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Development of a State Dependent Ricatti Equation Based Tracking Flight Controller for an Unmanned Aircraft

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A dual loop nonlinear State Dependent Riccati Equation (SDRE) control method is developed for the flight control of an unmanned aircraft. The outer loop addresses the attitude and altitude kinematics, while the inner loop handles the translational and rotational equations of motion. The control strategy utilizes a tracking control problem. The mismatch due to the SDC factorization of the inner loop is handled with a nonlinear compensator again derived from the tracking control formulation. The quadratic optimal control problems of the inner and outer loops are solved at discrete intervals in time. A nonlinear simulation model of the UAV is used to examine the performance of the SDRE controller. Two flight scenarios are considered: a coordinated turn maneuver and a high angle of attack flight. These simulation results show the effectiveness of the proposed nonlinear controller.

Nomenclature

u, v, w	Velocity components along X, Y and Z body axis, m/s
p, q, r	Angular rates, deg/s
ϕ, θ, ψ	Euler angles, deg
α	Angle of attack
X, Y, Z	Position in inertial frame, m
h	Altitude, m
m	Mass, kg
I_{xx}, I_{xy}, I_{xz}	Main moments of inertia
ρ	Air density
S	Wing reference area, m^2
$\bar{c}$	Mean aerodynamic chord, m
b	Wing reference area, m
$\bar{q}$	Dynamic pressure
C_L	Aerodynamic lift coefficient
C_D	Aerodynamic drag coefficient
C_l	Aerodynamic rolling moment coefficient
C_m	Aerodynamic pitching moment coefficient
C_n	Aerodynamic yawing moment coefficient

I. Introduction

Autonomous flight control systems for aerospace vehicles present significant challenges for nonlinear flight regimes such as, high angles of attack flight, asymmetric store separation, etc. For fixed-wing aircraft, linear controllers, together with gain scheduling usually provide sufficient flight control performance. However,

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while performing agile maneuvers, aircraft dynamics is highly nonlinear, and needs to be taken into account for precise control. For such cases, new control algorithms are needed.

The State-Dependent Riccati Equation (SDRE) control method is a nonlinear control technique, which became popular over the last decade. It provides an effective algorithm for synthesizing nonlinear feedback controls that allow considering nonlinearities in the system states, and additionally offers design flexibility through state-dependent weighting matrices. This was originally proposed by Pearson[1] and later expanded by Wernli and Cook[2], it also was independently studied by Mracek and Cloutier[3] and mentioned by Friedland[4]. SDRE controller design involves the factorization (in other words, parametrization) of the nonlinear dynamics into the state vector and the product of a matrix-valued function that depend on the states itself. The SDRE algorithm employs the nonlinear fully coupled equations of motion. The SDRE algorithm captures the nonlinearities of the system by converting it to a quasi-linear structure using state-dependent coefficient (SDC) matrices. This enables the re-computing of the controller gains in real time by minimizing a quasi-quadratic cost function. An algebraic Riccati equation (ARE) using the SDC matrices is then solved on-line to obtain the feedback gain. The algorithm thus involves solving an algebraic state-dependent Riccati equation, or SDRE, at every state. The non-uniqueness of the parameterization creates additional degrees of freedom, which may be used to enhance controller performance. It is important to note that methods using SDRE can be applied to minimum as well as a non-minimum phase nonlinear system. Furthermore, the weight may be adaptively changed to avoid actuator saturation problems. The SDRE control approach is applied to a number of control problems in aerospace applications, such as VTOL vehicles[5–7], and quadrotors [8]. Another wide area of SDRE technique application is a spacecraft attitude control[9–12]. However, there is a need to further study the application of SDRE control to the nonlinear flight regimes of a fixed-wing aircraft.

This paper focuses on the utilization of the SDRE control method for the flight control of a fixed wing unmanned aerial vehicle (UAV). The controller implemented as tracking, infinite horizon quadratic optimal control with concentric loops. In the outer loop attitude and altitude control problem is addressed. The inner loops uses rotational and translational equations of the aircraft. The design also includes a nonlinear compensator, which accounts for the mismatch between the full vehicle dynamics and its SDC parametrization of the inner loop. To demonstrate the performance of the designed control system two flight regimes are chosen for simulations. Thus, a harsh coordinated turn at a constant altitude with a given turn radius, and a high angle of attack flight phase are considered.

Section II provides background to the extended linearization and SDRE control. A tracking controller structure and the compensator formulations are given in Section III. Section IV presents the architecture of the control system used and the derivation of the corresponding state-dependent model. Application of the developed flight control system is given in Section V, with simulation results and discussed. Finally conclusions are given in Section VI.

II. Extended Linearization and SDRE Control

A. Extended Linearization

Extended linearization, also known as apparent linearization or SDC parameterization is the process of factorizing a nonlinear system into a linear-like structure, which contains SDC matrices [2,4]. Consider the system, which is full-state observable, autonomous, nonlinear in the state, and affine in the input, represented in the following form [13] :

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}) + \mathbf{B}(\mathbf{x})\mathbf{u}(t), \mathbf{x}(\mathbf{0}) = \mathbf{x}_\mathrm{o} \tag{1}$$

where $\mathbf{x} \in \mathbf{R}^n$ is the state vector, $\mathbf{u} \in \mathbf{R}^m$ is the input vector, function $\mathbf{f} : \mathbf{R}^n \to \mathbf{R}^n$, $\mathbf{B} : \mathbf{R}^n \to \mathbf{R}^{n\times m}$ and $\mathbf{B}(\mathbf{x}) \neq \mathbf{0}, \forall \mathbf{x}$.

Under the assumptions $\mathbf{f}(0) = 0$, and $\mathbf{f}(\cdot) \in \mathbf{C}^1(\mathbf{R}^n)$ a continuous nonlinear matrix- valued function $\mathbf{A}(\mathbf{x})$ always exists such that

$$\mathbf{f}(\mathbf{x}) = \mathbf{A}(\mathbf{x})\mathbf{x}, \tag{2}$$

where $\mathbf{A}$, the $n \times n$ matrix, is found by mathematical factorization and is non unique when $n > 1$. Hence, extended linearization of the input-affine nonlinear system (1) becomes

$$\dot{\mathbf{x}}(t) = \mathbf{A}(\mathbf{x})\mathbf{x}(t) + \mathbf{B}(\mathbf{x})\mathbf{u}(t), \mathbf{x}(\mathbf{0}) = \mathbf{x}_\mathrm{o} \tag{3}$$

which has a linear structure with SDC matrices $\mathbf{A}(\mathbf{x})$, $\mathbf{B}(\mathbf{x})$. The application of any linear control synthesis method to the linear-like SDC structure, where $\mathbf{A}(\mathbf{x})$ and $\mathbf{B}(\mathbf{x})$ are treated as constant matrices, forms an extended linearization control method [13].

