Information Sciences 198 (2012) 130–146

Contents lists available at SciVerse ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Adaptive fuzzy output feedback control of uncertain nonlinear systems with unknown backlash-like hysteresis

Yongming Li [a,b], Shaocheng Tong [a,*], Tieshan Li [b]

[a] Department of Basic Mathematics, Liaoning University of Technology, Jinzhou, Liaoning 121001, PR China
[b] Navigation College, Dalian Maritime University, Dalian, Liaoning 116026, PR China

ARTICLE INFO

Article history:
Received 17 July 2011
Received in revised form 14 February 2012
Accepted 26 February 2012
Available online 5 March 2012

Keywords:
Output feedback
Adaptive fuzzy control
Backlash-like hysteresis
Backstepping design
Stability analysis

ABSTRACT

In this paper, the problem of adaptive fuzzy output-feedback control is investigated for a class of uncertain nonlinear systems with unknown backlash-like hysteresis and unmeasured states. The fuzzy logic systems are used to approximate the nonlinear system functions, and a fuzzy state observer is designed to estimate the unmeasured states. By utilizing the fuzzy state observer, and combining the adaptive backstepping technique with adaptive fuzzy control design, an observer-based adaptive fuzzy output-feedback control approach is developed. It is proved that the proposed control approach can guarantee that all the signals in the closed-loop system are semi-globally uniformly ultimately bounded (SUUB), and both observer error and tracking error can converge to a small neighborhood of the origin. Two simulations are included to illustrate the effectiveness of the proposed approach.

1. Introduction

In the past decades, the control design of nonlinear systems preceded by hysteresis has become a challenging and yet rewarding problem. One of the main reasons is that the hysteresis phenomenon can be often encountered in a wide range of physical systems and devices [20]. On the other hand, since the hysteresis nonlinearity is non-differentiable, the system performance is often severely deteriorated and usually exhibits undesirable inaccuracies or oscillations and even instability [18,32].

To address such a challenge, it is important to find a model to describe the hysteresis nonlinearity and utilize this model for controller design. So far, various models have been proposed for hysteresis nonlinearity, for example, Ishlinskii hysteresis operator [9], Preisach model [17], Krasnoskl'skii–Pokrovskii hysteron [9], Duhem hysteresis operator [16], backlash [22], backlash-like hysteresis [21], and so on. Among them, the backlash-like hysteresis model was widely used due to its better representation of the hysteresis nonlinearity and its facilitation for the control design [9,16–18,20–22,32,34]. In [34], an adaptive state feedback control scheme was proposed for a class of nonlinear systems with unknown backlash-like hysteresis. Whereas in [32], two new decentralized adaptive output feedback schemes were explored for a class of interconnected nonlinear subsystems with the input of each loop preceded by unknown backlash-like hysteresis, where the local adaptive controllers were designed based on a general transfer function of the local subsystem with arbitrary relative degree, respectively. However, the proposed adaptive control schemes in [32,34] required the assumptions that the nonlinear functions either be known exactly or can be linearly parameterized, which are very strict and seldom satisfied in applications. Hence, the control schemes developed in [32,34] are not applicable in practice.

* Corresponding author. Tel.: +86 416 4199101; fax: +86 416 4199415.
E-mail address: jztsc@sohu.com (S. Tong).

0020-0255/$ - see front matter
doi:10.1016/j.ins.2012.02.050

It is well known that fuzzy logic system (FLS) has been found to be a particularly powerful tool for modeling uncertain nonlinear systems due to their universal approximation properties. In the past decades, the issues of utilizing fuzzy logic systems to approximate unknown nonlinear functions of systems have been well investigated, see for example, [2–5,8,11–15,23–27,29–31,33,35,36] and the references therein. For examples, the uncertain nonlinear systems considered in [4,5,8,11,15,31] were restricted to satisfy the matching conditions, that is, the unknown functions must appear on the same equation with the control input channel. Accordingly, to overcome this restriction, some backstepping-based adaptive fuzzy controllers were developed for a wide class of uncertain nonlinear systems without satisfying the matching conditions in [2,3,13,14,23–27,29,30,33,35,36], where the stabilities of the resulting closed loop systems were guaranteed with the help of the well-known Lyapunov direct method.

Recently, in order to control uncertain nonlinear systems with unknown backlash-like hysteresis, many adaptive fuzzy controllers have been developed by utilizing FLS's universal approximation properties [1,18,19,21]. For example, a stable adaptive fuzzy control approach was proposed in [21] for a class of SISO uncertain nonlinear systems with unknown backlash-like hysteresis. And two adaptive fuzzy state feedback control design approaches were developed in [1,19] for a class of interconnected uncertain systems and MIMO uncertain systems, respectively. Then, in [18], an output feedback controller was presented for a class of uncertain nonlinear systems preceded by unknown backlash-like hysteresis, where the hysteresis was modeled by a differential equation. The unmeasured states are estimated by a designed state observer. However, the aforementioned adaptive fuzzy control approaches [1,18,19,21] are only suitable for the nonlinear systems with the matching condition, they cannot be applied to those uncertain nonlinear systems without satisfying the matching condition.

Motivated by the above observations, an adaptive fuzzy output-feedback control approach is presented in this paper for a class of uncertain nonlinear systems under the conditions of unknown backlash-like hysteresis, unmeasured states and non-matching conditions. In the proposed scheme, based on backstepping technique, the fuzzy logic systems are used to approximate the nonlinear system functions, and a fuzzy state observer is designed to estimate the unmeasured states. It is proved that the proposed control approach can guarantee that all the signals of the resulting closed-loop system are SUUB, and the observer error and tracking error are proved to converge to a small neighborhood of the origin. The main advantages of the proposed control scheme are as follows: (i) it can be used to deal with those uncertain nonlinear systems with unknown backlash-like hysteresis and without satisfying the matching conditions. (ii) It does not require that all the states of the controlled system be measured directly, which is a common restriction in most existing adaptive fuzzy backstepping controller, such as [2,3,33,36]. (iii) Both the state observer and the controller are constructed simultaneously, rather than separately.

2. Problem formulations and preliminaries

2.1. System descriptions and some assumptions

Consider a class of SISO nth order nonlinear systems in the following form:

$$\begin{cases} \dot{x}_1 = x_2 + f_1(x_1) + d_1(x,t) \\ \dot{x}_2 = x_3 + f_2(\underline{x}_2) + d_2(x,t) \\ \vdots \\ \dot{x}_{n-1} = x_n + f_{n-1}(\underline{x}_{n-1}) + d_{n-1}(x,t) \\ \dot{x}_n = \phi(v) + f_n(x) + d_n(x,t) \\ y = x_1 \end{cases} \tag{1}$$

where $\underline{x}_i = [x_1, \ldots, x_i]^T \in R^i (x = [x_1, \ldots, x_n]^T \in R^n)$ is the states vector of the system, and $y \in R$ is the output, respectively. $f_i(\cdot)$ is an unknown smooth nonlinear function; $v \in R$ is the control input and $\phi(v)$ denotes hysteresis type of nonlinearity; d_i is an unknown but bounded external disturbance. This paper assumes that the states of the system (1) are unknown and only the output y is available for measurement.

Assumption 1. There exist an unknown positive constant d_{iM} and a large positive constant M such that $|d_i| \leqslant d_{iM}$, and $|v| \leqslant M$, respectively.

Assumption 2. There exists a known constant L_i such that

$$|f_i(\underline{x}_i) - f_i(\hat{\underline{x}}_i)| \leqslant L_i(|x_1 - \hat{x}_1| + \cdots + |x_i - \hat{x}_i|)$$

where $\hat{\underline{x}}_i$ is the estimate of $\underline{x}_i$.

Remark 1. Both Assumptions 1 and 2 are common, which can be founded in many existing literatures [7,10,33]. Due to the assumption of $|v| \leqslant M$ (M may be a large constant), the stability of the closed-loop control system of this paper is built in the sense of semi-globally uniformly ultimately bounded [2,3,6].

According to [18], the control input v and the hysteresis type of nonlinearity $\phi(v)$ in system (1) can be described by

$$\frac{d\phi}{dt} = \alpha\left|\frac{dv}{dt}\right|(cv - \phi) + B_1\frac{dv}{dt} \tag{2}$$

where α, c and B_1 are constants, satisfying $c > B_1$.

The dynamics (2) can be used to model a class of backlash-like hysteresis as shown in Fig. 1. In Fig. 1 parameters $\alpha = 1$, $c = 3.1635$ and $B_1 = 0.345$, the input signal $v(t) = 6.5\sin(2.3t)$ and the initial conditions $v(0) = 0$ and $\phi(0) = 0$ are chosen. In this paper the parameters of the hysteresis in (2), i.e., α, c and B_1 are completely unknown [18].

Now, the system (1) can be rewritten as follows

$$\begin{cases} \dot{x}_1 = x_2 + f_1(\hat{x}_1) + \Delta F_1 + d_1(x,t) \\ \dot{x}_2 = x_3 + f_2(\underline{\hat{x}}_2) + \Delta F_2 + d_2(x,t) \\ \vdots \\ \dot{x}_{n-1} = x_n + f_{n-1}(\underline{\hat{x}}_{n-1}) + \Delta F_{n-1} + d_{n-1}(x,t) \\ \dot{x}_n = \phi(v) + f_n(\hat{x}) + \Delta F_n + d_n(x,t) \\ y = x_1 \end{cases} \tag{3}$$

where $\Delta F_i = f_i(\underline{x}_i) - f_i(\underline{\hat{x}}_i)$.

2.1.1. Control objective

Given a reference signal $y_r(t)$, and assume that $y_r(t)$ has up to nth derivatives which are bounded. Our control objective is to design the output feedback controller v and parameters adaptive laws such that all the signals involved in the resulting closed-loop system are SUUB, and the output $y(t)$ tracks the given reference signal $y_r(t)$ as desired.

2.2. Fuzzy logic systems

A fuzzy logic system (FLS) consists of four parts: the knowledge base, the fuzzifier, the fuzzy inference engine working on fuzzy rules, and the defuzzifier [28]. The knowledge base for FLS comprises a collection of fuzzy If-Then rules of the following form:

$$R^l: \text{ If } x_1 \text{ is } F_1^l \text{ and } x_2 \text{ is } F_2^l \text{ and } \ldots \text{ and } x_n \text{ is } F_n^l,$$
$$\text{Then } y \text{ is } G^l, \ l = 1, 2, \ldots, N$$

where $x = [x_1, \ldots, x_n]^T$ and y are the FLS input and output, respectively. F_i^l and G^l are fuzzy sets associated with the fuzzy functions $\mu_{F_i^l}(x_i)$ and $\mu_{G^l}(y)$, respectively. N is the rules number.

