are no major changes in S_O . The readers can refer to [25] for more details on such "switching functions".

2.2. Wastewater treatment plant

The basic structure of the underlying wastewater treatment plant, including five reactors (i.e., two anoxic tanks and three aeration tanks) and a 10-layer secondary settler, is shown in Fig. 1 (see also [1] for more details). For ease of exposition, we assume that there is a reservoir before the reactors so that the influent flow rate Q varies in a small interval. In fact, as a simulation environment, the BSM1 is proposed based on ASM1 by the International Water Association (IWA) Taskgroup on Benchmarking of Control Strategies for wastewater treatment processes (Working Groups of COST Action 682 and 624) [1]. A rigorous performance evaluation methodology to enhance the acceptance of innovating control strategies can also be provided by BSM1.

The purposes of this paper are to derive a suitable fuzzy model for activated sludge wastewater treatment processes based on ASM1 and BSM1, and to design fuzzy model-based predictive controller for each aeration tank such that the concentration of dissolved oxygen can be maintained at the set-point.

3. Main results

In this section, the methods and algorithms used to obtain the fuzzy model of activated sludge processes in an aeration tank in

WWTP will be presented firstly. Then, a predictive controller will be designed based on the derived fuzzy model.

3.1. Predictive model

Consider an aeration tank shown in Fig. 1, a simplified fuzzy model will be given mathematically and tested by comparing with ASM1.

As described in Section 2, we can get the differential equations of dissolved oxygen dS_O/dt and active autotrophic biomass ?A3B2showscale86%? > dX_{BA}/dt according to rows eight and six of (1) based on ASM1:

$$\begin{split} \frac{dS_{O}}{dt} &= -\frac{1 - Y_{H}}{Y_{H}} \mu_{H} \left(\frac{S_{S}}{K_{S} + S_{S}} \right) \left(\frac{S_{O}}{K_{OH} + S_{O}} \right) X_{BH} \\ &- \frac{4.57 - Y_{A}}{Y_{A}} \mu_{A} \left(\frac{S_{NH}}{K_{NH} + S_{NH}} \right) \left(\frac{S_{O}}{K_{OA} + S_{O}} \right) X_{BA} + \frac{Q}{V} (S_{O_in} - S_{O}) \end{split} \tag{4}$$

$$\frac{dX_{BA}}{dt} = \mu_H \frac{S_{NH}}{K_{NH} + S_{NH}} \frac{S_O}{K_{OA} + S_O} X_{BA} - b_A X_{BA} + \frac{Q}{V} (X_{BA_in} - X_{BA})$$
 (5)

In order to take better advantage of discrete form of the data, (4) and (5) have been approximated by the following difference equations based on the first-order Euler approximation approach:

$$\begin{split} \frac{S_{O}(k+1) - S_{O}(k)}{T} &= -\frac{1 - Y_{H}}{Y_{H}} \mu_{H} \left(\frac{S_{S}}{K_{S} + S_{S}}\right) \left(\frac{S_{O}}{K_{OH} + S_{O}}\right) X_{BH} + \frac{Q}{V} (S_{O_in} - S_{O}) \\ &- \frac{4.57 - Y_{A}}{Y_{A}} \mu_{A} \left(\frac{S_{NH}}{K_{NH} + S_{NH}}\right) \left(\frac{S_{O}}{K_{OA} + S_{O}}\right) X_{BA} \end{split}$$

$$\begin{split} \frac{X_{BA}(k+1) - X_{BA}(k)}{T} &= \mu_H \bigg(\frac{S_{NH}}{K_{NH} + S_{NH}} \bigg) \bigg(\frac{S_O}{K_{OA} + S_O} \bigg) X_{BA} - b_A X_{BA} \\ &+ \frac{Q}{V} (X_{BA_in} - X_{BA}) \end{split}$$