B. SDRE Control

SDRE feedback control provides a similar approach to the nonlinear regulation problem for the input-affine system (1) as a linear control synthesis method (LQR). The performance cost function which is to be minimized is defined as follows:

$$\mathbf{J}(\mathbf{x}_o, \mathbf{u}) = \frac{\mathbf{1}}{\mathbf{2}} \int_o^{\infty} \left\{ \mathbf{x}^{\mathbf{T}}(t)\mathbf{Q}(\mathbf{x})\mathbf{x}(t) + \mathbf{u}^{\mathbf{T}}(t)\mathbf{R}(\mathbf{x})\mathbf{u}(t) \right\} dt \tag{4}$$

where $\mathbf{Q}(\mathbf{x}) \in \mathbf{R}^{n\times m}$ is symmetric positive semidefinite, $\mathbf{R}(\mathbf{x}) \in \mathbf{R}^{m\times m}$ is symmetric positive definite matrix, which in general may be state dependent. For the calculation of instantaneous feedback gains, the weighting matrices, $\mathbf{Q}$ and $\mathbf{R}$ as well as system matrices, $\mathbf{A}$ and $\mathbf{B}$ are assumed to be constant. Then, for the given sate, the feedback gain is calculated as it is done for an infinite horizon LQR controller [14, 15]:

$$\mathbf{u}(\mathbf{x}) = -\mathbf{K}(\mathbf{x})\mathbf{x} = -\mathbf{R}^{-\mathbf{1}}(\mathbf{x})\mathbf{B}^{\mathbf{T}}(\mathbf{x})\mathbf{P}(\mathbf{x})\mathbf{x} \tag{5}$$

where $\mathbf{P}(\mathbf{x})$ is the solution of the following Algebraic State Dependent Riccati Equation:

$$\mathbf{P}(\mathbf{x})\mathbf{A}(\mathbf{x})\mathbf{A}^{\mathbf{T}}(\mathbf{x})\mathbf{P}(\mathbf{x}) - \mathbf{P}(\mathbf{x})\mathbf{B}(\mathbf{x})\mathbf{R}^{-\mathbf{1}}(\mathbf{x})\mathbf{B}^{\mathbf{T}}(\mathbf{x})\mathbf{P}(\mathbf{x}) + \mathbf{Q}(\mathbf{x}) = \mathbf{0} \tag{6}$$

This approach is expected to have the usual robustness and asymptotic stability properties of the classical LQR. However, the controller generated this way is a nonlinear controller with the convenience that it does not require the linearization of the system equations. One important issue is to make sure that the system matrices $\mathbf{A}(\mathbf{x})$ and $\mathbf{B}(\mathbf{x})$ form a fully controllable pair. Although controllability depends on the physical nature of the problem, a physically controllable system may become uncontrollable from time to time due to an improper choice of the state dependent coefficient factorization. This in turn makes the solution of Eq. (6) impossible.

III. Tracking Controller with a Nonlinear Compensator

A trajectory following (tracking) linear quadratic optimal control (LQT)may be posed as follows. Given a linear dynamic system,

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{C}(t)\mathbf{x}(t) \end{aligned} \tag{7}$$

where $\mathbf{x}(t)$ is an n^{th} order state vector, $\mathbf{u}(t)$ is the r^{th} order control vector, and $\mathbf{y}(t)$ is the m^{th} order output vector. It is desired to control the system (7) such that the desired output $\mathbf{y}(t)$ tracks the reference input $\mathbf{z}(t)$ as close as possible whil minimizing the following quadratic cost function:[14, 15]

$$\mathbf{J} = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{e}^T(t_f)\mathbf{F}(t_f)\mathbf{e}(\mathbf{t_f}) + \frac{\mathbf{1}}{\mathbf{2}} \int_{t_\mathbf{0}}^{t_\infty} \left\{ \mathbf{e}^T(t)\mathbf{Q}(t)\mathbf{e}(t) + \mathbf{u}^T(t)\mathbf{R}(t)\mathbf{u}(t) \right\} dt \tag{8}$$

where $\mathbf{e}(t) = \mathbf{z}(t) - \mathbf{y}(t)$ is the error vector. It is assumed that $\mathbf{F}(t_f)$ and $\mathbf{Q}(t_f)$ are $m \times m$ symmetric positive semidefinite matrices, and $\mathbf{R}(t)$ is an $r \times r$ symmetric positive definite matrix. The solution of this optimal control problem is presented in the references cited. Now consider a system in the following form:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{u}(t) + \mathbf{f}(t) \\ \mathbf{y}(t) &= \mathbf{C}(t)\mathbf{x}(t) \end{aligned} \tag{9}$$

where $\mathbf{f}(t)$ represents the mismatch that appears as a result of the factorization of the nonlinear system equations in the form of (7) provided that $\mathbf{f}(t)$ is a slowly varying signal that may be assumed constant at certain time intervals, and $\mathbf{f}$ is bounded. If the performance index is defined by Eq. (19), the Hamiltonian then may be given as,

$$
\begin{aligned}
\mathbf{H}(\mathbf{x}(t),\mathbf{u}(t),\lambda(t)) = \tfrac{\mathbf{1}}{\mathbf{2}}[\mathbf{z}(t)-\mathbf{C}(t)\mathbf{x}(t)]^{\mathbf{T}}\mathbf{Q}(t)[\mathbf{z}(t)-\mathbf{C}(t)\mathbf{x}(t)] + \tfrac{\mathbf{1}}{\mathbf{2}}\mathbf{u}^{\mathbf{T}}(t)\mathbf{R}(t)\mathbf{u}(t) \\
+\lambda^{\mathbf{T}}(t)[\mathbf{A}(t)\mathbf{x}(t)+\mathbf{B}(t)\mathbf{u}(t)+\mathbf{f}(t)]
\end{aligned}
\tag{10}
$$