Selecting singleton function, center average defuzzification and product inference [28], the fuzzy logic system can be expressed as

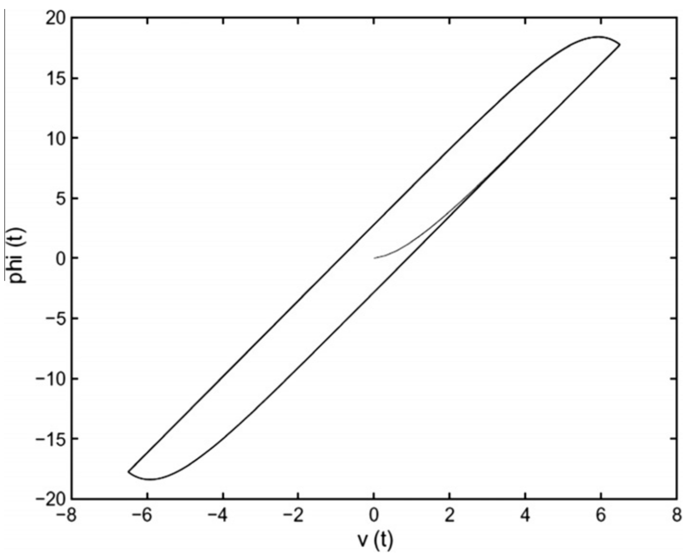

Fig. 1. Hysteresis curve given by (2) with $\alpha = 1$, $c = 3.1635$, $B_1 = 0.345$ and $v(t) = 6.5\sin(2.3t)$.

$$y(x) = \frac{\sum_{l=1}^{N} \bar{y}_l \prod_{i=1}^{n} \mu_{F_i^l}(x_i)}{\sum_{l=1}^{N} \left[\prod_{i=1}^{n} \mu_{F_i^l}(x_i)\right]} \tag{4}$$

where $\bar{y}_l = \max_{y \in R} \mu_{G^l}(y)$.

Define the fuzzy basis functions as

$$\varphi_l = \frac{\prod_{i=1}^{n} \mu_{F_i^l}(x_i)}{\sum_{l=1}^{N} \left(\prod_{i=1}^{n} \mu_{F_i^l}(x_i)\right)} \tag{5}$$

Denote $\theta = [\bar{y}_1, \bar{y}_2, \ldots, \bar{y}_N]^{\mathrm{T}} = [\theta_1, \theta_2, \ldots, \theta_N]^{\mathrm{T}}$ and $\varphi^{\mathrm{T}}(x) = [\bar{\varphi}_1(x), \ldots, \bar{\varphi}_N(x)]$, then fuzzy logic system (4) can be rewritten as

$$y(x) = \theta^{\mathrm{T}} \varphi(x) \tag{6}$$

Lemma 1 (28). *Let f(x) be a continuous function defined on a compact set Ω. Then for any constant ε > 0, there exists an FLS (6) such as*

$$\sup_{x \in \Omega} |f(x) - \theta^{\mathrm{T}} \varphi(x)| \leqslant \varepsilon \tag{7}$$

By Lemma 1, the FLS (6) is a universal approximator, i.e., it can approximate any unknown continuous function on a compact set. Therefore, it can be assumed that the unknown function $f_i(\hat{\underline{x}}_i)(i = 1, \ldots, n)$ in system (3) can be approximated by the following fuzzy logic system

$$\hat{f}_i(\hat{\underline{x}}_i | \theta_i) = \theta_i^{\mathrm{T}} \varphi_i(\hat{\underline{x}}_i) \tag{8}$$

Define the optimal parameter vector θ_i^* as

$$\theta_i^* := \arg \min_{\theta_i \in R^{N_i}} \left\{ \sup_{\hat{\underline{x}}_i \in D} \left| f_i(\hat{\underline{x}}_i) - \theta_i^{\mathrm{T}} \varphi_i(\hat{\underline{x}}_i) \right| \right\} \tag{9}$$

The fuzzy minimum approximation error and fuzzy approximation error. ε_i and δ_i are defined as

$$\varepsilon_i = f_i(\hat{\underline{x}}_i) - \theta_i^{*\mathrm{T}} \varphi_i(\hat{\underline{x}}_i), \quad \delta_i = f_i(\hat{\underline{x}}_i) - \theta_i^{\mathrm{T}} \varphi_i(\hat{\underline{x}}_i) \tag{10}$$

Assumption 3. $|\varepsilon_i| \leqslant \varepsilon_i^*$ and $|\delta_i| \leqslant \delta_i^*$, where $\varepsilon_i^* > 0$ and $\delta_i^* > 0$ are unknown constants, $i = 1, \ldots, n$.

Denote $w_i = \varepsilon_i - \delta_i$, by Assumption 3, we have $|w_i| \leqslant \varepsilon_i^* + \delta_i^* = w_i^*$.

Remark 2. The assumption of $|\delta_i| \leqslant \delta_i^*$ is reasonable. From the definitions of ε_i and δ_i, one has

$$\delta_i = f_i(\hat{\underline{x}}_i) - \theta_i^{\mathrm{T}} \varphi_i(\hat{\underline{x}}_i) = [f_i(\hat{\underline{x}}_i) - \theta_i^{*\mathrm{T}} \varphi_i(\hat{\underline{x}}_i)] + [\theta_i^{*\mathrm{T}} \varphi_i(\hat{\underline{x}}_i) - \theta_i^{\mathrm{T}} \varphi_i(\hat{\underline{x}}_i)] = \varepsilon_i + \left(\theta_i^* - \theta_i\right)^{\mathrm{T}} \varphi_i(\hat{\underline{x}}_i)$$

Since the minimum approximation error ε_i is bounded, $\theta_i, \theta_i^* \in R^{N_i}$, with R^{N_i} being a bounded compact, and fuzzy basis function $\varphi_i(\hat{\underline{x}}_i)$ satisfies $0 \leqslant \varphi_i^T(\hat{\underline{x}}_i)\varphi_i(\hat{\underline{x}}_i) \leqslant 1$. By Lemma 1, the FLS (6) has the approximation capability for any continuous smooth function, thus it is generally assumed that the fuzzy minimum approximation error ε_i and approximation errors δ_i are bounded by known supper bounds ε_i^* and δ_i^* [2,3,23,33,36]. However, the approximation accuracy depends on the number of the fuzzy rules N, that is, the larger N is, the smaller the approximation error is. In practice, it is difficult to determine the supper bounds of ε_i and δ_i. To avoid the conservative in control design, this paper assumes that the supper bounds ε_i^* and δ_i^* (or w_i^*) are unknown, which can be estimated via adaptation laws designed in the next section.

By (8), the system (3) can be rewritten as follows

$$\begin{cases} \dot{x}_1 = x_2 + \theta_1^{*\mathrm{T}} \varphi_1(\hat{x}_1) + \varepsilon_1(\hat{x}_1) + \Delta F_1 + d_1(x, t) \\ \dot{x}_2 = x_3 + \theta_2^{*\mathrm{T}} \varphi_2(\hat{\underline{x}}_2) + \varepsilon_2(\hat{\underline{x}}_2) + \Delta F_2 + d_2(x, t) \\ \vdots \\ \dot{x}_{n-1} = x_n + \theta_{n-1}^{*\mathrm{T}} \varphi_{n-1}(\hat{\underline{x}}_{n-1}) + \varepsilon_{n-1}(\hat{\underline{x}}_{n-1}) + \Delta F_{n-1} + d_{n-1}(x, t) \\ \dot{x}_n = \phi(v) + \theta_n^{*\mathrm{T}} \varphi_n(\hat{x}) + \varepsilon_n(\hat{x}) + \Delta F_n + d_n(x, t) \\ y = x_1 \end{cases} \tag{11}$$

Based on the analysis in [18,20], (2) can be solved explicitly as

$$\begin{aligned}&\phi(v) = cv(t) + \underline{d}_1(v),\\ &\underline{d}_1(v) = [\phi_0 - cv_0]e^{-\alpha(v-v_0)\mathrm{sgn}\dot{v}} + e^{-\alpha v\mathrm{sgn}\dot{v}}\int_{v_0}^{v}[B_1 - c]e^{\alpha\eta\mathrm{sgn}\dot{v}}d\eta\end{aligned} \tag{12}$$

where $v(0) = v_0$ and $\phi(v_0) = \phi_0$.

Based on above solution it is shown in [18,20] that $\underline{d}_1(v)$ is bounded. Thus using (12) and (11) can be reformulated as follows

$$\begin{cases}\dot{x}_1 = x_2 + \theta_1^{*\mathrm{T}}\varphi_1(\hat{x}_1) + \varepsilon_1(\hat{x}_1) + \Delta F_1 + d_1(x,t)\\ \dot{x}_2 = x_3 + \theta_2^{*\mathrm{T}}\varphi_2(\underline{\hat{x}}_2) + \varepsilon_2(\underline{\hat{x}}_2) + \Delta F_2 + d_2(x,t)\\ \vdots\\ \dot{x}_{n-1} = x_n + \theta_{n-1}^{*\mathrm{T}}\varphi_{n-1}(\underline{\hat{x}}_{n-1}) + \varepsilon_{n-1}(\underline{\hat{x}}_{n-1}) + \Delta F_{n-1} + d_{n-1}(x,t)\\ \dot{x}_n = cv(t) + \theta_n^{*\mathrm{T}}\varphi_n(\hat{x}) + \varepsilon_n(\hat{x}) + \Delta F_n + d_g\\ y = x_1\end{cases} \tag{13}$$

where $d_g = \underline{d}_1(v) + d_n(x,t)$. Since $\underline{d}_1(v)$ and $d_n(x,t)$ are bounded, thus d_g is bounded.

2.3. Fuzzy adaptive observer design

Note that the states of system (11) are not available for feedback, thus, a state observer must be designed to estimate the unmeasured states. In this paper, a fuzzy adaptive observer is designed for (13) as follows

$$\begin{cases}\dot{\hat{x}}_1 = \hat{x}_2 + \theta_1^{\mathrm{T}}\varphi_1(\hat{x}_1) + k_1(y - \hat{x}_1)\\ \dot{\hat{x}}_2 = \hat{x}_3 + \theta_2^{\mathrm{T}}\varphi_2(\underline{\hat{x}}_2) + k_2(y - \hat{x}_1)\\ \vdots\\ \dot{\hat{x}}_{n-1} = \hat{x}_n + \theta_{n-1}^{\mathrm{T}}\varphi_{n-1}(\underline{\hat{x}}_{n-1}) + k_{n-1}(y - \hat{x}_1)\\ \dot{\hat{x}}_n = \hat{c}v(t) + \theta_n^{\mathrm{T}}\varphi_n(\hat{x}) + k_n(y - \hat{x}_1)\end{cases} \tag{14}$$

Rewriting (14) in the following form:

$$\begin{cases}\dot{\hat{x}} = A\hat{x} + Ky + F(\hat{x}|\theta_n) + E_n\hat{c}v(t)\\ \hat{y} = E_1^{\mathrm{T}}\hat{x}\end{cases} \tag{15}$$

where $A = \begin{bmatrix}-k_1 & \\ \vdots & I_{n-1}\\ -k_n & \cdots 0\end{bmatrix}$, $K = [k_1, \ldots, k_n]^{\mathrm{T}}$, $F(\hat{x}|\theta_n) = \left[\theta_1^{\mathrm{T}}\varphi_1(\hat{x}_1), \ldots, \theta_n^{\mathrm{T}}\varphi_n(\hat{x})\right]^{\mathrm{T}}, E_1^{\mathrm{T}} = [1, 0, \ldots, 0], E_n^{\mathrm{T}} = [0, \ldots, 0, 1]$, and $\hat{c}$ is the estimate of c.

