A Dynamic Model for Facility Location in Closed-Loop Supply Chain Design

Orapadee Joochim

1 Introduction

At the strategic level, closed-loop supply chain (CLSC) management involves long-term decisions regarding the location and capacity allocation of forward/reverse logistics facilities, the assignment of products to facilities, and the distribution of products between facilities and their end users or suppliers. Both demand and return handling must be taken into account, and the overall problem becomes more complicated than a separate forward supply chain (FSC) or reverse supply chain (RSC). Thus, the system should be scalable enough for being able to facilitate different kinds of requirements without any potential disruption of supply chain activities.

In this paper, a mathematical model is developed to comprehensively determine strategic solutions for the capacitated facility location problem in closed-loop supply chains.

2 The Proposed Model

Figure 1 schematically illustrates the type of supply chain (SC) modeled in this paper. The mathematical symbols firstly introduced in Fig. 1 are provided along with a short description of each in Table 1. The presented SC system consists of four critical processes: (1) production, (2) distribution, (3) collection and (4) disassembly and remanufacturing.

The production and disassembly-remanufacturing centers could be located at the same site (as bidirectional facility) or different places (as unidirectional facility). It is possible to locate both the distribution center and collection center at the same

O. Joochim (✉)
Institut für Produktionswirtschaft, Leibniz Universität Hannover, Königsworther Platz 1, 30167 Hannover, Germany
e-mail: orapadee.joochim@prod.uni-hannover.de

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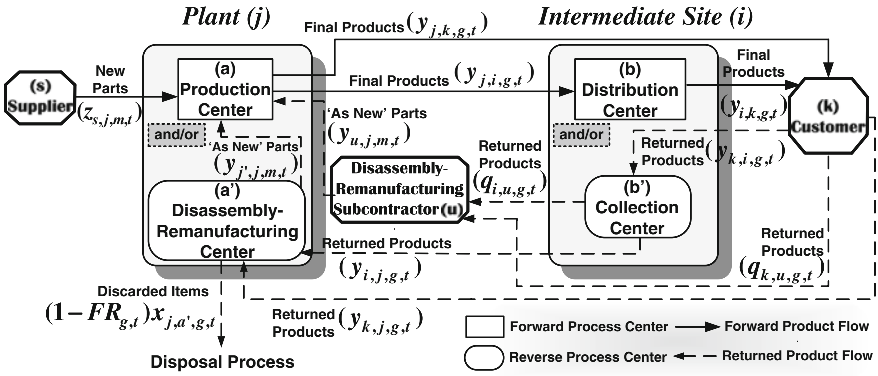

Fig. 1 Configuration of the proposed model

Table 1 Notation in the formulation of the proposed model

Indexsets			
$l \in \mathscr{L}$	Location sites	$c \in \mathscr{C}$	Center types for SC processes
$o \in \mathscr{O}$	Selectable location sites, $\mathscr{O} \subset \mathscr{L}$	$f \in \mathscr{F}$	Center types for FSC processes, $\mathscr{F} \subset \mathscr{C}$
$e \in \mathscr{E}$	Existing location sites, $\mathscr{E} \subset \mathscr{O}$	$r \in \mathscr{R}$	Center types for RSC processes, $\mathscr{R} \subset \mathscr{C}$
$n \in \mathscr{N}$	New location sites, $\mathscr{N} \subset \mathscr{O}$	$a \in \mathscr{A}$	Center types at plant sites, $\mathscr{A} \subset \mathscr{C}$
$j \in \mathscr{J}$	Plant sites, $\mathscr{J} \subset \mathscr{O}$	$b \in \mathscr{B}$	Center types at intermediate sites, $\mathscr{B} \subset \mathscr{C}$
$i \in \mathscr{I}$	Intermediate sites, $\mathscr{I} \subset \mathscr{O}$	$p \in \mathscr{P}$	Product types
$s \in \mathscr{S}$	External suppliers, $\mathscr{S} \subset \mathscr{L}$	$g \in \mathscr{G}$	Final products, $\mathscr{G} \subset \mathscr{P}$
$k \in \mathscr{K}$	Customers, $\mathscr{K} \subset \mathscr{L}$	$m \in \mathscr{M}$	Parts/components, $\mathscr{M} \subset \mathscr{P}$
$u \in \mathscr{U}$	External subcontractors, $\mathscr{U} \subset \mathscr{L}$	$t \in \mathscr{T}$	Periods in the planning horizon
Parameters			
$DP_{k,g,t}$	Demand of g by k in t	$KM_{o,c}$	Fixed expanding/relocating size for c at o
$RC_{k,g,t}$	Fraction of g returned from k in t	$AM_{m,g}$	Amount of m for assembling a unit of g
KO_o^{max}	Maximum capacity of o	$RM_{m,g}$	Amount of m from disassembling a unit of g
$KI_{o,c}$	Initial capacity of c at o	$FR_{g,t}$	Fraction of g satisfying specifications in t
$KC_{o,c}^{max}$	Maximum capacity of c at o	$FC_{o,c}$	Fraction of capacity of c allowed in o
$KC_{o,c}^{min}$	Minimum capacity of c at o	IR	Interest rate
Decisionvariables			
Non-negative integer			
$x_{j,a,g,t}$	Processing quantity	$exp_{o,c,t}$	Amount of expanded capacity
$y_{l,l',p,t}$	Transported products from l to l'	$mov_{o,o',c,t}$	Amount of relocated capacity
$z_{s,j,m,t}$	Purchased parts/components	$w_{o,c,t}$	Number of fixed sizes for expansion
$q_{l,u,g,t}$	Subcontracted returned products	$v_{e,n,c,t}$	Number of fixed sizes for relocation
Binary			
$\varphi_{o,t}$	1, if o is operated; 0 otherwise	$\rho_{e,c}$	1, if c is expanded at e; 0 otherwise
$\delta_{o,c,t}$	1, if c at o is operated; 0 otherwise		