The optimal control is obtained from $\frac{\partial \mathbf{H}}{\partial \mathbf{u}} = \mathbf{0}$, which giving,

$$
\mathbf{u}^*(t) = -\mathbf{R}^{-\mathbf{1}}(t)\mathbf{B}^{\mathbf{T}}(t)\lambda^*(t) \tag{11}
$$

The remaining equations for the states and costates may be obtained as,

$$
\begin{bmatrix} \dot{\mathbf{x}}^*(t) \\ \dot{\lambda}^*(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A}(t) & -\mathbf{E}(t) \\ -\mathbf{V}(t) & -\mathbf{A}^{\mathbf{T}}(t) \end{bmatrix} \begin{bmatrix} \mathbf{x}^*(t) \\ \lambda^*(t) \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{W}(t) \end{bmatrix} \mathbf{z}(t) + \begin{bmatrix} \mathbf{f}(t) \\ \mathbf{0} \end{bmatrix} \tag{12}
$$

where

$$
\begin{aligned}
\mathbf{E}(t) &= \mathbf{B}(t)\mathbf{R}^{-\mathbf{1}}(t)\mathbf{B}^{\mathbf{T}}(t) \\
\mathbf{V}(t) &= \mathbf{C}^{\mathbf{T}}(t)\mathbf{Q}(t)\mathbf{C}(t) \\
\mathbf{W}(t) &= \mathbf{C}^{\mathbf{T}}(t)\mathbf{Q}(t)
\end{aligned}
\tag{13}
$$

The boundary conditions for the state and costate equations are defined by the initial condition on the state: $\mathbf{x}(t=t_0) = x(t_0)$ and the final condition on the costate:

$$
\lambda(t_f) = \frac{\partial}{\partial(\mathbf{x}(t_f))}\left[\frac{\mathbf{1}}{\mathbf{2}}\mathbf{e}^{\mathbf{T}}(t_f)\mathbf{F}(t_f)\mathbf{e}(t_f)\right] = \mathbf{C}^{\mathbf{T}}(t_f)\mathbf{F}(t_f)\mathbf{C}(t_f)\mathbf{x}(t_f) - \mathbf{C}^{\mathbf{T}}(t_f)\mathbf{F}(\mathbf{t_f})\mathbf{z}(t_f) \tag{14}
$$

Assuming a linear relation between the sate and co-state of the following form [14]:

$$
\lambda^*(t) = \mathbf{P}(t)\mathbf{x}^*(t) - \mathbf{g}(t) \tag{15}
$$

where $\mathbf{P}(t)$ is a square matrix of size n and $\mathbf{g}(t)$ is a vector of length n, are to be determined such that the canonical system (12) is satisfied. As a result it may be shown that if $\mathbf{P}(t)$ can be found as solution to a matrix differential Riccati equation (16), and $\mathbf{g}(t)$ is a solution to a vector differential equation (17):

$$
\dot{\mathbf{P}}(t) = -\mathbf{P}(t)\mathbf{A}(t) - \mathbf{A}^{\mathbf{T}}(t)\mathbf{P}(t) + \mathbf{P}(t)\mathbf{E}(t)\mathbf{P}(t) - \mathbf{V}(t) \tag{16}
$$

$$
\dot{\mathbf{g}}(t) = \left[\mathbf{P}(t)\mathbf{E}(t) - \mathbf{A}^{\mathbf{T}}(t)\right]\mathbf{g}(t) - \mathbf{W}(t)\mathbf{z}(t) + \mathbf{P}(t)\mathbf{f}(t) \tag{17}
$$

The optimal control is obtained in the form given by Eq. (18).

$$
\mathbf{u}^*(t) = -\mathbf{R}^{-\mathbf{1}}(t)\mathbf{B}^{\mathbf{T}}(t)\left[\mathbf{P}(t)\mathbf{x}^*(t) - \mathbf{g}(t)\right] = -\mathbf{K}(t)\mathbf{x}^*(t) + \mathbf{R}^{-\mathbf{1}}(t)\mathbf{B}^{\mathbf{T}}(t)\mathbf{g}(t) \tag{18}
$$

For the infinite-horizon problem formulation, consider the system Eq. (9) but with the system matrices being time invariant, and the performance index chosen as

$$
\lim_{\mathbf{t}_f\to\infty}\mathbf{J} = \lim_{\mathbf{t}_f\to\infty}\frac{\mathbf{1}}{\mathbf{2}}\int_{\mathbf{t_0}}^{\mathbf{t}_f}\left\{\mathbf{e}^{\mathbf{T}}(t)\mathbf{Q}(t)\mathbf{e}(t) + \mathbf{u}^{\mathbf{T}}(t)\mathbf{R}(t)\mathbf{u}(t)\right\}\mathbf{dt} \tag{19}
$$

Using the results for a finite-time case above and let $\mathbf{t}_f \to \infty$ will lead to the infinite-time case solution. Thus, the matrix function $\mathbf{P}(t)$ in Eq. (16) will result to the steady-state value $\mathbf{P}$ as the solution of the following algebraic Riccati equation:

$$
-\mathbf{PA} - \mathbf{A^TP} + \mathbf{PBR^{-1}B^TP} - \mathbf{C^TQC} = \mathbf{0} \tag{20}
$$

For slowly varying input signals $\mathbf{z}(t)$, solution of a matrix differential equation (17) can be obtained by setting the derivative to zero and solving Eq. (17) for $\mathbf{g}(t)$:

$$
\mathbf{g}(t) = \left[\mathbf{PE} - \mathbf{A^T}\right]^{-\mathbf{1}}(\mathbf{Wz}(t) + \mathbf{Pf}(t)) \tag{21}
$$

where

$$
\begin{aligned}
\mathbf{E} &= \mathbf{BR^{-1}B^T} \\
\mathbf{W} &= \mathbf{C^TQ}
\end{aligned}
\tag{22}
$$

Then the optimal control is:

$$\mathbf{u}(t) = \mathbf{K}\mathbf{x}(t) + \mathbf{K_z}\mathbf{z}(t) + \mathbf{K_f}\mathbf{f}(t) \tag{23}$$

and the corresponding controller gains are defined as:

$$\begin{aligned} \mathbf{K} &= -\mathbf{R^{-1}BP} \\ \mathbf{K_z} &= \mathbf{R^{-1}B}\left[\mathbf{PE} - \mathbf{A^T}\right]^{-1}\mathbf{W} \\ \mathbf{K_f} &= -\mathbf{R^{-1}B}\left[\mathbf{PE} - \mathbf{A^T}\right]^{-1}\mathbf{P} \end{aligned} \tag{24}$$

State dependent formulation to be used for tracking control formulation is presented next.