The coefficient k_i is chosen such that the polynomial $p(s) = s^n + k_1s^{n-1} + \cdots + k_{n-1}s + k_n$ is a Hurwitz. Thus, given a $Q^{\mathrm{T}} = Q > 0$, there exists a positive definite matrix $P^{\mathrm{T}} = P$ such that

$$A^{\mathrm{T}}P + PA = -Q \tag{16}$$

Let $e = x - \hat{x}$ be observer error, then from (3), (13), (14), (15), the observer error equation can be obtained as follows

$$\dot{e} = Ae + \delta + \Delta F + E_n\tilde{c}v + d \tag{17}$$

where $\Delta F = [\Delta F_1, \ldots, \Delta F_n]^{\mathrm{T}}$, $\delta = [\delta_1, \ldots, \delta_n]^{\mathrm{T}}$, $d = [d_1, \ldots, d_{n-1}, d_g]^{\mathrm{T}}$ and $\tilde{c} = c - \hat{c}$.

Consider the following Lyapunov candidate V_0 for (17) as follows

$$V_0 = e^{\mathrm{T}}Pe \tag{18}$$

Then the time derivative of V_0 along the solutions of (17) is

$$\dot{V}_0 = -e^{\mathrm{T}}Qe + 2e^{\mathrm{T}}P(\delta + \Delta F + E_n\tilde{c}v + d) \tag{19}$$

Utilizing Young's inequality and considering Assumptions 1–3, one has

$$2e^{\mathrm{T}}P(\Delta F + \delta + d) \leqslant 2\|e\|\|P\|\left(\sum_{i=1}^{n}L_i(|x_1 - \hat{x}_1| + \cdots + |x_i - \hat{x}_i|) + \delta^* + d_M\right) \leqslant (2 + \overline{L})\|e\|^2 + \overline{D} \tag{20}$$

where $\bar{L} = 2\|P\|\sum_{i=1}^{n} iL_i, \bar{D} = \|P\|^2(\|\delta^*\|^2 + \|d_M\|^2), \delta^* = [\delta_1^*, \ldots, \delta_n^*]^{\mathrm{T}}$ and $d_M = [d_{1M}, \ldots, d_{nM}]^{\mathrm{T}}$.

Substituting the Eq. (20) into (19), one has

$$\dot{V}_0 \leqslant -e^{\mathrm{T}}Qe + (2+\bar{L})\|e\|^2 + \bar{D} + 2e^{\mathrm{T}}PE_n\tilde{c}\upsilon \tag{21}$$

3. Adaptive fuzzy control design and stability analysis

In this section, an adaptive fuzzy controller and parameters adaptive laws will be developed in the framework of the backstepping technique, so that all the signals in the closed-loop system are SUUB and, both the tracking error and the observer error can be made as small as desired.

The n-step adaptive fuzzy backstepping output feedback control design is based on the following change of coordinates:

$$\begin{aligned} \chi_1 &= y - y_r \\ \chi_i &= \hat{x}_i - \alpha_{i-1}, \quad i = 2, \ldots, n \end{aligned} \tag{22}$$

where α_{i-1} is called the intermediate control function, which will be given later.

Step1: The time derivative of χ_1 along the solutions of (11) is

$$\begin{aligned} \dot{\chi}_1 &= x_2 + \theta_1^{*\mathrm{T}}\varphi_1(\hat{x}_1) + \varepsilon_1 + \Delta F_1 + d_1 - \dot{y}_r = \hat{x}_2 + e_2 + \theta_1^{\mathrm{T}}\varphi_1(\hat{x}_1) + \varepsilon_1 + \tilde{\theta}_1^{\mathrm{T}}\varphi_1(\hat{x}_1) + \Delta F_1 + d_1 - \dot{y}_r \\ &= \chi_2 + \alpha_1 + e_2 + \theta_1^{\mathrm{T}}\varphi_1(\hat{x}_1) + \varepsilon_1 + \tilde{\theta}_1^{\mathrm{T}}\varphi_1(\hat{x}_1) + \Delta F_1 + d_1 - \dot{y}_r \end{aligned} \tag{23}$$

where $\tilde{\theta}_1 = \theta_1^* - \theta_1$. Consider the Lyapunov function candidate V_1 as follows

$$V_1 = V_0 + \chi_1^2 + \frac{1}{2\gamma_1}\tilde{\theta}_1^{\mathrm{T}}\tilde{\theta}_1 + \frac{1}{2\bar{\gamma}_1}\tilde{\varepsilon}_1^2 \tag{24}$$

where γ_1 and $\bar{\gamma}_1$ are positive design constants and $\tilde{\varepsilon}_1 = \varepsilon_1^* - \hat{\varepsilon}_1$. $\hat{\varepsilon}_1$ is the estimate of ε_1^*.

The time derivative of V_1 along (21) and (23) is

$$\begin{aligned} \dot{V}_1 &\leqslant -e^{\mathrm{T}}Qe + (2+\bar{L})\|e\|^2 + \bar{D} + 2e^{\mathrm{T}}PE_n\tilde{c}\upsilon + 2\chi_1(\chi_2 + \alpha_1 + e_2 + \theta_1^{\mathrm{T}}\varphi_1(\hat{x}_1) + \Delta F_1 + d_1 - \dot{y}_r) + 2|\chi_1|\varepsilon_1^* \\ &\quad + \frac{1}{\gamma_1}\tilde{\theta}_1^{\mathrm{T}}(2\gamma_1\chi_1\varphi_1(\hat{x}_1) - \dot{\theta}_1) + \frac{1}{\bar{\gamma}_1}\tilde{\varepsilon}_1\dot{\hat{\varepsilon}}_1 \\ &\leqslant -e^{\mathrm{T}}Qe + (2+\bar{L})\|e\|^2 + \bar{D} + 2e^{\mathrm{T}}PE_n\tilde{c}\upsilon + 2\chi_1\left(\chi_2 + \alpha_1 + e_2 + \theta_1^{\mathrm{T}}\varphi_1(\hat{x}_1) + \hat{\varepsilon}_1\tanh\left(\frac{2\chi_1}{\kappa}\right) + \Delta F_1 + d_1 - \dot{y}_r\right) \\ &\quad + 2|\chi_1|\varepsilon_1^* - 2\chi_1\varepsilon_1^*\tanh\left(\frac{2\chi_1}{\kappa}\right) + \frac{1}{\gamma_1}\tilde{\theta}_1^{\mathrm{T}}(2\gamma_1\chi_1\varphi_1(\hat{x}_1) - \dot{\theta}_1) + \frac{1}{\bar{\gamma}_1}\tilde{\varepsilon}_1\left(2\bar{\gamma}_1\chi_1\tanh\left(\frac{2\chi_1}{\kappa}\right) - \dot{\hat{\varepsilon}}_1\right) \end{aligned} \tag{25}$$

Note that for any $\kappa > 0$, the following inequality holds

$$2|\chi_1| - 2\chi_1\tanh(2\chi_1/\kappa) \leqslant 0.2785\kappa = \kappa' \tag{26}$$

By using Young's inequality under the Assumptions 1 and 3, one has

$$2\chi_1(e_2 + \Delta F_1 + d_1) \leqslant \left(2 + L_1^2\right)\chi_1^2 + 2\|e\|^2 + d_{1M}^2 \tag{27}$$

Substituting (26) and (27) into (25) results in

$$\begin{aligned} \dot{V}_1 &\leqslant -e^{\mathrm{T}}Qe + (4+\bar{L})\|e\|^2 + \bar{D} + 2e^{\mathrm{T}}PE_n\tilde{c}\upsilon + 2\chi_1\left(\chi_2 + \alpha_1 + \theta_1^{\mathrm{T}}\varphi_1(\hat{x}_1) + \hat{\varepsilon}_1\tanh\left(\frac{2\chi_1}{\kappa}\right) - \dot{y}_r + \frac{2+L_1^2}{2}\chi_1\right) \\ &\quad + \kappa'\varepsilon_1^* + \frac{1}{\gamma_1}\tilde{\theta}_1^{\mathrm{T}}(2\gamma_1\chi_1\varphi_1(\hat{x}_1) - \dot{\theta}_1) + \frac{1}{\bar{\gamma}_1}\tilde{\varepsilon}_1\left(2\bar{\gamma}_1\chi_1\tanh\left(\frac{2\chi_1}{\kappa}\right) - \dot{\hat{\varepsilon}}_1\right) + d_{1M}^2 \end{aligned} \tag{28}$$

Choose the intermediate control function α_1, parameter adaptive laws θ_1 and $\hat{\varepsilon}_1$ as follows

$$\alpha_1 = -\frac{1}{2}c_1\chi_1 - \theta_1^{\mathrm{T}}\varphi_1(\hat{x}_1) - \hat{\varepsilon}_1\tanh\left(\frac{2\chi_1}{\kappa}\right) + \dot{y}_r - \frac{2+L_1^2}{2}\chi_1 \tag{29}$$

$$\dot{\theta}_1 = 2\gamma_1\chi_1\varphi_1(\hat{x}_1) - \sigma_1\theta_1 \tag{30}$$

$$\dot{\hat{\varepsilon}}_1 = 2\bar{\gamma}_1\chi_1\tanh\left(\frac{2\chi_1}{\kappa}\right) - \bar{\sigma}_1\hat{\varepsilon}_1 \tag{31}$$

where c_1, σ_1 and $\bar{\sigma}_1$ are positive design constants.

Now, substituting (29)–(31) into (28) yields

$$\dot{V}_1 \leqslant -e^{\mathrm{T}}Qe + \bar{L}_1\|e\|^2 + 2e^{\mathrm{T}}PE_n\tilde{c}v - c_1\chi_1^2 + 2\chi_1\chi_2 + \frac{\sigma_1}{\gamma_1}\tilde{\theta}_1^{\mathrm{T}}\theta_1 + \frac{\bar{\sigma}_1}{\bar{\gamma}_1}\tilde{\varepsilon}_1\hat{\varepsilon}_1 + D_1 \tag{32}$$

where $\bar{L}_1 = 4 + \bar{L}$ and $D_1 = \overline{D} + \kappa'\varepsilon_1^* + d_{1M}^2$.

Step 2: The time derivative of χ_2 along the solutions of (14) and (22) is

$$\begin{aligned}\dot{\chi}_2 = \dot{\hat{x}}_2 - \dot{\alpha}_1 &= \hat{x}_3 + \theta_2^{\mathrm{T}}\varphi_2(\underline{\hat{x}}_2) + k_2(y-\hat{x}_1) - \frac{\partial\alpha_1}{\partial x_1}\dot{x}_1 - \frac{\partial\alpha_1}{\partial\theta_1}\dot{\theta}_1 - \frac{\partial\alpha_1}{\partial y_r}\dot{y}_r - \frac{\partial\alpha_1}{\partial\dot{y}_r}\ddot{y}_r - \frac{\partial\alpha_1}{\partial\hat{\varepsilon}_1}\dot{\hat{\varepsilon}}_1 - \frac{\partial\alpha_1}{\partial\hat{x}_1}\dot{\hat{x}}_1\\ &= \chi_3 + \alpha_2 + \theta_2^{\mathrm{T}}\varphi_2(\underline{\hat{x}}_2) + w_2 + \tilde{\theta}_2^{\mathrm{T}}\varphi_2(\underline{\hat{x}}_2) + k_2(y-\hat{x}_1) - \frac{\partial\alpha_1}{\partial x_1}(e_2 + \delta_1 + \Delta F_1 + d_1) + H_2\end{aligned} \tag{33}$$

where

$$H_2 = -\frac{\partial\alpha_1}{\partial x_1}(\hat{x}_2 + \theta_1^{\mathrm{T}}\varphi_1(\hat{x}_1)) - \frac{\partial\alpha_1}{\partial\theta_1}\dot{\theta}_1 - \frac{\partial\alpha_1}{\partial y_r}\dot{y}_r - \frac{\partial\alpha_1}{\partial\dot{y}_r}\ddot{y}_r - \frac{\partial\alpha_1}{\partial\hat{\varepsilon}_1}\dot{\hat{\varepsilon}}_1 - \frac{\partial\alpha_1}{\partial\hat{x}_1}\dot{\hat{x}}_1$$

Remark 3. From the definitions of $w_i = \varepsilon_i - \delta_i$ and $\tilde{\theta}_i = \theta_i^* - \theta_i$, one obtains $w_2 = -(\theta_2^* - \theta_2)^{\mathrm{T}}\varphi_2(\underline{\hat{x}}_i)$, that is, $w_2 + \tilde{\theta}_2^{\mathrm{T}}\varphi_2(\underline{\hat{x}}_2) = 0$.