site or to locate the distribution center and the collection center at different intermediate sites, i.e. bidirectional or unidirectional intermediate site.[1] In our framework, production centers have three alternatives for acquiring parts/components used to manufacture final products: (1) ordering the required parts/components from external suppliers, (2) re-processing the returned products and bringing those back 'as new' parts/components, and (3) outsourcing to subcontractors for disassembled and remanufactured parts/components. The manufactured products from production centers are initially transported to distribution centers and/or directly transported to customers. The distribution centers will then store the products until needed by customers. Whereas the collection points, which receive the used goods from customers, are used as storage for the reverse channel before the returned products are shipped to disassembly-remanufacturing centers and/or disassembly-remanufacturing subcontractors. Both disassembly-remanufacturing centers and disassembly-remanufacturing subcontractors can also receive the returned products straight from customers. The disassembly-remanufacturing centers are responsible for some essential activities of recovering, in which the returns are disassembled, tested, sorted and cleaned for reuse, repair and remanufacturing. Some discarded items from the disassembly-remanufacturing centers will be sent for the disposal process.

The objective function of the proposed model[2] as shown in (1) is based on cash flows. The total expenses in period t ($total\ expenses_t$) include purchasing expenses of parts/components, subcontracting expenses of disassembled and remanufactured parts/components, processing expenses at plant sites, transportation expenses of products, expenses of operating facilities, expenses of closing and opening facilities, expansion and relocation expenses, and disposal expenses.

$$\mathbf{MAX} \sum_{t \in \mathcal{T}} \frac{1}{(1+IR)^t} \left[total\ revenue_t - total\ expenses_t \right] \tag{1}$$

Forward and Reverse Flow Constraints Constraint (2) provides the amount of parts/components required for manufacturing. Constraint (3) assures the connection between the manufacturing process at any production center, and the outbound flows to distribution centers and directly to customers. Constraint (4) is the flow conservation at the distribution center. Constraint (5) ensures that all customer demands must be met. In constraint (6), the predefined return rate of final products is used as the return amount from customers. Constraint (7) is the flow conservation at the collection center. The volume of returned products sent to any disassembly-remanufacturing center is ensured by constraint (8). Constraints (9) and (10) establish the requirement for the outgoing flows of reusable parts/components from disassembly-remanufacturing process.

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[1] These uni/bidirectional facilities are developed from the idea of Sahyouni et al. [3].

[2] The model is developed from the models of Demirel and Gökçen [1], and Melo et al. [2].

$$\sum_{s \in \mathscr{S}} z_{s,j,m,t} + \sum_{j' \in \mathscr{J}} y_{j',j,m,t} + \sum_{u \in \mathscr{U}} y_{u,j,m,t} = \sum_{g \in \mathscr{G}} x_{j,a,g,t} AM_{m,g} \ , \quad \forall j, \ a \in \mathscr{F}, \ m, \ t \tag{2}$$

$$x_{j,a,g,t} = \sum_{i \in \mathscr{I}} y_{j,i,g,t} + \sum_{k \in \mathscr{K}} y_{j,k,g,t} \ , \quad \forall j, \ a \in \mathscr{F}, \ g, \ t \tag{3}$$

$$\sum_{j \in \mathscr{J}} y_{j,i,g,t} = \sum_{k \in \mathscr{K}} y_{i,k,g,t} \ , \quad \forall i, \ g, \ t \tag{4}$$