Thus, we have

$$\begin{split} S_{O}(k+1) &= T \left[-\frac{1-Y_{H}}{Y_{H}} \mu_{H} \left(\frac{S_{S}}{K_{S}+S_{S}} \right) \left(\frac{S_{O}}{K_{OH}+S_{O}} \right) X_{BH} + \frac{Q}{V} (S_{O_in} - S_{O}) \right. \\ &\left. - \frac{4.57 - Y_{A}}{Y_{A}} \mu_{A} \left(\frac{S_{NH}}{K_{NH}+S_{NH}} \right) \left(\frac{S_{O}}{K_{OA}+S_{O}} \right) X_{BA} \right] + S_{O}(k), \end{split}$$

$$\begin{split} X_{BA}(k+1) &= T \left[\mu_H \left(\frac{S_{NH}}{K_{NH} + S_{NH}} \right) \left(\frac{S_O}{K_{OA} + S_O} \right) X_{BA} - b_A X_{BA} \right. \\ &\left. + \frac{Q}{V} (X_{BA_in} - X_{BA}) \right] + X_{BA}(k). \end{split}$$

The above difference equations can be rewritten into vector form as

$$\begin{bmatrix} S_{O}(k+1) \\ X_{BA}(k+1) \end{bmatrix} = \begin{bmatrix} a & \hat{c} \\ 0 & \hat{q} \end{bmatrix} \begin{bmatrix} S_{O}(k) \\ X_{BA}(k) \end{bmatrix} + \begin{bmatrix} \hat{b} & d & 0 \\ 0 & 0 & w \end{bmatrix} \begin{bmatrix} X_{BH} \\ S_{O_in} \\ X_{BA} & in \end{bmatrix}$$
(6)

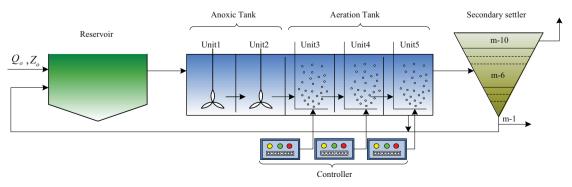


Fig. 1. Schematic representation of the wastewater treatment plant.

where

$$\begin{split} a &\triangleq 1 + T \cdot \frac{Q}{V}, \hat{b} \triangleq -T \cdot \frac{1 - Y_H}{Y_H} \mu_H \left(\frac{S_S}{K_S + S_S} \right) \left(\frac{S_O}{K_{OH} + S_O} \right), \\ \hat{c} &\triangleq -T \cdot \frac{4.57 - Y_A}{Y_A} \mu_A \left(\frac{S_{NH}}{K_{NH} + S_{NH}} \right) \left(\frac{S_O}{K_{OA} + S_O} \right), \quad d \triangleq T \cdot \frac{Q}{V}, \\ \hat{q} &\triangleq T \left[\mu_H \left(\frac{S_{NH}}{K_{NH} + S_{NH}} \right) \left(\frac{S_O}{K_{OA} + S_O} \right) - b_A - \frac{Q}{V} + \frac{1}{T} \right], \quad w \triangleq T \cdot \frac{Q}{V}. \end{split}$$

Let us presume that a, \hat{b} , \hat{c} , d, \hat{q} and w are constant parameters, then the nonlinear matrix equation (6) becomes a linear matrix equation which is desirable for controller design. Note also that Y_H , Y_A are Stoichiometric parameters, μ_H , μ_A , K_S , K_{OH} , K_{NH} , K_{OA} and b_A are Kinetic parameters, all of which are fixed when the plant and environmental conditions are determined. Due to the reservoir, the influent flow rate Q can be seen as a constant. Therefore, if we choose S_S , S_{NH} as premise variables, divide S_S , S_{NH} into 2 fuzzy sets labeled as NB, PB (stand for negative big and positive big, respectively), then the 2-dimensional space will be divided into 2×2 fuzzy subspaces and the nonlinear equation can be approximated by a linear matrix equation for each subspace, respectively.

In particular, for aeration tanks, the concentration of dissolved oxygen S_O is so high that we can assume that the outputs of switching functions do not change when S_O has a little variation. Also, we add variable $z = [z_1 \ z_2]^T$ to represent the error between the obtained fuzzy model and the real ASPs. The difference equations of concentrations of dissolved oxygen S_O and active autotrophic biomass X_{BA} can therefore be simplified as

$$\begin{bmatrix} S_{O}(k+1) \\ X_{BA}(k+1) \end{bmatrix} = \begin{bmatrix} a & c \\ 0 & q \end{bmatrix} \begin{bmatrix} S_{O}(k) \\ X_{BA}(k) \end{bmatrix} + \begin{bmatrix} d & 0 \\ 0 & w \end{bmatrix} \begin{bmatrix} S_{O_in} \\ X_{BA_in} \end{bmatrix} + z$$