Consider the Lyapunov function candidate V_2 as follows

$$V_2 = V_1 + \chi_2^2 + \frac{1}{2\gamma_2}\tilde{\theta}_2^{\mathrm{T}}\tilde{\theta}_2 + \frac{1}{2\bar{\gamma}_2}\tilde{w}_2^2 \tag{34}$$

where γ_i and $\bar{\gamma}_i$ are positive design constants. $\tilde{\theta}_i = \theta_i^* - \theta_i$ and $\tilde{w}_i = w_i^* - \hat{w}_i$. $\hat{w}_i$ is the estimate of $w_i^*(i = 2,\ldots,n)$. Then the time derivative of V_2 along the solutions of (33) is

$$\dot{V}_2 = \dot{V}_1 + 2\chi_2\left(\chi_3 + \alpha_2 + \theta_2^{\mathrm{T}}\varphi_2(\underline{\hat{x}}_2) + k_2(y-\hat{x}_1) + w_2 + \tilde{\theta}_2^{\mathrm{T}}\varphi_2(\underline{\hat{x}}_2) - \frac{\partial\alpha_1}{\partial x_1}(e_2 + \delta_1 + \Delta F_1 + d_1) + H_2\right) + \frac{1}{\gamma_2}\tilde{\theta}_2^{\mathrm{T}}\dot{\tilde{\theta}}_2 + \frac{1}{\bar{\gamma}_2}\tilde{w}_2\dot{\tilde{w}}_2 \tag{35}$$

Similarly, one has

$$-2\chi_2\frac{\partial\alpha_1}{\partial x_1}(e_2 + \delta_1 + \Delta F_1 + d_1) \leqslant \left(3 + L_1^2\right)\left(\chi_2\frac{\partial\alpha_1}{\partial x_1}\right)^2 + 2\|e\|^2 + \delta_1^{*2} + d_{1M}^2 \tag{36}$$

Substituting (32) and (36) into (35) results in

$$\begin{aligned}\dot{V}_2 &\leqslant -e^{\mathrm{T}}Qe + \bar{L}_2\|e\|^2 + 2e^{\mathrm{T}}PE_n\tilde{c}v - c_1\chi_1^2 + \frac{\sigma_1}{\gamma_1}\tilde{\theta}_1^{\mathrm{T}}\theta_1 + \frac{\bar{\sigma}_1}{\bar{\gamma}_1}\tilde{\varepsilon}_1\hat{\varepsilon}_1 + D_1\\ &\quad + 2\chi_2\left(\chi_1 + \chi_3 + \alpha_2 + \theta_2^{\mathrm{T}}\varphi_2(\underline{\hat{x}}_2) + k_2(y-\hat{x}_1) + \frac{1}{2}\left(3 + L_1^2\right)\chi_2\left(\frac{\partial\alpha_1}{\partial x_1}\right)^2 + H_2 + \hat{w}_2\tanh\left(\frac{2\chi_2}{\kappa}\right)\right) + 2|\chi_2|w_2^*\\ &\quad - 2\chi_2 w_2^*\tanh\left(\frac{2\chi_2}{\kappa}\right) + 2\chi_2\tilde{\theta}_2^{\mathrm{T}}\varphi_2(\underline{\hat{x}}_2) + \delta_1^{*2} + d_{1M}^2 + \frac{1}{\gamma_2}\tilde{\theta}_2^{\mathrm{T}}\dot{\tilde{\theta}}_2 + \frac{1}{\bar{\gamma}_2}\tilde{w}_2\left(2\bar{\gamma}_2\chi_2\tanh\left(\frac{2\chi_2}{\kappa}\right) - \dot{\hat{w}}_2\right)\\ &\leqslant -e^{\mathrm{T}}Qe + \bar{L}_2\|e\|^2 + 2e^{\mathrm{T}}PE_n\tilde{c}v - c_1\chi_1^2 + \frac{\sigma_1}{\gamma_1}\tilde{\theta}_1^{\mathrm{T}}\theta_1 + \frac{\bar{\sigma}_1}{\bar{\gamma}_1}\tilde{\varepsilon}_1\hat{\varepsilon}_1 + D_2\\ &\quad + 2\chi_2\left(\chi_1 + \chi_3 + \alpha_2 + \theta_2^{\mathrm{T}}\varphi_2(\underline{\hat{x}}_2) + k_2(y-\hat{x}_1) + \frac{1}{2}\left(3 + L_1^2\right)\chi_2\left(\frac{\partial\alpha_1}{\partial x_1}\right)^2 + H_2 + \hat{w}_2\tanh\left(\frac{2\chi_2}{\kappa}\right)\right)\\ &\quad + \frac{1}{\gamma_2}\tilde{\theta}_2^{\mathrm{T}}(2\gamma_2\chi_2\varphi_2(\underline{\hat{x}}_2) - \dot{\theta}_2) + \frac{1}{\bar{\gamma}_2}\tilde{w}_2\left(2\bar{\gamma}_2\chi_2\tanh\left(\frac{2\chi_2}{\kappa}\right) - \dot{\hat{w}}_2\right)\end{aligned} \tag{37}$$

where $\bar{L}_2 = \bar{L}_1 + 2$ and $D_2 = D_1 + \kappa' w_2^* + \delta_1^{*2} + d_{1M}^2$.

Choose the intermediate control function α_2, parameter adaptive laws θ_2 and $\hat{w}_2$ as follows

$$\alpha_2 = -\frac{1}{2}c_2\chi_2 - \chi_1 - \theta_2^{\mathrm{T}}\varphi_2(\underline{\hat{x}}_2) - k_2(y-\hat{x}_1) - \hat{w}_2\tanh\left(\frac{2\chi_2}{\kappa}\right) - H_2 - \frac{1}{2}\left(3 + L_1^2\right)\chi_2\left(\frac{\partial\alpha_1}{\partial x_1}\right)^2 \tag{38}$$

$$\dot{\theta}_2 = 2\gamma_2\chi_2\varphi_2(\underline{\hat{x}}_2) - \sigma_2\theta_2 \tag{39}$$

$$\dot{\hat{w}}_2 = 2\bar{\gamma}_2\chi_2\tanh\left(\frac{2\chi_2}{\kappa}\right) - \bar{\sigma}_2\hat{w}_2 \tag{40}$$

where c_2, σ_2 and $\bar{\sigma}_2$ are positive design constants. Then substituting (38)–(40) into (37) will result in

$$\dot{V}_2 \leqslant -e^T Q e + \bar{L}_2\|e\|^2 + 2e^T P E_n \tilde{c} v - c_1\chi_1^2 - c_2\chi_2^2 + \frac{\sigma_1}{\gamma_1}\tilde{\theta}_1^T\theta_1 + \frac{\bar{\sigma}_1}{\bar{\gamma}_1}\tilde{\varepsilon}_1\hat{\varepsilon}_1 + D_2 + 2\chi_2\chi_3 + \frac{\sigma_2}{\gamma_2}\tilde{\theta}_2^T\theta_2 + \frac{\bar{\sigma}_2}{\bar{\gamma}_2}\tilde{w}_2\hat{w}_2 \tag{41}$$

Step $i(3 \leqslant i \leqslant n-1)$: The time derivative of χ_i along (14) and (22) is

$$\dot{\chi}_i = \chi_{i+1} + \alpha_i + \theta_i^T\varphi_i(\underline{\hat{x}}_i) + w_i + \tilde{\theta}_i^T\varphi_i(\underline{\hat{x}}_i) + k_i(y - \hat{x}_1) - \frac{\partial\alpha_{i-1}}{\partial x_1}(e_2 + \delta_1 + \Delta F_1 + d_1) + H_i \tag{42}$$

where

$$H_i = -\frac{\partial\alpha_{i-1}}{\partial x_1}(\hat{x}_2 + \theta_1^T\varphi_1(\hat{x}_1)) - \sum_{l=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\theta_l}\dot{\theta}_l - \sum_{l=1}^{i}\frac{\partial\alpha_{i-1}}{\partial y_r}y_r^{(l)} - \frac{\partial\alpha_{i-1}}{\partial\hat{\varepsilon}_1}\dot{\hat{\varepsilon}}_1 - \sum_{l=2}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{w}_l}\dot{\hat{w}}_l - \sum_{l=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{x}_l}\dot{\hat{x}}_l$$

Consider the Lyapunov function candidate V_i as follows

$$V_i = V_{i-1} + \chi_i^2 + \frac{1}{2\gamma_i}\tilde{\theta}_i^T\tilde{\theta}_i + \frac{1}{2\bar{\gamma}_i}\tilde{w}_i^2 \tag{43}$$

Following the similar manipulations of (35) and (36) in step 2, one can obtain the following inequality

$$\begin{aligned}\dot{V}_i \leqslant{}& -e^T Q e + L_i\|e\|^2 + 2e^T P E_n\tilde{c}v - \sum_{l=1}^{i-1}c_l\chi_l^2 + \sum_{l=1}^{i-1}\frac{\sigma_l}{\gamma_l}\tilde{\theta}_l^T\theta_l + \frac{\bar{\sigma}_1}{\bar{\gamma}_1}\tilde{\varepsilon}_1\hat{\varepsilon}_1 + \sum_{l=2}^{i-1}\frac{\bar{\sigma}_l}{\bar{\gamma}_l}\tilde{w}_l\hat{w}_l + D_i \\ &+ 2\chi_i\left(\chi_{i-1} + \chi_{i+1} + \alpha_i + \theta_i^T\varphi_i(\underline{\hat{x}}_i) + k_i(y-\hat{x}_1) + \frac{1}{2}\left(3 + L_1^2\right)\chi_i\left(\frac{\partial\alpha_{i-1}}{\partial x_1}\right)^2 + H_i + \hat{w}_i\tanh\left(\frac{2\chi_i}{\kappa}\right)\right) \\ &+ \frac{1}{\gamma_i}\tilde{\theta}_i^T(2\gamma_i\chi_i\varphi_i(\underline{\hat{x}}_i) - \dot{\theta}_i) + \frac{1}{\bar{\gamma}_i}\tilde{w}_i\left(2\bar{\gamma}_i\chi_i\tanh\left(\frac{2\chi_i}{\kappa}\right) - \dot{\hat{w}}_i\right)\end{aligned} \tag{44}$$

where $\bar{L}_i = \bar{L}_{i-1} + 2$ and $D_i = D_{i-1} + \kappa' w_i^* + \delta_1^{*2} + d_{1M}^2$.