$$\sum_{j \in \mathscr{J}} y_{j,k,g,t} + \sum_{i \in \mathscr{I}} y_{i,k,g,t} = DP_{k,g,t} \ , \quad \forall k, \ g, \ t \tag{5}$$

$$\left(\sum_{j \in \mathscr{J}} y_{j,k,g,t} + \sum_{i \in \mathscr{I}} y_{i,k,g,t} \right) RC_{k,g,t} = \sum_{i \in \mathscr{I}} y_{k,i,g,t} + \sum_{j \in \mathscr{J}} y_{k,j,g,t} + \sum_{u \in \mathscr{U}} q_{k,u,g,t} \ , \quad \forall k, \ g, \ t \tag{6}$$

$$\sum_{k \in \mathscr{K}} y_{k,i,g,t} = \sum_{j \in \mathscr{J}} y_{i,j,g,t} + \sum_{u \in \mathscr{U}} q_{i,u,g,t} \ , \quad \forall i, \ g, \ t \tag{7}$$

$$\sum_{k \in \mathscr{K}} y_{k,j,g,t} + \sum_{i \in \mathscr{I}} y_{i,j,g,t} = x_{j,a,g,t} \ , \quad \forall j, \ a \in \mathscr{R}, \ g, \ t \tag{8}$$

$$\sum_{g \in \mathscr{G}} \left[FR_{g,t} \left(x_{j,a,g,t} RM_{m,g} \right) \right] = \sum_{j' \in \mathscr{J}} y_{j,j',m,t} \ , \quad \forall j, \ a \in \mathscr{R}, \ m, \ t \tag{9}$$

$$\sum_{g \in \mathscr{G}} \left\{ FR_{g,t} \left[\left(\sum_{k \in \mathscr{K}} q_{k,u,g,t} + \sum_{i \in \mathscr{I}} q_{i,u,g,t} \right) RM_{m,g} \right] \right\} = \sum_{j \in \mathscr{J}} y_{u,j,m,t} \ , \quad \forall u, \ m, \ t \tag{10}$$

Capacity Expansion and Relocation Constraints It is possible to expand the capacity at some existing location sites e. Constraint (11) limits the capacity for further expansion at each center c of any existing location site e. Constraint (12) restricts the full expanded capacity at any existing location site e. Constraint (13) limits the capacity that can be relocated from each center c at any existing location site e to one or more new location sites n. Moreover, constraint (11) together with (13) make sure that an existing capacity can either relocate to new sites ($\rho_{e,c} = 0$) or expand its capacity ($\rho_{e,c} = 1$). Constraint (14) imposes that by period t center c has been established at the new location site n for expanding the additional capacity and/or relocating the capacity from one or more existing location sites e. The additional capacity allowed at any new location site n is restricted by constraint (15). For each time period t, the allowable amount of capacity added to every center c at any selectable location site o is bounded by constraints (16) and (17).

$$\sum_{t \in \mathcal{T}} exp_{e,c,t} \leq \left(KC^{max}_{e,c} - KI_{e,c}\right) \rho_{e,c} \ , \quad \forall e, \ c \tag{11}$$

$$\sum_{c \in \mathcal{C}} \left[FC_{e,c} \left(\sum_{\tau=1}^{t} exp_{e,c,\tau} + KI_{e,c} \rho_{e,c} \right) \right] \leq KO^{max}_{e} \varphi_{e,t} \ , \quad \forall e, \ t \tag{12}$$

$$\sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{T}} mov_{e,n,c,t} \leq KI_{e,c} \left(1 - \rho_{e,c}\right) \ , \quad \forall e, \ c \tag{13}$$

$$\left(\sum_{\tau=1}^{t} exp_{n,c,\tau} + \sum_{e \in \mathcal{E}} \sum_{\tau=1}^{t} mov_{e,n,c,\tau} \right) \leq KC^{max}_{n,c} \delta_{n,c,t} \ , \quad \forall n, \ c, \ t \tag{14}$$

$$\sum_{c \in \mathcal{C}} \left[FC_{n,c} \left(\sum_{\tau=1}^{t} exp_{n,c,\tau} + \sum_{e \in \mathcal{E}} \sum_{\tau=1}^{t} mov_{e,n,c,\tau} \right) \right] \leq KO^{max}_{n} \varphi_{n,t} \ , \quad \forall n, \ t \tag{15}$$

$$exp_{o,c,t} = w_{o,c,t} KM_{o,c} \ , \quad \forall o, \ c, \ t \tag{16}$$

$$mov_{e,n,c,t} = v_{e,n,c,t} KM_{e,c} \ , \quad \forall e, \ n, \ c, \ t \tag{17}$$

Several additional constraints can not be provided in this paper due to space limitation. Among them are, e.g., maximum and minimum capacity constraints of facilities, facility configuration constraints allowing facilities to change their status (opened or closed) at most once, a logical constraint deciding whether facilities' capacity is to be added, and constraints enforcing non-negativity and binary conditions on the decision variables.