where

$$\begin{split} a &\triangleq 1 + T \cdot \frac{Q}{V}, \quad c \triangleq -T \cdot \frac{4.57 - Y_A}{Y_A} \mu_A \bigg(\frac{S_{NH}}{K_{NH} + S_{NH}} \bigg), \quad d \triangleq T \cdot \frac{Q}{V} \\ q &\triangleq T \bigg[\mu_H \bigg(\frac{S_{NH}}{K_{NH} + S_{NH}} \bigg) - b_A - \frac{Q}{V} + \frac{1}{T} \bigg], \quad w \triangleq T \cdot \frac{Q}{V}. \end{split}$$

According to Table 1, we can get the fuzzy model for dissolved oxygen S_O and active autotrophic biomass X_{BA} with 4 rules $Rule\ r$: IF $S_S(k)$ is C_i , $S_{NH}(k)$ is D_i , THEN

$$\begin{bmatrix} S_{Or}(k+1) \\ X_{BAr}(k+1) \end{bmatrix} = \begin{bmatrix} a_r & c_r \\ 0 & q_r \end{bmatrix} \begin{bmatrix} S_{Or}(k) \\ X_{BAr}(k) \end{bmatrix} + \begin{bmatrix} d_r & 0 \\ 0 & w_r \end{bmatrix} \begin{bmatrix} S_{O_in} \\ X_{BA_in} \end{bmatrix} + Z_r$$
 (7)

where r = 1, 2, ..., 4, i = 1, 2, j = 1, 2. a_r, c_r, d_r, q_r, w_r and z_r are the constants to be identified (the algorithm is given in Appendix B). We employ the Gauss-shaped fuzzy sets with the membership

function as follows:

$$\mu_{\text{Ci}}(S_{\text{S}}(k)) \triangleq e^{-(S_{\text{S}}(k) - f_1)^2/2g_1^2}, \quad \mu_{\text{Dj}}(S_{\text{NH}}(k)) \triangleq e^{-(S_{\text{NH}}(k) - f_2)^2/2g_2^2}$$

where f_1, f_2, g_1, g_2 are the parameters to be identified (the details can be found in Appendix A). The degree of compatibility of each rule $\mu_r = \mu_{Ci} \cdot \mu_{Dj}$. The model output is defined by

$$\begin{bmatrix} S_O(k) \\ X_{BA}(k) \end{bmatrix} = \sum_{r=1}^4 \hat{\mu}_r \begin{bmatrix} S_{Or}(k) \\ X_{BAr}(k) \end{bmatrix}$$

where $\hat{\mu}_r = \mu_r / \sum_{t=1}^4 \mu_t$, r = 1, 2, ..., 4; i = 1, 2; j = 1, 2.

Table 1Fuzzy rules for modeling of activated sludge processes.

$S_S(k)$	$S_{NH}(k)$	$S_{NH}(k)$		
	$D_1 = NB$	$D_2 = PB$		
$C_1 = NB$ $C_2 = PB$	Rule 1 Rule 3	Rule 2 Rule 4		

3.2. Testification results

In this subsection, we shall testify the performance of proposed modeling approach by comparing the simulation results of fuzzy model and the simulation data supplied by ASM1. In order to represent the dynamic behavior of the S_O and X_{BA} , the values of influent parameters were set as the values shown in Table 2. It is important to note that the values of S_S , S_{NH} , S_O and X_{BA} were not constants, they were given as random values with variances 30, 20, 2 and 30, respectively. The other parameters and initial values were given equal to the BSM1. The testification results of an aerobic

Table 2Values of influent parameters.

Parameter	Value	Parameter	Value	Parameter	Value
S_S X_P S_{ND} S_O	69.5 mg/L 10 mg/L 6.95 mg/L 2 mg/L	X_S S_{NO} X_{ND} Q	202.32 mg/L 10 mg/L 10.59 mg/L 18,446 L	X _{BH} S _{NH} X _{BA} V	2000 mg/L 31.56 mg/L 60 mg/L 1333 m ²

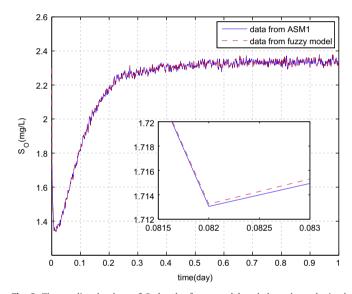


Fig. 2. The predicted values of S_{O} by the fuzzy model and the values obtained by ASM1.