IV. Controller Structure and SDC Model

The flight control equations are divided into kinematic and dynamic equations. The dynamic equations describe the translational and rotational motion of the rigid body aircraft. The kinematic equations, on the other hand, relates the rotational velocities to the aircraft attitude and aircraft velocities to the position and altitude. The dynamic equations are treated in the inner loop. The equations related to the aircraft attitude and altitude are treated in the outer loop, forming a two loop structure. This two loop system is shown in Fig. 1. As it may be observed from the figure, there are two feedback loops. Each loop gain is designed using seperate SDRE controller. The main advantage of this two loop system is the reduction in the dimensions of state vectors, and computational cost associated with the calculation of the feedback gain.

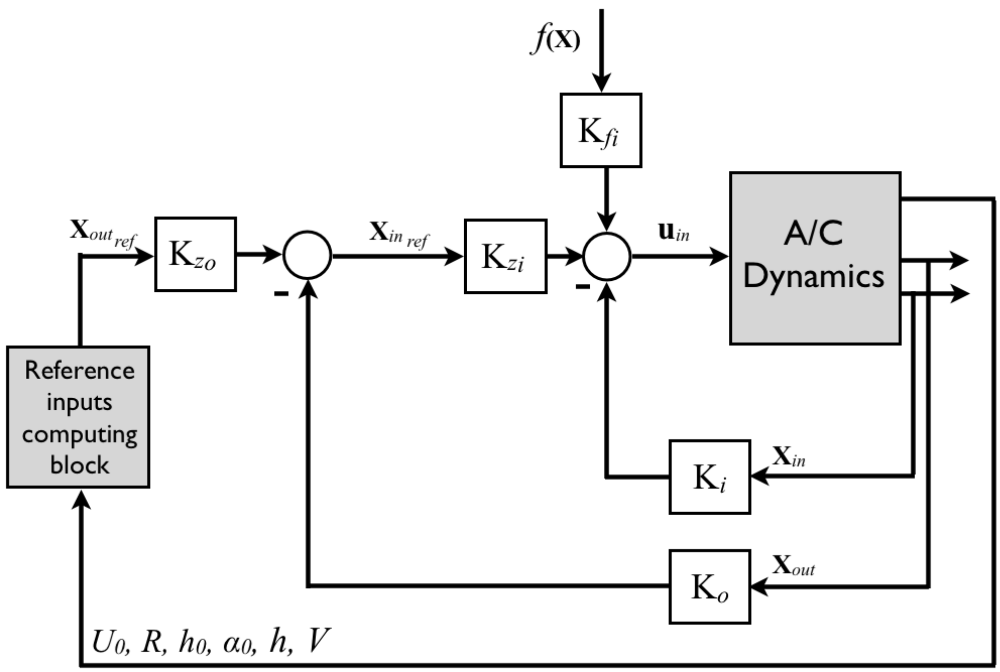

Figure 1. Control System Block Diagram.

The 6 degrees-of-freedom equations of motion of an aircraft written in the body-fixed coordinate system are used to obtain the SDC model[16]. As a result the kinematic equations relate the body fixed measurements to the altitude and attitude. Then the outer loop state and control vectors are defined as follows:

$$\mathbf{x_{out}} = [\phi, \theta, \psi, h]^T, \mathbf{u_{out}} = [u, v, w, p, q, r]^T \tag{25}$$

A possible set of SDC matrices for the outer loop dynamics may be written as:

$$\mathbf{A_{out}}(\mathbf{x_{out}}) = [\mathbf{0}] \tag{26}$$

$$\mathbf{B_{out}}(\mathbf{x_{out}}) = \begin{bmatrix} 0 & 0 & 0 & 1 & \tan\theta\sin\phi & \tan\theta\cos\phi \\ 0 & 0 & 0 & 0 & \cos\phi & -\sin\phi \\ \sin\theta & \sin\phi\cos\theta & -\cos\theta & 0 & 0 & 0 \end{bmatrix} \tag{27}$$

and,

$$\mathbf{f_{out}} = [\mathbf{0}] \tag{28}$$

The inner loop state and control vectors are defined as follows:

$$\mathbf{x_{in}} = [u, v, w, p, q, r]^T, \mathbf{u_{in}} = [\delta_a, \delta_e, \delta_r, \delta_T]^T \tag{29}$$

Following relations are assumed for the aerodynamic coefficients[16]:

$$\begin{gathered} C_D = C_{D_0} + C_{D_\alpha}\alpha + C_{D_{\delta_e}}\delta_e \\ C_L = C_{L_0} + C_{L_\alpha}\alpha + C_{L_{\dot\alpha}}\tfrac{\bar c}{2V_0}\dot\alpha + C_{L_q}\tfrac{\bar c}{2V_0}q + C_{L_{\delta_e}}\delta_e \\ C_m = C_{m_0} + C_{m_\alpha}\alpha + C_{m_{\dot\alpha}}\tfrac{\bar c}{2V_0}\dot\alpha + C_{m_q}\tfrac{\bar c}{2V_0}q + C_{m_{\delta_e}}\delta_e \\ C_l = C_{l_0} + C_{l_\beta}\beta + C_{l_p}\tfrac{b}{2V_0}p + C_{l_r}\tfrac{b}{2V_0}r + C_{l_{\delta_a}}\delta_a + C_{l_{\delta_r}}\delta_r \\ C_Y = C_{Y_0} + C_{Y_\beta}\beta + C_{Y_p}\tfrac{b}{2V_0}p + C_{Y_r}\tfrac{b}{2V_0}r + C_{Y_{\delta_a}}\delta_a + C_{Y_{\delta_r}}\delta_r \\ C_n = C_{n_0} + C_{n_\beta}\beta + C_{n_p}\tfrac{b}{2V_0}p + C_{n_r}\tfrac{b}{2V_0}r + C_{n_{\delta_a}}\delta_a + C_{n_{\delta_r}}\delta_r \end{gathered} \tag{30}$$