Choose the intermediate control function α_i, parameter adaptive laws θ_i and $\hat{w}_i$ as follows

$$\alpha_i = -\frac{1}{2}c_i\chi_i - \chi_{i-1} - \theta_i^T\varphi_i(\underline{\hat{x}}_i) - k_i(y - \hat{x}_1) - \hat{w}_i\tanh\left(\frac{2\chi_i}{\kappa}\right) - H_i - \frac{1}{2}\left(3 + L_1^2\right)\chi_i\left(\frac{\partial\alpha_{i-1}}{\partial x_1}\right)^2 \tag{45}$$

$$\dot{\theta}_i = 2\gamma_i\chi_i\varphi_i(\underline{\hat{x}}_i) - \sigma_i\theta_i \tag{46}$$

$$\dot{\hat{w}}_i = 2\bar{\gamma}_i\chi_i\tanh\left(\frac{2\chi_i}{\kappa}\right) - \bar{\sigma}_i\hat{w}_i \tag{47}$$

where c_i, σ_i and $\bar{\sigma}_i$ are positive design constants.

Similarly, substituting (45)–(47) into (44) yields

$$\dot{V}_i \leqslant -e^T Q e + \bar{L}_i\|e\|^2 + 2e^T P E_n\tilde{c}v - \sum_{l=1}^{i}c_l\chi_l^2 + \sum_{l=1}^{i}\frac{\sigma_l}{\gamma_l}\tilde{\theta}_l^T\theta_l + \frac{\bar{\sigma}_1}{\bar{\gamma}_1}\tilde{\varepsilon}_1\hat{\varepsilon}_1 + \sum_{l=2}^{i}\frac{\bar{\sigma}_l}{\bar{\gamma}_l}\tilde{w}_l\hat{w}_l + D_i + 2\chi_i\chi_{i+1} \tag{48}$$

Step n: The time derivative of χ_n along (14) and (22) is

$$\dot{\chi}_n = \hat{c}v(t) + \theta_n^T\varphi_n(\underline{\hat{x}}_n) + w_n + \tilde{\theta}_n^T\varphi_n(\underline{\hat{x}}_n) + k_n(y - \hat{x}_1) - \frac{\partial\alpha_{n-1}}{\partial x_1}(e_2 + \delta_1 + \Delta F_1 + d_1) + H_n \tag{49}$$

where

$$H_n = -\frac{\partial\alpha_{n-1}}{\partial x_1}(\hat{x}_2 + \theta_1^T\varphi_1(\hat{x}_1)) - \sum_{l=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\theta_l}\dot{\theta}_l - \sum_{l=1}^{n}\frac{\partial\alpha_{n-1}}{\partial y_r}y_r^{(l)} - \frac{\partial\alpha_{n-1}}{\partial\hat{\varepsilon}_1}\dot{\hat{\varepsilon}}_1 - \sum_{l=2}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{w}_l}\dot{\hat{w}}_l - \sum_{l=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{x}_l}\dot{\hat{x}}_l$$

Consider the Lyapunov function candidate V as follows

$$V = V_{n-1} + \chi_n^2 + \frac{1}{2\gamma_n}\tilde{\theta}_n^T\tilde{\theta}_n + \frac{1}{2\bar{\gamma}_n}\tilde{w}_n^2 + \frac{1}{2\bar{\bar{\gamma}}}\tilde{c}^2 \tag{50}$$

where $\bar{\bar{\gamma}}$ is a positive design constants.

Now, following the similar manipulations in step i yields

$$\dot{V} \leqslant -e^{\mathrm{T}}Qe + \bar{L}_n\|e\|^2 + 2e^{\mathrm{T}}PE_n\tilde{c}v - \sum_{l=1}^{n-1} c_l\chi_l^2 + \sum_{l=1}^{n-1}\frac{\sigma_l}{\gamma_l}\tilde{\theta}_l^{\mathrm{T}}\theta_l + \frac{\bar{\sigma}_1}{\bar{\gamma}_1}\tilde{\varepsilon}_1\hat{\varepsilon}_1 + \sum_{l=2}^{n-1}\frac{\bar{\sigma}_l}{\bar{\gamma}_l}\tilde{w}_l\hat{w}_l + D_n$$
$$+ 2\chi_n\left(\chi_{n-1} + \hat{c}v + \theta_n^{\mathrm{T}}\varphi_n(\hat{\underline{x}}_n) + k_n(y-\hat{x}_1) + \frac{1}{2}\left(3+L_1^2\right)\chi_n\left(\frac{\partial\alpha_{n-1}}{\partial x_1}\right)^2 + H_n + \hat{w}_n\tanh\left(\frac{2\chi_n}{\kappa}\right)\right)$$
$$+ \frac{1}{\gamma_n}\tilde{\theta}_n^{\mathrm{T}}\left(2\gamma_n\chi_n\varphi_n(\hat{\underline{x}}_n) - \dot{\theta}_n\right) + \frac{1}{\bar{\gamma}_n}\tilde{w}_n\left(2\bar{\gamma}_n\chi_n\tanh\left(\frac{2\chi_n}{\kappa}\right) - \dot{\hat{w}}_n\right) - \frac{1}{\bar{\bar{\gamma}}}\tilde{c}\dot{\hat{c}} \quad (51)$$

where $\bar{L}_n = \bar{L}_{n-1} + 2$ and $D_n = D_{n-1} + \kappa' w_n^* + \delta_1^{*2} + d_{1M}^2$.

By using Young's inequality, one has

$$2e^{\mathrm{T}}PE_n\tilde{c}v \leqslant \frac{1}{\varsigma}\|e\|^2 + \varsigma\tilde{c}(c-\hat{c})\|P\|^2v^2 \leqslant \frac{1}{\varsigma}\|e\|^2 + \frac{\varsigma}{2}\tilde{c}^2\|P\|^2v^2 + \frac{\varsigma}{2}c^2\|P\|^2v^2 - \varsigma\tilde{c}\hat{c}\|P\|^2v^2 \quad (52)$$

where ς is a positive design constant. Then substituting (52) into (51) gives

$$\dot{V} \leqslant -e^{\mathrm{T}}Qe + (\bar{L}_n + \frac{1}{\varsigma})\|e\|^2 - \sum_{l=1}^{n-1} c_l\chi_l^2 + \sum_{l=1}^{n-1}\frac{\sigma_l}{\gamma_l}\tilde{\theta}_l^{\mathrm{T}}\theta_l + \frac{\bar{\sigma}_1}{\bar{\gamma}_1}\tilde{\varepsilon}_1\hat{\varepsilon}_1 + \sum_{l=2}^{n-1}\frac{\bar{\sigma}_l}{\bar{\gamma}_l}\tilde{w}_l\hat{w}_l + D_n$$
$$+ 2\chi_n\left(\chi_{n-1} + \hat{c}v + \theta_n^{\mathrm{T}}\varphi_n(\hat{\underline{x}}_n) + k_n(y-\hat{x}_1) + \frac{1}{2}\left(3+L_1^2\right)\chi_n\left(\frac{\partial\alpha_{n-1}}{\partial x_1}\right)^2 + H_n + \hat{w}_n\tanh\left(\frac{2\chi_n}{\kappa}\right)\right)$$
$$+ \frac{1}{\gamma_n}\tilde{\theta}_n^{\mathrm{T}}(2\gamma_n\chi_n\varphi_n(\hat{\underline{x}}_n) - \dot{\theta}_n) + \frac{1}{\bar{\gamma}_n}\tilde{w}_n\left(2\bar{\gamma}_n\chi_n\tanh\left(\frac{2\chi_n}{\kappa}\right) - \dot{\hat{w}}_n\right) + \frac{1}{\bar{\bar{\gamma}}}\tilde{c}\left(-\dot{\hat{c}} - \varsigma\bar{\bar{\gamma}}\hat{c}\|P\|^2v^2\right) + \frac{\varsigma}{2}\tilde{c}^2\|P\|^2v^2$$
$$+ \frac{\varsigma}{2}c^2\|P\|^2v^2 \quad (53)$$

Choose the control input v, parameter adaptive laws θ_n, $\hat{w}_n$ and $\hat{c}$ as follows

$$v = \frac{1}{\hat{c}}\left(-\frac{1}{2}c_n\chi_n - \chi_{n-1} - \theta_n^{\mathrm{T}}\varphi_n(\hat{\underline{x}}_n) - k_n(y-\hat{x}_1) - \hat{w}_n\tanh\left(\frac{2\chi_n}{\kappa}\right) - H_n - \frac{1}{2}\left(3+L_1^2\right)\chi_n\left(\frac{\partial\alpha_{n-1}}{\partial x_1}\right)^2\right) \quad (54)$$

$$\dot{\theta}_n = 2\gamma_n\chi_n\varphi_n(\hat{\underline{x}}_n) - \sigma_n\theta_n \quad (55)$$
$$\dot{\hat{w}}_n = 2\bar{\gamma}_n\chi_n\tanh\left(\frac{2\chi_n}{\kappa}\right) - \bar{\sigma}_n\hat{w}_n \quad (56)$$
$$\dot{\hat{c}} = -\varsigma\bar{\bar{\gamma}}\hat{c}\|P\|^2v^2 - \bar{\bar{\sigma}}\hat{c} \quad (57)$$

where c_n, σ_n, $\bar{\sigma}_n$ and $\bar{\bar{\sigma}}$ are positive design constants.

Substituting (54)–(57) into (53) results in

$$\dot{V} \leqslant -\left(\lambda_{\min}(Q) - \bar{L}_n - \frac{1}{\varsigma}\right)\|e\|^2 - \sum_{l=1}^{n} c_l\chi_l^2 + \sum_{l=1}^{n}\frac{\sigma_l}{\gamma_l}\tilde{\theta}_l^{\mathrm{T}}\theta_l + \frac{\bar{\sigma}_1}{\bar{\gamma}_1}\tilde{\varepsilon}_1\hat{\varepsilon}_1 + \sum_{l=2}^{n}\frac{\bar{\sigma}_l}{\bar{\gamma}_l}\tilde{w}_l\hat{w}_l + \frac{\bar{\bar{\sigma}}}{\bar{\bar{\gamma}}}\tilde{c}\hat{c} + \frac{\varsigma}{2}\tilde{c}^2\|P\|^2v^2 + \frac{\varsigma}{2}c^2\|P\|^2v^2 + D_n \quad (58)$$

where $\lambda_{\min}(Q)$ is the minimum eigenvalue of Q.