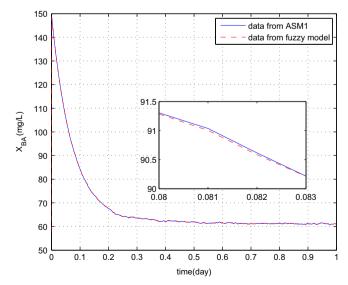


Fig. 3. The predicted values of X_{BA} by the fuzzy model and the values obtained by ASM1.

tank are shown in Figs. 2 and 3, where the solid line shows the data derived by ASM1, the dotted line shows the predicted data using the obtained fuzzy model. It can be seen from the figures that the predictions approximate the data obtained by ASM1 well with quite minor errors shown in the subgraphs (magnifying the regions marked in the original curves). This demonstrates that the model established by our approach can effectively describe the nonlinear dynamics in the ASPs, which lays a good basis for further control tasks in the WWTPs.

3.3. Fuzzy model-based predictive controller design

In the following, we will develop the fuzzy model-based predictive control law for the activated sludge wastewater treatment processes based on the previously obtained result [21].

Without loss of generality, we can define $\zeta(k) = [S_O(k) \ X_{BA}(k)]^T$. Then, the fuzzy model (7) can be rewritten as

$$\zeta(k+1) = A(k) \cdot \zeta(k) + B(k) \cdot u(k) + \theta(k),$$

$$v(k) = C \cdot \zeta(k)$$

where

$$\begin{split} A(k) &\triangleq \begin{bmatrix} \sum_{r=1}^4 \hat{\mu}_r(k) a_r & \sum_{r=1}^4 \hat{\mu}_r(k) c_r \\ 0 & \sum_{r=1}^4 \hat{\mu}_r(k) q_r \end{bmatrix}, \quad \theta(k) \triangleq \sum_{r=1}^4 \hat{\mu}_r Z_r, \\ B(k) &\triangleq \begin{bmatrix} \sum_{r=1}^4 \hat{\mu}_r(k) d_r & 0 \\ 0 & \sum_{r=1}^4 \hat{\mu}_r(k) w_r \\ 0 & \sum_{r=1}^4 \hat{\mu}_r(k) w_r \end{bmatrix}, \quad C \triangleq [1 \ 0]. \end{split}$$

By means of the previously obtained fuzzy model, the future *P*-step outputs of the activate sludge wastewater treatment system can be predicted and given by

$$\hat{y}(k+P|k) = \hat{A} \cdot \zeta(k) + \hat{B} \cdot \hat{u}(k) + \hat{\theta}$$

where

$$\hat{A} \triangleq \begin{bmatrix} CA(k) \\ CA(k+1)A(k) \\ \vdots \\ C\prod_{i=0}^{P-1} A(k+i) \end{bmatrix}, \quad \hat{\theta}(k) \triangleq \begin{bmatrix} C\theta(k) \\ C\theta(k+1) \\ \vdots \\ C\theta(k+P) \end{bmatrix}, \quad \hat{u}(k) \triangleq \begin{bmatrix} u(k) \\ u(k+1) \\ \vdots \\ u(k+M) \end{bmatrix}, \quad \hat{y}(k+P|k) \triangleq \begin{bmatrix} y(k+1|k) \\ y(k+2|k) \\ \vdots \\ y(k+P|k) \end{bmatrix},$$

$$\hat{B} \triangleq \begin{bmatrix} CB(k) & 0 & \dots & 0 \\ CA(k+1)B(k) & CB(k+1) & \dots & 0 \\ CA(k+2)A(k+1)B(k) & CA(k+2)B(k+1) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C\left(\prod_{i=1}^{P} A(k+i)\right)B(k) & C\left(\prod_{i=2}^{P} A(k+i)\right)B(k+1) & \dots & CB(k+M-1) \end{bmatrix}$$

with P > 0 and $P \ge M > 0$ are the prediction horizon and the control horizon, respectively.