3 Numerical Example and Conclusion

The model is illustrated with a numerical example comprised of two existing plant sites (pl1 and pl2), one existing intermediate site (in1), one potential new plant site (pl3) and one potential new intermediate site (in2). Before the planning horizon starts, pl1 and pl2 have both production center (a1) and disassembly-remanufacturing center (a2), and in1 has both distribution center (b1) and collection center (b2). It is possible to open a1 and a2 at pl3, and b1 and b2 at in2. The model is solved using GAMS/CPLEX.

To evaluate the model, two scenarios are defined in terms of the percentage of products returned from customers over 10 year periods. In scenario L, low rates of returns are considered. Product demands of 10–30 % are assumed to return to the supply chain. Scenario H considers high rates of returns. Product returns of 70–90 %

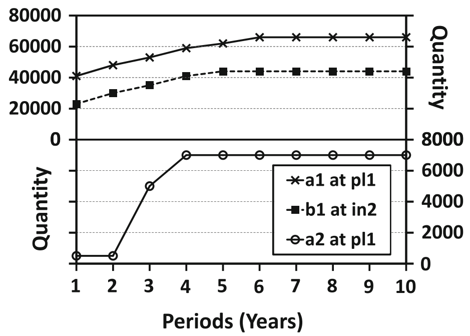

Fig. 2 Capacity expansion (scenario L)

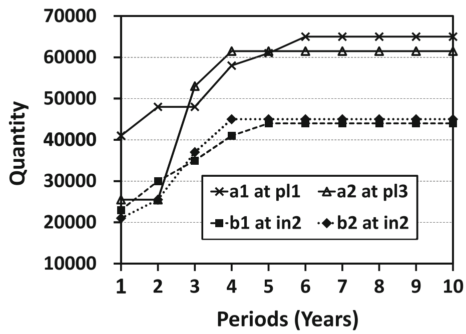

Fig. 3 Capacity expansion (scenario H)

from customers are assumed. Both scenario cases consider that customers' product demands of approximately 5–10 % gradually increase every year.

Figures 2, 3, 4, 5 show the capacity expansion and relocation at both existing and new location sites. There are investments in expanding the capacity of a1 at pl1 and b1 at in2 to meet increasing demands (both scenarios). For scenario L, the capacity of a2 at pl1 is expanded. In scenario H, it is more profitable to open a2 at pl3 and b2 at in2 for capacity expansion. There are also investments in capacity relocation from some existing facilities due to their high processing and shipping expenses. For

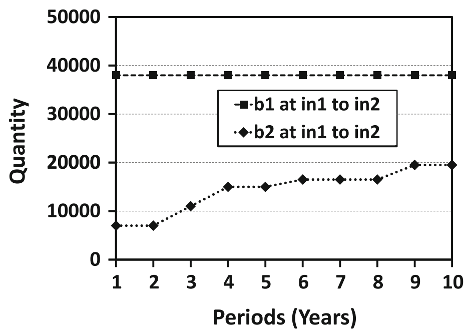

Fig. 4 Capacity relocation (scenario L)

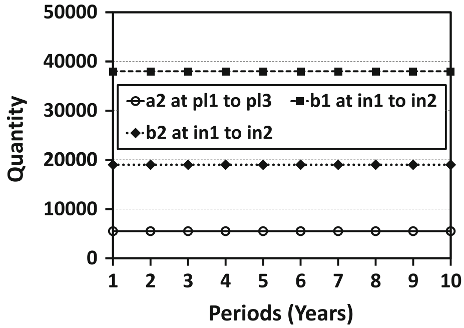

Fig. 5 Capacity relocation (scenario H)

both scenarios, the capacity of b1 and b2 are relocated from in1 to in2. Since returns increase in scenario H, the capacity of a2 is relocated from pl1 to pl3. The model selected to close a2 at pl2 due to high operating expenses at this site.

The configuration of both forward and reverse channels has a strong influence on the performance of each other. Bidirectional facilities eliminate substantial investment in infrastructure, equipment, and human resources. Only isolated, stand-alone forward and reverse facilities might be beneficial, if transportation and processing expenses are a large portion of the total expenses. The present model can be used to get better insight into the quantitative aspects of strategic planning within the CLSC context.

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