By completing squares, one has

$$\frac{\sigma_l}{\gamma_l}\tilde{\theta}_l^{\mathrm{T}}\theta_l = \frac{\sigma_l}{\gamma_l}\tilde{\theta}_l^{\mathrm{T}}\left(\theta_l^* - \tilde{\theta}_l\right) \leqslant -\frac{\sigma_l}{2\gamma_l}\tilde{\theta}_l^{\mathrm{T}}\tilde{\theta}_l + \frac{\sigma_l}{2\gamma_l}\theta_l^{*\mathrm{T}}\theta_l^* \quad (59)$$
$$\frac{\bar{\sigma}_1}{\bar{\gamma}_1}\tilde{\varepsilon}_1\hat{\varepsilon}_1 \leqslant -\frac{\bar{\sigma}_1}{2\bar{\gamma}_1}\tilde{\varepsilon}_1^2 + \frac{\bar{\sigma}_1}{2\bar{\gamma}_1}\varepsilon_1^{*2} \quad (60)$$
$$\frac{\bar{\sigma}_l}{\bar{\gamma}_l}\tilde{w}_l\hat{w}_l \leqslant -\frac{\bar{\sigma}_l}{2\bar{\gamma}_l}\tilde{w}_l^2 + \frac{\bar{\sigma}_l}{2\bar{\gamma}_l}w_l^{*2} \quad (61)$$

$$\frac{\bar{\bar{\sigma}}}{\bar{\bar{\gamma}}}\tilde{c}\hat{c} \leqslant -\frac{\bar{\bar{\sigma}}}{2\bar{\bar{\gamma}}}\tilde{c}^2 + \frac{\bar{\bar{\sigma}}}{2\bar{\bar{\gamma}}}c^2 \quad (62)$$

Then, substituting (59)–(62) into (58) yields

$$\dot{V} \leqslant -\left(\lambda_{\min}(Q) - \bar{L}_n - \frac{1}{\varsigma}\right)\|e\|^2 - \sum_{l=1}^{n} c_l \chi_l^2 - \sum_{l=1}^{n} \frac{\sigma_l}{2\gamma_l}\tilde{\theta}_l^{\mathrm{T}}\tilde{\theta}_l - \frac{\bar{\sigma}_1}{2\bar{\gamma}_1}\tilde{\varepsilon}_1^2 - \sum_{l=2}^{n}\frac{\bar{\sigma}_l}{2\bar{\gamma}_l}\tilde{w}_l^2 - \frac{1}{2\bar{\bar{\gamma}}}(\bar{\bar{\sigma}} - \varsigma\|P\|^2 v^2)\tilde{c}^2 + \frac{\bar{\bar{\sigma}}}{2\bar{\bar{\gamma}}}c^2$$
$$+ \frac{\varsigma}{2}c^2\|P\|^2 v^2 + D_n + \frac{\bar{\sigma}_1}{2\bar{\gamma}_1}\varepsilon_1^{*2} + \sum_{l=1}^{n}\frac{\sigma_l}{2\gamma_l}\theta_l^{*\mathrm{T}}\theta_l^* + \sum_{l=2}^{n}\frac{\bar{\sigma}_l}{2\bar{\gamma}_l}w_l^{*2} \tag{63}$$

Choose $\rho = \lambda_{\min}(Q) - \bar{L}_n - \frac{1}{\varsigma} > 0$ and $\rho_1 = \bar{\bar{\sigma}} - \varsigma\|P\|^2 M^2 > 0$.
Let

$$\beta = \min\{\rho/\lambda_{\max}(P), c_1, \ldots, c_n, \sigma_1, \ldots, \sigma_n, \bar{\sigma}_1, \ldots, \bar{\sigma}_n, \bar{\bar{\sigma}}, \rho_1\},$$
$$\pi = \frac{\bar{\bar{\sigma}}}{2\bar{\bar{\gamma}}}c^2 + \frac{\varsigma}{2}c^2\|P\|^2 M^2 + D_n + \frac{\bar{\sigma}_1}{2\bar{\gamma}_1}\varepsilon_1^{*2} + \sum_{l=1}^{n}\frac{\sigma_l}{2\gamma_l}\theta_l^{*\mathrm{T}}\theta_l^* + \sum_{l=2}^{n}\frac{\bar{\sigma}_l}{2\bar{\gamma}_l}w_l^{*2}$$

Then (63) can be rewritten as follows

$$\dot{V} \leqslant -\beta V + \pi \tag{64}$$

And (64) can be further rewritten as follows

$$V(t) \leqslant V(0)e^{-\beta t} + \frac{\pi}{\beta} \tag{65}$$

From (65), it can be shown for each $i = 1, \ldots, n$, that the signals x, e_i, χ_i, θ_i, $\hat{\varepsilon}_1$, $\hat{w}_i$ and u are SUUB. Meanwhile, one can also obtain $|y(t) - y_r(t)| \leqslant \sqrt{V(0)e^{-\beta t}} + \sqrt{\pi/\beta}$ and $\|e(t)\| \leqslant (\sqrt{V(0)e^{-\beta t}} + \sqrt{\pi/\beta})/\sqrt{\lambda_{\max}(P)}$. Note that as $t \to \infty$, $e^{-\beta t} \to 0$, therefore, it follows that $\lim_{t\to\infty}|y(t) - y_r(t)| = \sqrt{\pi/\beta}$ and $\lim_{t\to\infty}\|e(t)\| = \sqrt{\pi/\beta}/\sqrt{\lambda_{\max}(P)}$. According to [2,3,19,22,24], the observer error and tracking error can be made as small as desired by appropriate choice of the design parameters.

Theorem 1. *For nonlinear systems* (1), *under the* Assumptions 1–3, *the controller* (54) *along with the state observer* (14), *the intermediate controllers* (29) and (45) *and parameter adaptation laws* (30), (31), (46), (47), (55), (56), (57), *can guarantees that all the signals in the closed-loop system are SUUB. Moreover, the observer error and tracking error can be made as small as desired by appropriate choice of design parameters.*

Remark 4. If the last equation in the system (1) becomes $\dot{x}_n = g(x)\phi(v) + f_n(x) + d_n(x,t)$, where $g(x)(g(x) \neq 0)$ is an unknown smooth function, the above adaptive control scheme can be also applicable if the following slight modifications are made. That is, the control (54) and parameter adaptive law (56) are modified as follows

$$v = \frac{1}{\theta_g^{\mathrm{T}}\varphi_g(\hat{x})}\left(-\frac{1}{2}c_n\chi_n - \chi_{n-1} - \theta_n^{\mathrm{T}}\varphi_n(\underline{\hat{x}}_n) - k_n(y - \hat{x}_1) - \hat{w}_n\tanh\left(\frac{2\chi_n}{\kappa}\right) - H_n - \frac{1}{2}\left(3 + L_1^2\right)\chi_n\left(\frac{\partial\alpha_{n-1}}{\partial x_1}\right)^2\right) \tag{66}$$

$$\dot{\theta}_g = -\varsigma\bar{\bar{\gamma}}\theta_g\|P\|^2\|\varphi_g\|^2 v^2 - \bar{\bar{\sigma}}\theta_g \tag{67}$$

4. Simulation examples

In this section, the feasibility and the control performances of the proposed method will be illustrated by two examples.

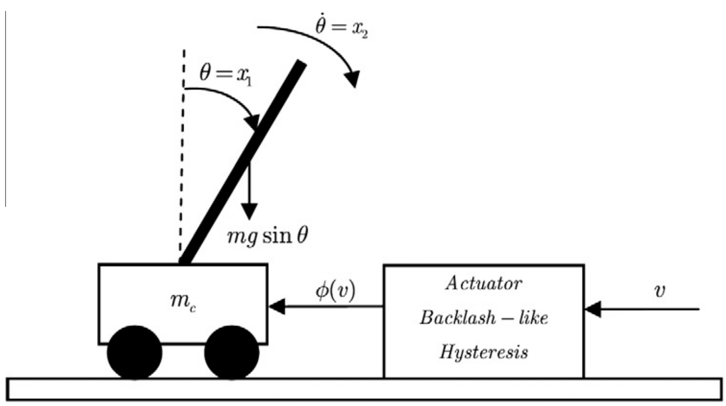

Fig. 2. Inverted pendulum system with backlash-like hysteresis actuator.

Example 1. Consider the inverted pendulum with backlash-like hysteresis actuator depicted in Fig. 2. Denoting $x_1 = \theta$ and $x_2 = \dot{\theta}$, a second-order model of inverted pendulum can be depicted as follows [18],

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(\underline{x}_2) + g(\underline{x}_2)\phi(v) + d(t) \end{cases} \tag{68}$$

where

$$f(\underline{x}_2) = \frac{9.8\sin x_1 - \frac{mlx_2^2\cos x_1 \sin x_1}{m_c+m}}{l\left(\frac{4}{3} - \frac{m\cos^2 x_1}{m_c+m}\right)}, \quad g(\underline{x}_2) = \frac{\frac{\cos x_1}{m_c+m}}{l\left(\frac{4}{3} - \frac{m\cos^2 x_1}{m_c+m}\right)}, \quad d(t) = 3 + 2\cos(2t).$$

m_c is the mass of the cart, m is the mass of the pole, $2l$ is the pole's length, $d(t)$ is the external disturbance, $\phi(v)$ is the output of the backlash-like hysteresis actuator and $v(t)$ is the applied force (control). In the simulation that follows $m_c = 1$ kg, $m = 0.1$ kg and $l = 0.5$ m will be chosen. The nonlinear functions $f(\underline{x}_2)$ and $g(\underline{x}_2)$ are assumed to be unknown.

Choosing fuzzy membership functions as follows

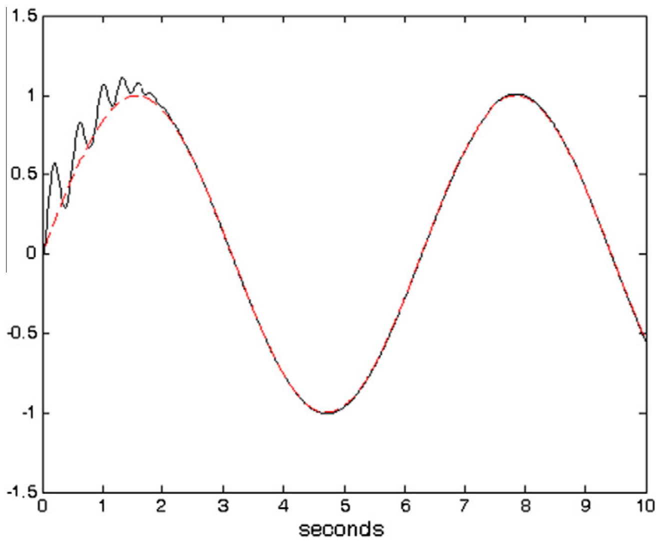

Fig. 3. Trajectories of y (solid line) and y_r (deshed line).

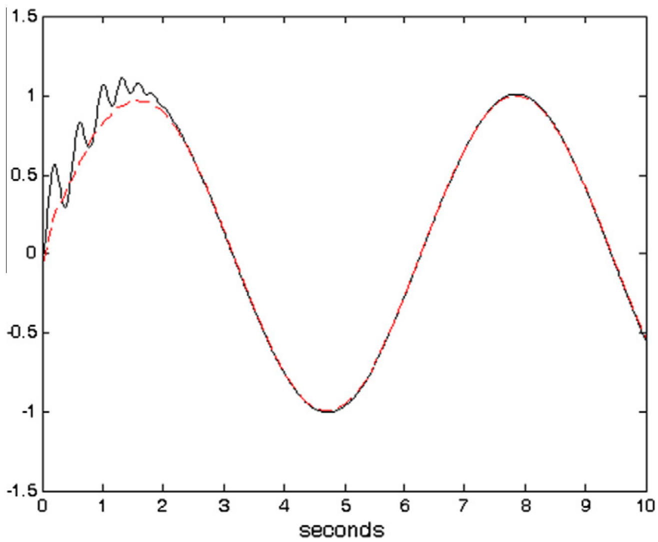

Fig. 4. Trajectories of x_1 (solid line) and $\hat{x}_1$ (deshed line).

$$\mu_{F_1^l}(\hat{x}_1) = \exp\left[-\frac{(\hat{x}_1 - 6 + 2l)^2}{2}\right], \quad \mu_{F_2^l}(\underline{\hat{x}}_2) = \exp\left[-\frac{(\hat{x}_1 - 6 + 2l)^2}{2}\right] \times \exp\left[-\frac{(\hat{x}_2 - 3 + l)^2}{5}\right],$$

$$\mu_{F_g^l}(\hat{x}_1) = \exp\left[-\frac{(\hat{x}_1 - 6 + 2l)^2}{2}\right], \quad l = 1, \ldots, 5.$$

And define fuzzy basis functions as follows

$$\varphi_{1,l}(\hat{x}_1) = \mu_{F_1^l} \Bigg/ \sum_{k=1}^{5} \mu_{F_1^k}, \quad \varphi_{2,l}(\underline{\hat{x}}_2) = \mu_{F_2^l} \Bigg/ \sum_{k=1}^{5} \mu_{F_2^k} \quad \text{and} \quad \varphi_{g,l}(\hat{x}_1) = \mu_{F_g^l} \Bigg/ \sum_{k=1}^{5} \mu_{F_g^k}, \quad l = 1, \ldots, 5.$$

Choose the first intermediate control function α_1 as (29) and the input of the backlash-like v as (54) and the parameters update laws as (30), (31), (55), (56) and (57), respectively.