The predictive error \hat{e} is given by the difference between the real output and the predictive output of the system, i.e.,

$$\hat{e}(k+1) \triangleq y(k+1) - \tilde{y}(k+1|k)$$

Using the predictive error at sample time k, the predictive output at the next time k+1 can be corrected, for example,

$$\tilde{y}(k+P) \triangleq \hat{y}(k+P|k) + h\hat{e}(k+1)$$

where

$$\tilde{y}(k+P) \triangleq \begin{bmatrix} \tilde{y}(k+1|k+1) \\ \tilde{y}(k+2|k+1) \\ \vdots \\ \tilde{y}(k+P|k+1) \end{bmatrix}, \quad h \triangleq [h_1 \ h_2 \ \dots \ h_P]^T.$$

At each sample time, the following quadratic cost function is optimized in order to determine the sequence of M future actuation signals u(k+i), i = 1, 2, ..., M.

$$J \triangleq \sum_{i=1}^{P} \|y_r(k+i) - \tilde{y}(k+i|k)\|_{W}^{2} + \sum_{j=1}^{M} \|u(k+i)\|_{R}^{2}$$

where

$$y_r(k+i) \triangleq y_{set}(k+i) - v^i(y_{set}(k) - y(k)), \quad 0 \le v < 1$$

and $W > 0, R \ge 0$ are the weighting matrixes. Further, we can get

$$J = \|\hat{y}_r(k) - \tilde{y}(k+P)\|_W^2 + \|\hat{u}(k)\|_R^2$$

where

$$\hat{y}_r(k) \triangleq \begin{bmatrix} y_r(k+1) \\ y_r(k+2) \\ \vdots \\ y_r(k+P) \end{bmatrix}, \quad \hat{u}(k) \triangleq \begin{bmatrix} u(k+1) \\ u(k+2) \\ \vdots \\ u(k+M) \end{bmatrix}.$$

Note that this is a common quadratic programming problem. Specially, when P = M = 1, the above cost function can be rewritten as

$$J = \|y_r(k+1) - \tilde{y}(k+1|k)\|_W^2 + \|u(k)\|_R^2$$

= $\|y_{set}(k+1) - v(y_{set}(k) - y(k)) - CA(k-1)\zeta(k)$
- $C(B(k)u(k) + \theta(k)) - h\hat{y}(k|k-1) + hy(k)\|_W^2 + \|u(k)\|_R^2$

If we define $E \triangleq y_{set}(k+1) - v(y_{set}(k) - y(k)) - CA(k)\zeta(k) - h\hat{y}(k|k-1) + hy(k)$, then, we can get

$$J = ||E - C(B(k)u(k) + \theta(k))||_{W}^{2} + ||u(k)||_{R}^{2}$$

$$t = [E^{T} - (u^{T}(k)B^{T}(k)) + \theta^{T}(k)C^{T}]W[E - C(B(k)u(k) + \theta(k))]$$

$$+ u^{T}(k)Ru(k).$$

The dJ/du(k) can now be written as

$$\frac{dJ}{du(k)} = -E^T W C B(k) - B^T(k) C^T W E + 2B^T(k) C^T W C B(k) u(k)$$
$$+ B^T(k) C^T W C \theta(k) + \theta^T(k) C^T W C B(k) + (R + R^T) u(k).$$

By setting

$$\frac{dJ}{du(k)} = 0$$

and deriving the control variable u(k), we get the following equation:

$$(B^{T}(k)C^{T}WCB(k) + R)u(k) = E^{T}WCB(k) + B^{T}(k)C^{T}WC\theta(k)$$

Then, the control law can be explicitly expressed as

$$u(k) = (B^{T}(k)C^{T}WCB(k) + R)^{-1}(E^{T}WCB(k) + B^{T}(k)C^{T}WC\theta(k)).$$

For general cases (i.e., P > 1 and M > 1), the control law can be implicitly obtained by solving the above-mentioned quadratic programming problem by letting the degree of membership be equivalent to the one at the previous sampling time. It is noted that such degrees will vary and lead to a variation of model for prediction.