A possible set of the state dependent matrices for the inner loop dynamics model may be obtained in the following form:

$$\mathbf{A_{in}}(\mathbf{x_{in}}) = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \tag{31}$$

where,

$$A_{11} = \begin{bmatrix} \frac{\frac{1}{2}\rho VS(-C_{D_0} - C_{D_\alpha}\alpha)}{m} & 0 & \frac{\bar qS(C_{L_\alpha}\alpha + C_{L_0})}{mu} \\ 0 & \frac{\bar qSC_{Y_\beta}}{mu} & 0 \\ \frac{\frac{1}{2}\rho VS(-C_{D_0}\alpha - C_{L_0})}{m} & 0 & \frac{\bar qS(-C_{D_\alpha}\alpha - C_{L_\alpha})}{mu} \end{bmatrix} \tag{32}$$

$$A_{12} = \begin{bmatrix} 0 & \frac{\bar qS\bar c(C_{L_q} + C_{L_{\dot\alpha}})\alpha}{m}\frac{\bar c}{2V_0} - w & v \\ \frac{\bar qS\bar cC_{Y_p}}{m}\frac{\bar c}{2V_0} + w & 0 & \frac{\bar qS\bar cC_{Y_r}}{m}\frac{\bar c}{2V_0} - u \\ -v & \frac{\bar qS\bar c(-C_{L_q} - C_{L_{\dot\alpha}}\alpha}{m}\frac{\bar c}{2V_0} + u & 0 \end{bmatrix} \tag{33}$$

$$A_{21} = \begin{bmatrix} 0 & \frac{\bar qSb(c_3C_{l_\beta} + c_4C_{n_\beta})}{u} & 0 \\ \frac{\frac{1}{2}\rho VS\bar cC_{m_0}}{I_{yy}} & 0 & \frac{\bar qS\bar cC_{m_\alpha}}{I_{yy}u} \\ 0 & \frac{\bar qSb(c_4C_{l_\beta} + c_9C_{n_\beta})}{u} & 0 \end{bmatrix} \tag{34}$$

$$A_{22} = \begin{bmatrix} \bar qSb(c_3C_{l_p} + c_4C_{n_p})\frac{b}{2V_0} + c_2q & 0 & \bar qSb(c_3C_{l_r} + c_4C_{n_r})\frac{b}{2V_0} + c_1q \\ 0 & \frac{\bar qS\bar c(C_{m_q} + C_{m_{\dot\alpha}})}{I_{yy}}\frac{\bar c}{2V_0} & 0 \\ \bar qSb(c_4C_{l_p} + c_9C_{n_p})\frac{b}{2V_0} + c_8q & 0 & \bar qSb(c_4C_{l_r} + c_9C_{n_r})\frac{b}{2V_0} - c_2q \end{bmatrix} \tag{35}$$

$$\mathbf{B_{in}}(\mathbf{x_{in}}) = \begin{bmatrix} 0 & \frac{\bar qS(C_{L_{\delta_e}}\alpha - C_{D_{\delta_e}})}{m} & 0 & \frac{C_T}{m} \\ 0 & 0 & \frac{\bar qSC_{Y_{\delta_r}}}{m} & 0 \\ 0 & \frac{\bar qS(-C_{L_{\delta_e}} - C_{D_{\delta_e}}\alpha)}{m} & 0 & 0 \\ \bar qSb(c_3C_{l_{\delta_a}} + c_4C_{n_{\delta_a}}) & 0 & \bar qSb(c_3C_{l_{\delta_r}} + c_4C_{n_{\delta_r}}) & 0 \\ 0 & \frac{\bar qS\bar cC_{m_{\delta_e}}}{I_{yy}} & 0 & 0 \\ \bar qSb(c_4C_{l_{\delta_a}} + c_9C_{n_{\delta_a}}) & 0 & \bar qSb(c_4C_{l_{\delta_r}} + c_9C_{n_{\delta_r}}) & 0 \end{bmatrix} \tag{36}$$

and,

$$\begin{array}{lll} c_1 = \frac{(I_{yy}-I_{zz})I_{zz}-I_{xz}^2}{I_{xx}I_{zz}-I_{xz}^2}; & c_2 = \frac{(I_{xx}-I_{yy}+I_{zz})I_{xz}}{I_{xx}I_{zz}-I_{xz}^2}; & c_3 = \frac{I_{zz}}{I_{xx}I_{zz}-I_{xz}^2} \\ c_4 = \frac{I_{xz}}{I_{xx}I_{zz}-I_{xz}^2}; & c_5 = \frac{(I_{zz}-I_{xx})}{I_{yy}}; & c_6 = \frac{I_{xz}}{I_{yy}} \\ c_7 = \frac{1}{I_{yy}}; & c_8 = \frac{(I_{xx}-I_{yy})I_{xx}-I_{xz}^2}{I_{xx}I_{zz}-I_{xz}^2}; & c_9 = \frac{I_{xx}}{I_{xx}I_{zz}-I_{xz}^2} \end{array} \tag{37}$$

The mismatch between the original dynamics and the SDC parametrization includes terms that appear due to the gravitational acceleration is modelled as a slowly varying external input.

$$\mathbf{f_{in}} = \begin{bmatrix} -g\sin\theta \\ g\cos\theta\sin\phi \\ g\cos\theta\cos\phi \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{38}$$

Note that the above vector of gravitational effects does not contain any states of the inner loop. Since the attitude change is much slower than the inner loop parameters, the slowly changing external input is justifiable.

V. Simulation Results

To demonstrate the effectiveness of the control algorithm proposed two simulations are carried out. The first one is the coordinated turn maneuver, the second one is the high angle of attack flight. For this purpose, a nonlinear simulation of a fixed wing UAV is used. The aircraft has a mass of $105kg$, wing span of $4.3m$, and chord length $0.53m$. Although not modeled in the SDRE formualtion, the actuators are assumed to have a first order lag dynamics.