Parameters in controller and adaptive laws are chosen as $c_1 = 20$, $c_2 = 20$, $k_1 = 14$, $k_2 = 24$, $\kappa_1 = 0.1$, $\kappa_2 = 6$, $\sigma_1 = 0.1$, $\bar{\sigma}_1 = 0.1$, $\sigma_2 = 0.1$, $\bar{\sigma}_2 = 0.1, \varsigma = 0.00001, \gamma_1 = 1$, $\bar{\gamma}_1 = 4$, $\gamma_2 = 1, \bar{\gamma}_2 = 4, \bar{\bar{\gamma}} = 0.1, \bar{\bar{\sigma}} = 0.1$, $Q = diag\{10^4, 10^4\}$.

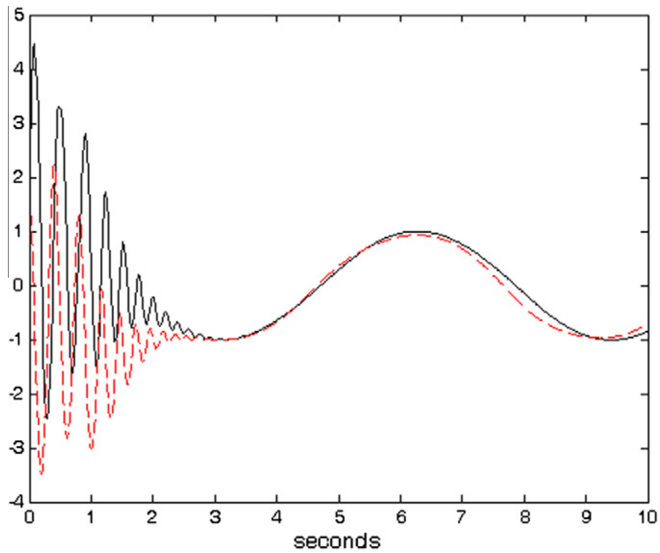

Fig. 5. Trajectories of x_2 (solid line) and $\hat{x}_2$ (deshed line).

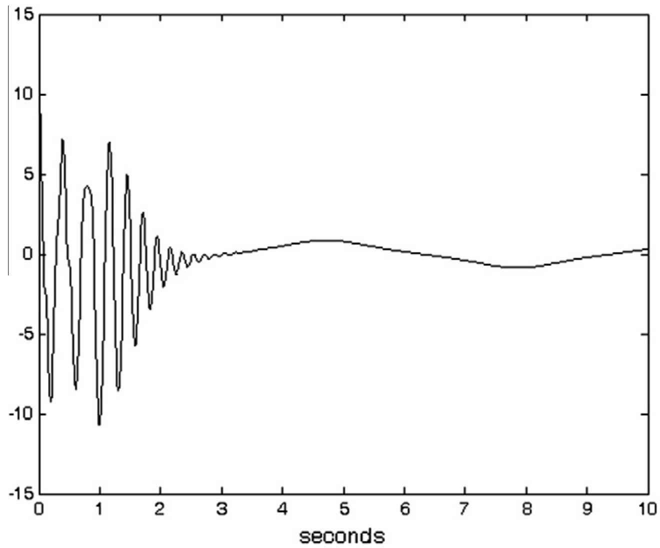

Fig. 6. Trajectory of ϕ.

$\phi(v)$ represents an output of the following backlash-like hysteresis:

$$\frac{d\phi}{dt} = \alpha\left|\frac{dv}{dt}\right|(cv - \phi) + B_1\frac{dv}{dt}$$

with $\alpha = 5$, $c = 3.1635$ and $B_1 = 0.345$.

The initial conditions are chosen as $x_1(0) = -\pi/60, x_2(0) = 0, \hat{x}_1(0) = -\pi/30, \hat{x}_2(0) = -1$, $\theta_g^T(0) = [1,1,1,1,1]$ and the others initial values are chosen as zeros. The reference is given as $y_r(t) = \sin(t)$.

The simulation results are shown in Figs. 3–7, where Fig. 3 illustrates the trajectories of the output and tracking signal; Figs. 4 and 5 illustrate the trajectories of the states and their estimates; Figs. 6 and 7 show the trajectory of ϕ and v, respectively.

Remark 5. From Figs. 3–7 it can be seen that our control scheme can achieve good control performances as those in [18]. Moreover, comparing with these two control schemes, we can find that our control scheme has removed the restrictions imposed on [18] that the controlled systems must satisfy the strictly positive real (SPR) condition and the matching condition.

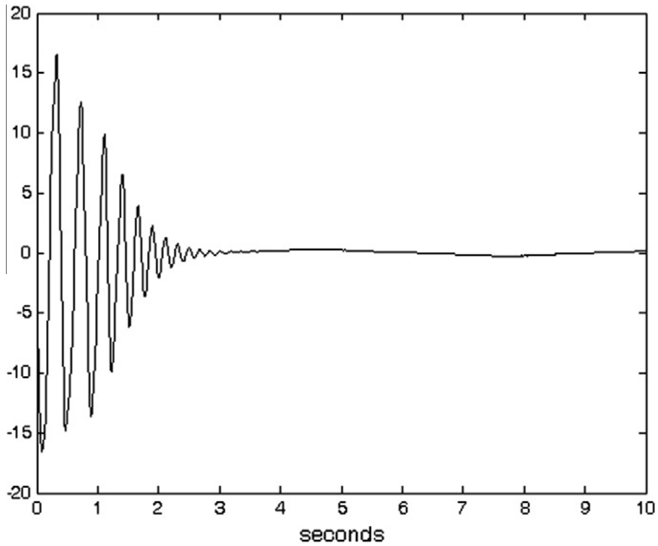

Fig. 7. Trajectory of v.

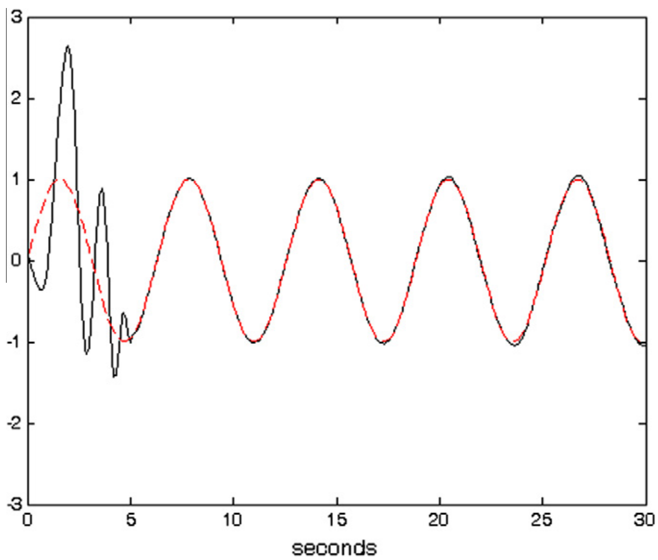

Fig. 8. Trajectories of y (solid line) and y_r (deshed line).

Example 2. Consider a nonlinear plant with two relative degrees as follows:

$$\begin{aligned} \dot{x}_1 &= x_2 + f_1(x_1) + d_1(x,t) \\ \dot{x}_2 &= \phi(v) + f_2(\underline{x}_2) + d_2(x,t) \\ y &= x_1 \end{aligned} \tag{69}$$

where $f_1(x_1) = -\cos(x_1)$, $f_2(\underline{x}_2) = -\sin(x_1\ x_2)$, $d_1(x,t) = 0.01\sin(e^{-x_1^2}x_1)$ and $d_2(x,t) = \frac{0.01x_1^2}{1+x_1^2}$.

Choosing fuzzy membership functions as follows

$$\mu_{F_1^l}(\hat{x}_1) = \exp\left[-\frac{(\hat{x}_1 - 6 + 2l)^2}{2}\right], \quad \mu_{F_2^l}(\hat{\underline{x}}_2) = \exp\left[-\frac{(\hat{x}_1 - 6 + 2l)^2}{2}\right] \times \exp\left[-\frac{(\hat{x}_2 - 3 + l)^2}{5}\right]$$

$l = 1, \ldots, 5$. Define fuzzy basis functions as follows

$$\varphi_{1,l}(\hat{x}_1) = \mu_{F_1^l} \Big/ \sum_{k=1}^{5} \mu_{F_1^k}, \quad \varphi_{2,l}(\hat{\underline{x}}_2) = \mu_{F_2^l} \Big/ \sum_{k=1}^{5} \mu_{F_2^k}, \quad l = 1, \ldots, 5.$$

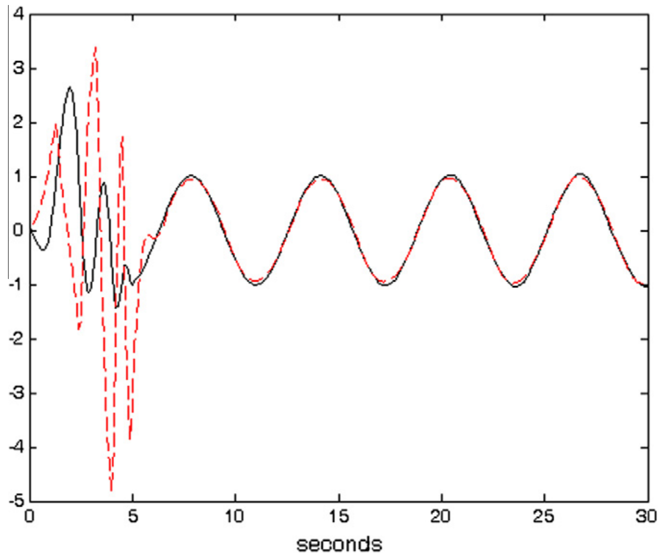

Fig. 9. Trajectories of x_1 (solid line) and $\hat{x}_1$ (deshed line).

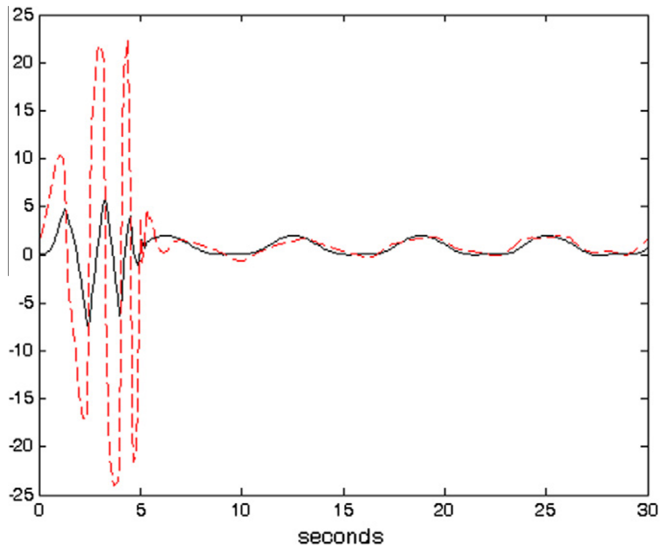

Fig. 10. Trajectories of x_2 (solid line) and $\hat{x}_2$ (deshed line).