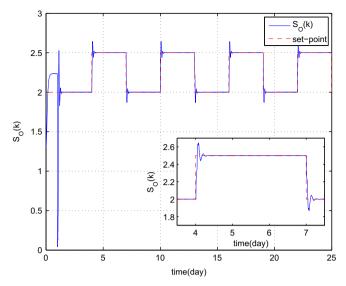


Fig. 4. The PID control of the activated sludge wastewater treatment processes.

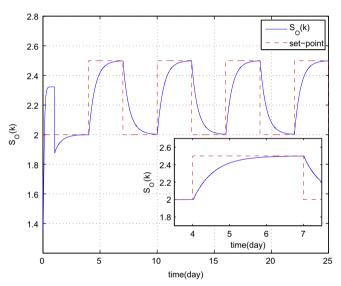


Fig. 5. The DMC control of the activated sludge wastewater treatment processes.

3.4. Fuzzy model-based predictive control of dissolved oxygen

In Section 3.3 the fuzzy model-based predictive control law had been systematically developed, now some simulation results using the derived predictive control law are presented in order to illustrate the effectiveness of the proposed design method. Consider an aeration tank shown in Fig. 1, whose fuzzy modeling was done in Section 3.1. The values of influent parameters were given in Section 3.2 without disturbance. The purpose here is to prove that the fuzzy model-based predictive control law can stabilize the concentration of dissolved oxygen S_0 at the set-point. At first, the activated sludge system was operated without control until the concentration of dissolved oxygen S_O was stable (after 1 day). Then, the control law was completed and it can be seen that the concentration of dissolved oxygen So persistently tracked the setpoints well. The control parameters were given as P = M = 1, W=60, R=0.1, v=0, h=0.718 (set-point is 2 mg/L) and h=0.685(set-point is 2.5 mg/L). Fig. 6 shows the changing curves of the concentration of dissolved oxygen S_0 . For the sake of comparison, the state responses when using PID and dynamic matrix control (DMC) algorithms, considering the same set-up for the proposed fuzzy model-based predictive controller design, have also been

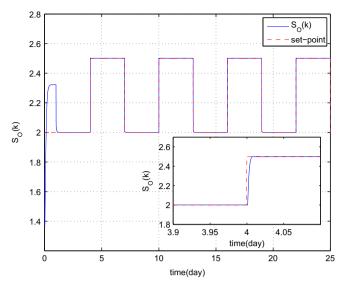


Fig. 6. The fuzzy model-based predictive control of the activated sludge wastewater treatment processes.

provided (see Figs. 4 and 5). In the PID control, the proportional gain \mathbf{K}_p , integral gain \mathbf{K}_i and derivative gain \mathbf{K}_d are chosen as $K_p=2.6$, $K_i=10$ and $K_d=1$. The configuration of the used DMC controller, in which the step response model of an aeration tank is employed as the predictive model with model horizon N=23, is given by P=15, M=2, $W=I\in\mathbb{R}^{15\times15}$, $R=I\in\mathbb{R}^{2\times2}$, v=0, $h=[0.5;0.5;...;0.5]\in\mathbb{R}^{15}$. By comparison, one can clearly see that the designed fuzzy model-based control law can realize the control goal with improved transient performance including small overshoot and short settling time.

4. Conclusions

In this paper, a fuzzy model-based predictive controller was proposed to control the concentration of dissolved oxygen in the activated sludge wastewater treatment processes. The aim is to maintain the concentration of dissolved oxygen at the set-point. In order to get the suitable fuzzy model of activated sludge processes, the structure of fuzzy rules required in the approach was first identified based on ASM1. Then, by combining the fuzzy c-means cluster algorithm and the method of least squares, the fuzzy space of input variables in the approach were further partitioned and the consequent parameters can be identified using the data derived by ASM1. Compared with the conventional PID and DMC controllers, it has been shown that the fuzzy modelbased predictive controller provided significant performance benefits and can be effectively used for dissolved oxygen control in activated sludge wastewater treatment plants. It is expected that the methods and ideas behind the paper could be applied to solve multi-objective control issues for the underlying system.