An important step in designing a SDRE controller is a choice of the weighting matrices $\mathbf{Q}$ and $\mathbf{R}$. As it was mentioned, in general these matrices may be state-dependent. For the purposes of this work, matrices $\mathbf{Q}$ and $\mathbf{R}$ are chosen to be constant diagonal matrices. The nominal values for the weighting matrices are:

$$\mathbf{Q}_{out} = diag\left[1\times10^2, 1\times10^2, 1\times10^2, 0.6\right] \tag{39}$$

$$\mathbf{R}_{out} = diag\left[1, 1, 3, 1, 1, 1\right] \tag{40}$$

$$\mathbf{Q}_{in} = diag\left[1\times10^3, 1\times10^3, 1\times10^3, 1\times10^4, 2\times10^5, 1\times10^4\right] \tag{41}$$

$$\mathbf{R}_{in} = diag\left[1, 1, 1, 10^{-3}\right] \tag{42}$$

A. Coordinated Turn Maneuver

In this first example the aircraft is commanded to perform a coordinated turn maneuver for a given turn radius, while keeping the altitude constant. In order to avoid saturation of the control surfaces a command filter is utilized in the outer loop to shape the necessary reference inputs. Simulation results for a coordinated turn maneuver with a turn radius equal to $200m$ are presented below. Responses of the aircraft states are shown in Fig. 2 - 5. The response as well as the reference commands for Euler angles are presented in Fig. 2, and for the altitude in, Fig. 3. Slight steady state errors are present in the responses, however, the overall tracking performance of the controller may be considered to be quite satisfactory. Time responses of the inner loop states of the aircraft translational velocities (Fig. 4), and angular rates (Fig. 5). Figure 6 shows the time history of the actuators positions and a throttle response. During the simulation studies it is realized that more penalty must be imposed on the actuators to avoid saturation of the control surfaces. Thus, the coefficients of the $\mathbf{R_{in}}$ matrix are increased. The update rate of the SDC system matrices coefficients in the simulations are set to $2Hz$.

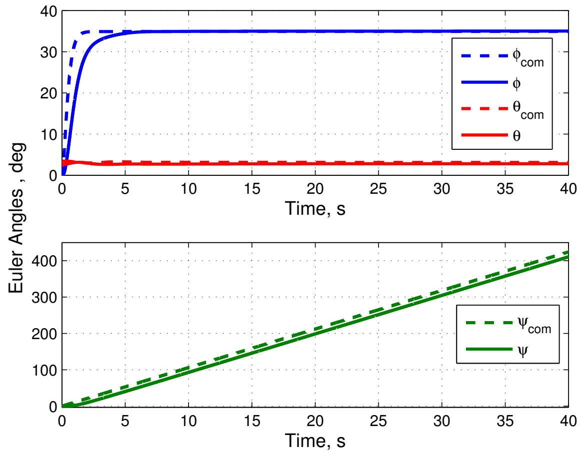

Figure 2. Euler Angles.

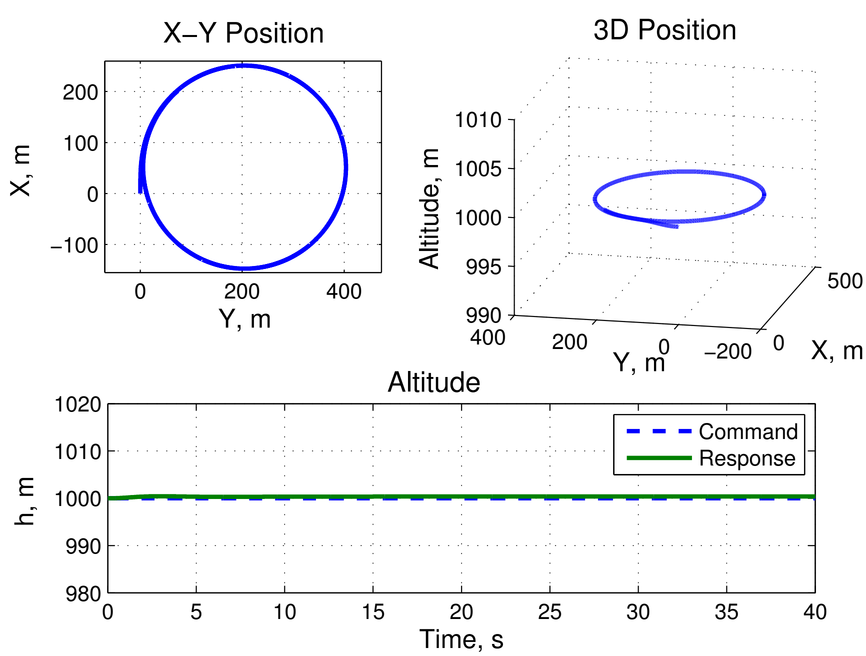

Figure 3. Aircraft Position: X-Y coordinates, 3D View, and Altitude.

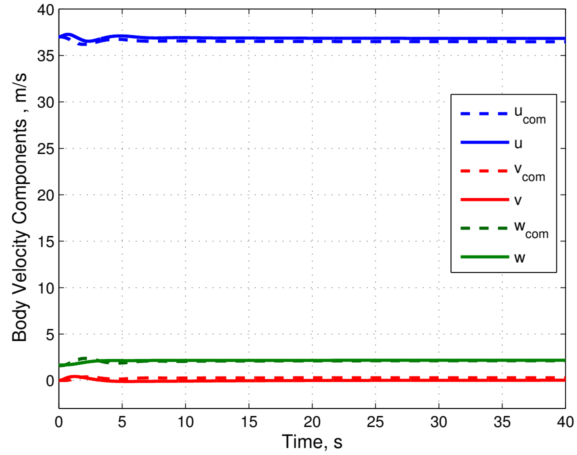

Figure 4. Body Velocity Components.

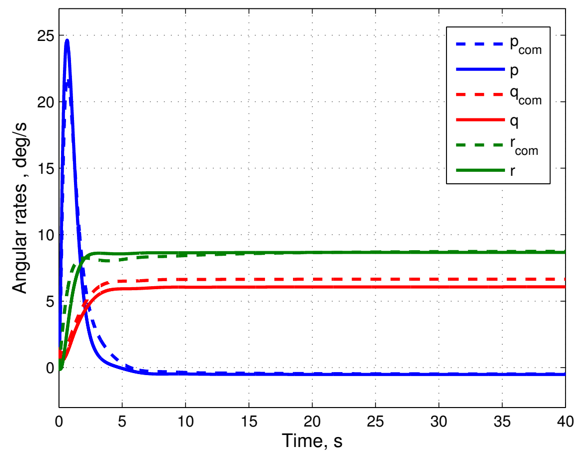

Figure 5. Angular Rates Responses.

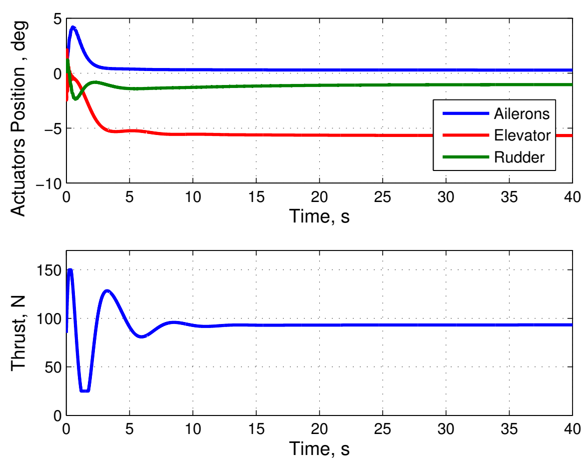

Figure 6. Actuators Positions and Thrust.

B. High Angle of Attack Flight Phase

To demonstrate effectiveness of the designed flight control system in flight regimes that cover the nonlinear regions of the aerodynamic lift coefficient curve, a level flight at a high angle of attack is considered. This flight regime is achieved by requesting a high pitch angle reference and holding the altitude constant. The aerodynamic lift coefficient versus angle of attack plot is shown in Fig. 7, from which it may be observed that for a given aircraft, the stall angle of attack value is around 10 deg.

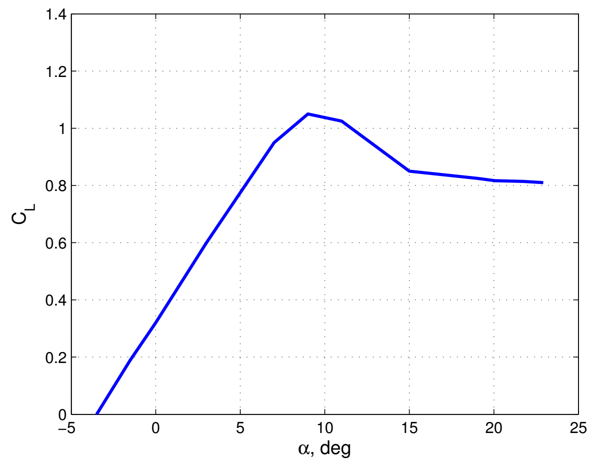

Figure 7. Lift Coefficient vs AOA

The reference pitch attitude is set to $18deg$, altitude must be hold at $1000m$, and zero roll and yaw angles are requested. These are the commanded inputs to the outer loop states. References for the inner loop will be generated by the corresponding control requirements of the outer loop. Figure 8 illustrates the pitch angle and the altitude responses. A steady state error in the altitude may be observed, however, the level flight condition is achieved. Responses of the inner loop states are shown in Fig. 9. From this figure it may be observed that the velocity is properly adjusted to realize the level flight condition at high angle of attack. The actuator time histories are presented in Fig. 10. Since this is a high angle of attack maneuver, the elevator saturation angle is increased. In an actual UAV, this may be achieved by increasing the tail area instead, to realize the pitch moment needed during the high angle of attack flight. The angle of attack time history is given in Fig. 11, from which it may be observed that the aircraft operates at the high angle of attack flight regime of requested $18deg$. Time histories of inner and outer loop controller gains are given in Fig. 12 - 16, it may be observed in the gains are re-adjusted according to the flight regime, ensuring sufficient tracking performance of the controller.

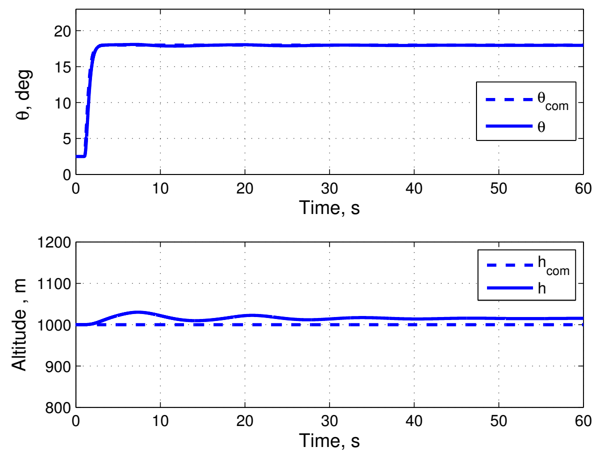

Figure 8. Pitch Angle and Altitude.

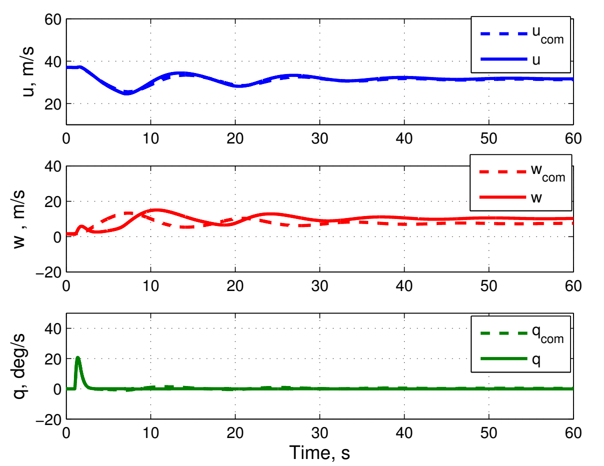

Figure 9. Linear and Angular Velocity Components.

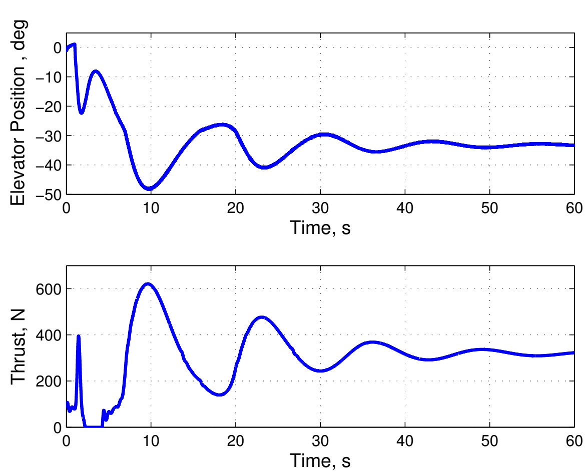

Figure 10. Actuators Positions and Thrust.