Appendix A. Data processing

The fuzzy c-means cluster algorithm [4,27,35,36] is adopted to process the data in order to get the membership function whose shape is fixed upon the cluster centers z and the distances between two nearby cluster centers. The cluster centers of premise variables (i.e., S_S , S_{NH}) need to be identified. The algorithm is as follows:

Step 1: Initialize the number of clusters C=2, the exponent m=2 and the cluster centers

$$z^{(0)} = [data_{\min} \ data_{\max}]$$

where $data_{\min}/data_{\max}$ is the min/max value of the data to be clustered. The 'data' here mean the premise variables S_S , S_{NH} . Step 2: Repeat

$$r = r + 1$$
.

Compute the elements of partition matrix, which will be used to calculate the cluster centers, $\nu_{i:}^{(r)}$:

$$\nu_{ik}^{(r)} = \left(\sum_{j=i}^{C} \left(\frac{d_{ik}}{d_{jk}}\right)^{2/(m-1)}\right)^{-1}, \quad 1 \le i \le C, \quad 1 \le k \le N$$

where $d_{ik}^2 = \|x_k - z_i^{(r-1)}\|^2$. If $d_{jk} = 0$, $\nu_{ik} = 1$. N is the number of data and x_k is the k_{th} set of data. Compute the cluster centers $z_i^{(r)}$:

$$z_{i}^{(r)} = \frac{\sum_{k=1}^{N} x_{k}(\nu_{ik})^{m}}{\sum_{k=1}^{N} (\nu_{ik})^{m}}, \quad 1 \leq i \leq C, \quad 1 \leq k \leq N.$$

Step 3: Output cluster centers until $\|\nu_{ik}^{(r)} - \nu_{ik}^{(r-1)}\| < \epsilon$ where $\epsilon > 0$ is the termination tolerance. In verifications in the paper, we set $\epsilon = 0.01$.

Appendix B. Parameters estimation

The least squares algorithm will be used in the paper for the purpose of parameters estimation as follows.

Let us rewrite (7) as

$$y(k) = h(k)\theta + e(k)$$

where

$$y(k) \triangleq S_0(k+1) \in \mathbb{R}^1 \tag{8}$$

$$\theta(k) \triangleq [a_1(k) \ c_1(k) \ d_1(k) \ e_1(k) \ a_2(k) \ c_2(k) \ \dots \ e_4(k)]^T \in \mathbb{R}^{16},$$
 (9)

$$h(k) \triangleq [\widehat{\mu}_1 S_0(k) \ \widehat{\mu}_1 X_{BA}(k) \ \widehat{\mu}_1 S_{0,n}(k) \ \widehat{\mu}_1 \ \widehat{\mu}_2 S_0(k) \ \dots \ \widehat{\mu}_4] \in \mathbb{R}^{1 \times 16}$$
(10)

01

$$y(k) \triangleq X_{BA}(k+1) \in \mathbb{R}^1 \tag{11}$$

$$\theta(k) \triangleq [q_1(k) \ w_1(k) \ ee_1(k) \ q_2(k) \ w_2(k) \ \dots \ ee_4(k)]^T \in \mathbb{R}^{12},$$
 (12)

$$h(k) \triangleq [\widehat{\mu}_1 X_{BA}(k) \ \widehat{\mu}_1 X_{BA,n}(k) \ \widehat{\mu}_1 \ \widehat{\mu}_2 X_{BA}(k) \ \dots \ \widehat{\mu}_4] \in \mathbb{R}^{1 \times 12}$$
 (13)

y(k) and h(k) are observable variables which can be got from the historical data (the data of two text files discussed in Section 2.2). The criterion function is set as

$$J(\theta) \triangleq \sum_{k=1}^{n} [e(k)]^2 = \sum_{k=1}^{n} [y(k) - h(k)\theta]^2 = (Y - H\theta)^T (Y - H\theta)$$

where n is the number of data used to identify the parameters, $Y \in \mathbb{R}^n$, $H \in \mathbb{R}^{n \times 16}$ for S_O and $H \in \mathbb{R}^{n \times 12}$ for X_{BA} . Minimize the criterion function, we can get the regular equation:

$$(H^T H)\widehat{\theta} = H^T Y.$$

Thus, the estimation of θ can be readily derived by

$$\hat{\theta} = (H^T H)^{-1} H^T Y$$

which gives the parameters identification needed in the proposed fuzzy modeling approach.

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