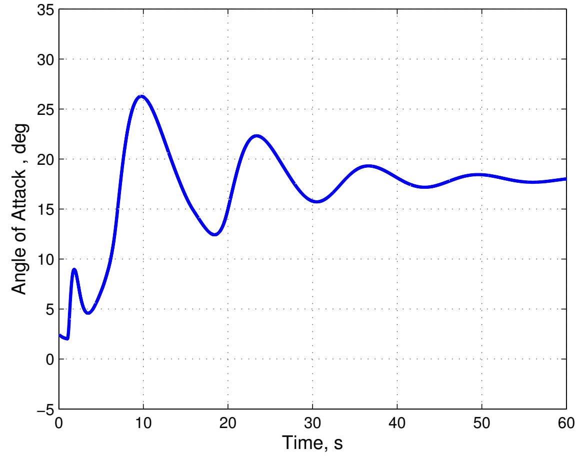

Figure 11. Angle of Attack.

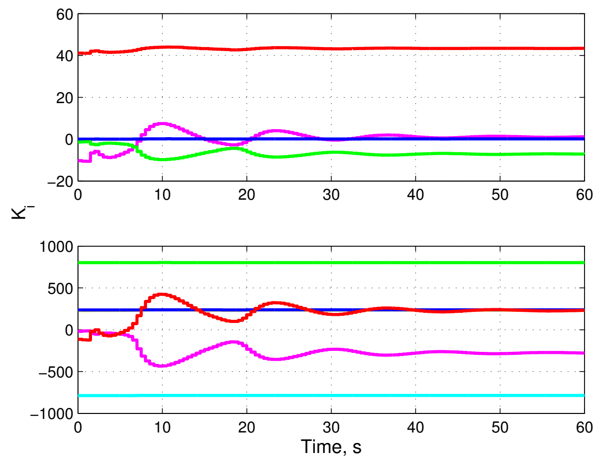

Figure 12. K Inner Loop.

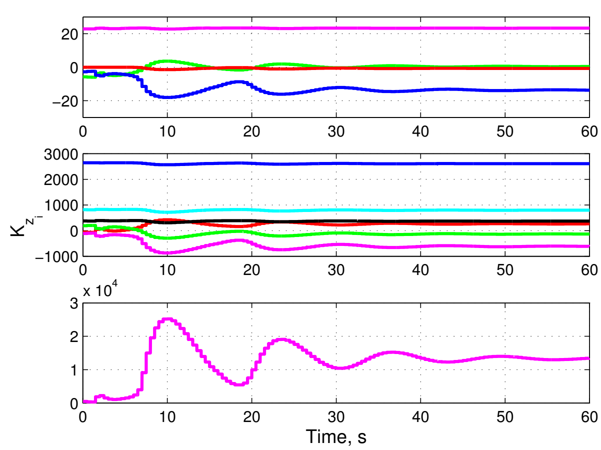

Figure 13. K_z Inner Loop.

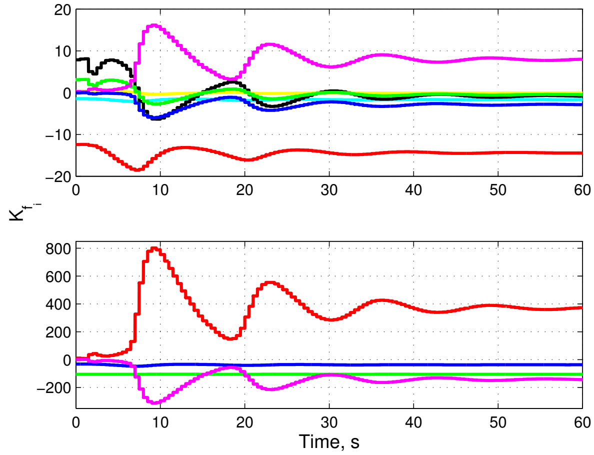

Figure 14. K_f Inner Loop.

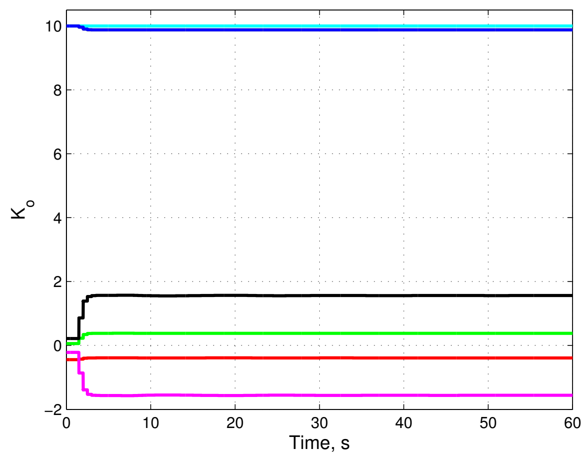

Figure 15. K Outer Loop.

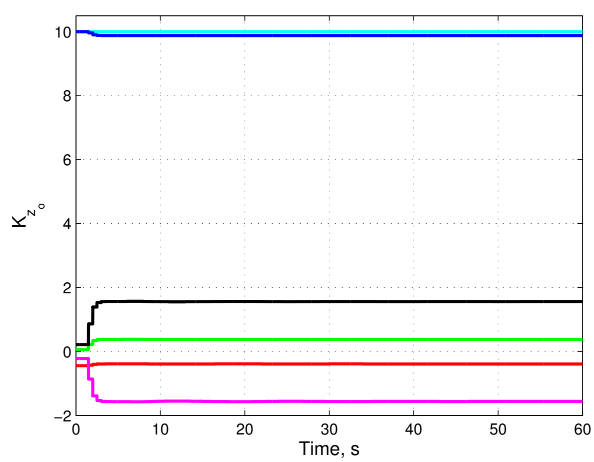

Figure 16. K_z Outer Loop.

VI. Conclusions

In this paper, SDRE control technique is applied to the flight control of a UAV in extreme flight conditions, to demonstrate the effectiveness of the nonlinear SDRE control method. A two loop SDRE control scheme is proposed where the outer loop addresses the kinematic variables while the inner loop handles flight equations. A tracking control algorithm is developed to handle this two loop structure with a nonlinear compensator for the gravity terms acting on the flight equations. The simulation results show that the proposed approach is quite suitable for the nonlinear control of aerospace vehicles eliminating the linearization and gain scheduling commonly used in flight control systems.

Acknowledgments

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