Choose the first intermediate control function α_1 as (29) and the input of the backlash-like v as (54) and the parameters update laws as (30), (31), (55), (56) and (57), respectively.

Parameters in controller and adaptive laws are chosen as $c_1 = 4$, $c_2 = 6$, $k_1 = 4$, $k_2 = 20$, $\kappa_1 = 0.1$, $\kappa_2 = 0.2$, $\sigma_1 = 0.1$, $\bar{\sigma}_1 = 0.2$, $\sigma_2 = 1$, $\bar{\sigma}_2 = 0.1$, $\varsigma = 0.0001$, $\gamma_1 = 3$, $\bar{\gamma}_1 = 4$, $\gamma_2 = 4, \bar{\gamma}_2 = 5, \bar{\bar{\gamma}} = 0.001, \bar{\bar{\sigma}} = 0.01$, $Q = diag\{10^5, 10^5\}$.

$\phi(v)$ represents an output of the following backlash-like hysteresis:

$$\frac{d\phi}{dt} = \alpha\left|\frac{dv}{dt}\right|(cv - \phi) + B_1\frac{dv}{dt}$$

with $\alpha = 6$, $c = 3.1635$ and $B_1 = 0.345$.

The initial conditions are chosen as $x_1(0) = 0.1, x_2(0) = 0.1, \hat{c}(0) = 15$ and the others initial values are chosen as zeros. The reference is given as $y_r(t) = \sin(t)$.

The simulation results are shown in Figs. 8–12, where Fig. 8 illustrates the trajectories of the output and tracking signal; Figs. 9 and 10 exhibit the trajectories of the states and their estimates; Figs. 11 and 12 show the trajectory of ϕ and v, respectively.

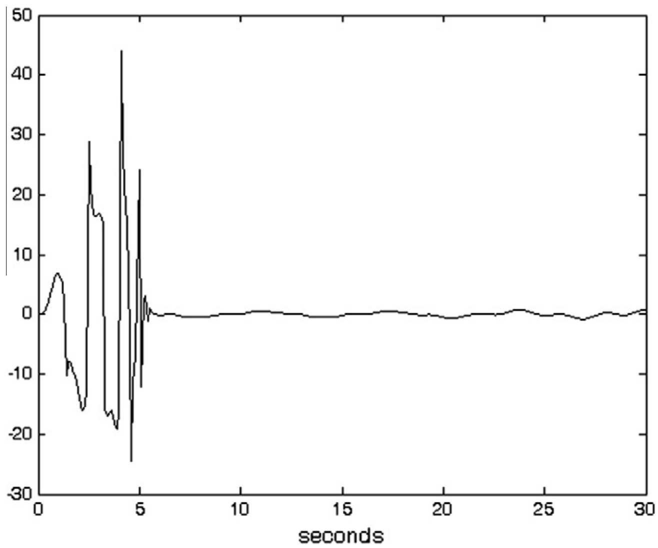

Fig. 11. Trajectory of ϕ.

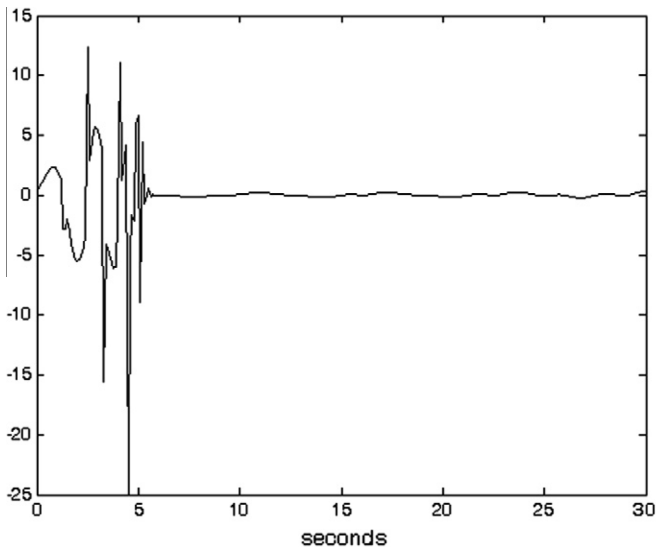

Fig. 12. Trajectory of v.

5. Conclusions

For a class of uncertain nonlinear systems without the measurements of the states and with unknown backlash-like hysteresis, an adaptive fuzzy output-feedback control approach has been developed. The fuzzy logic systems are used to approximate the nonlinear functions and a fuzzy state observer is designed for the unmeasured states. The proposed control scheme mainly solved the following three problems, that is, the restriction of unknown backlash-like hysteresis, the unmeasurement of the system states, and the non-matching condition of the nonlinear systems.. It is proved that the proposed control approach can guarantee that all the signals of the closed-loop system are SUUB, and both the observer and the tracking errors can be made as small as desired by appropriate choice of design parameters.

Acknowledgements

This work was supported in part by the National Natural Science Foundation of China (Nos. 61074014, 51179019, 60874056), the Outstanding Youth Funds of Liaoning Province (No. 2005219001), the Natural Science Foundation of Liaoning Province (No. 20102012) and China Postdoctoral Special Science Foundation (No. 200902241).

References

[1] A. Boulkroune, M. M'Saadand, H. Chekireb, Design of a fuzzy adaptive controller for MIMO nonlinear time-delay systems with unknown actuator nonlinearities and unknown control direction, Information Sciences 180 (2010) 5041–5059.
[2] B. Chen, X. P Liu, K.F. Liu, P. Shi, C. Lin, Direct adaptive fuzzy control for nonlinear systems with time-varying delays, Information Sciences 180 (2010) 776–792.
[3] B. Chen, X.P. Liu, Adaptive fuzzy output tracking control of MIMO nonlinear uncertain systems, IEEE Transactions on Fuzzy System 15 (2007) 287–300.
[4] B.S. Chen, C.H. Lee, Y.C. Chang, H_∞ tracking design of uncertain nonlinear SISO systems: adaptive fuzzy approach, IEEE Transactions on Fuzzy System 4 (1996) 32–43.
[5] C.S. Chiu, Mixed feedforward/feedback based adaptive fuzzy control for a class of MIMO nonlinear systems, IEEE Transactions on Fuzzy System 14 (2006) 716–727.
[6] S.S. Ge, C. Wang, Adaptive neural control of uncertain MIMO nonlinear systems, IEEE Transactions on Neural Networks 15 (2004) 674–692.
[7] C.C. Hua, X.P. Guan, P. Shi, Robust output tracking control for a class of time-delay nonlinear using neural network, IEEE Transactions on Neural Networks 18 (2007) 495–505.
[8] Y.S. Huang, M. Wu, Robust decentralized direct adaptive output feedback fuzzy control for a class of large-sale nonaffine nonlinear systems, Information Sciences 181 (2011) 2392–2404.
[9] M.A. Krasnoskl'skii, A.V. Pokrovskii, Systems with Hysteresis, Nauka, Moscow, Russia, 1983.
[10] P. Krishnamurthy, F. Khorrami, R.S. Chandra, Global high-gain-based observer and backstepping controller for generalized output-feedback canonical form, IEEE Transactions on Automatic Control 48 (2003) 2277–2284.
[11] H.X. Li, S.C. Tong, A hybrid adaptive fuzzy control for a class of nonlinear MIMO systems, IEEE Transactions on Fuzzy Systems 11 (2003) 24–34.
[12] T.S. Li, S.C. Tong, G. Feng, A novel robust adaptive fuzzy tracking control for a class of MIMO systems, IEEE Transactions on Fuzzy Systems 18 (2010) 150–160.
[13] T.S. Li, D. Wang, N.X. Chen, Adaptive fuzzy control of uncertain MIMO nonlinear systems in block-triangular forms, Nonlinear Dynamics 63 (2011) 105–123.
[14] C.L. Liu, S.C. Tong, Y.M. Li, Adaptive fuzzy backstepping output feedback control for nonlinear systems with unknown sign of high-frequency gain, ICIC Express Letters 4 (2010) 1689–1694.
[15] Y.J. Liu, W. Wang, Adaptive fuzzy control for a class of uncertain nonaffine nonlinear systems, Information Sciences 177 (2007) 3901–3917.
[16] J.W. Macki, P. Nistri, P. Zecca, Mathematical models for hysteresis, SIAM Review 35 (1993) 94–123.
[17] I.D. Mayergoyz, The Preisach Model for Hysteresis, Springer-Verlag, Berlin, Germany, 1991.
[18] R. Shahnazi, N. Pariz, A.V. Kamyad, Adaptive fuzzy output feedback control for a class of uncertain nonlinear systems with unknown backlash-like hysteresis, Communications in Nonlinear Science and Numerical Simulation 15 (2010) 2206–2221.
[19] Q.K. Shen, T.P. Zhang, C.Y. Zhou, Decentralized adaptive fuzzy control of time-delayed interconnected systems with unknown backlash-like hysteresis, Journal of Systems Engineering and Electronics 19 (2008) 1235–1242.
[20] C.Y. Su, Y. Stepanenko, J. Svoboda, T.P. Leung, Robust adaptive control of a class of nonlinear systems with unknown backlash-like hysteresis, IEEE Transactions on Automatic Control 45 (2000) 2427–2432.
[21] C.Y. Su, M. Oya, H. Hong, Stable adaptive fuzzy control of nonlinear systems proceeded by unknown backlash-like hysteresis, IEEE Transactions on Fuzzy Systems 11 (2003) 1–8.
[22] G. Tao, P.V. Kokotovic, Adaptive control of systems with backlash, Automatica 29 (1993) 323–335.
[23] S.C. Tong, Y.M. Li, Observer-based fuzzy adaptive control for strict-feedback nonlinear systems, Fuzzy Sets and Systems 160 (2009) 1749–1764.
[24] S.C. Tong, Y.M. Li, Fuzzy adaptive robust backstepping stabilization for SISO nonlinear systems with unknown virtual control direction, Information Sciences 180 (2010) 4619–4640.
[25] S. Tong, Y. Li, P. Shi, Fuzzy adaptive backstepping robust control for SISO nonlinear system with dynamic uncertainties, Information Sciences 179 (2009) 1319–1332.
[26] S.C. Tong, Y.M. Li, T. Wang, Adaptive fuzzy backstepping fault-tolerant control for uncertain nonlinear systems based on dynamic surface, International Journal of Innovative Computing, Information and Control 5 (2009) 3249–3262.
[27] S.C. Tong, N. Sheng, Y.M. Li, Observer-based adaptive fuzzy backstepping control for strict-feedback nonlinear systems with unknown time delays, International Journal of Innovative Computing, Information and Control 7 (2011) 6949–6964.
[28] L.X. Wang, Adaptive Fuzzy Systems and Control, Prentice Hall, Englewood Cliffs, 1994.
[29] M. Wang, X. Liu, P. Shi, Adaptive neural control of pure-feedback nonlinear time-delay systems via dynamic surface technique, IEEE Transactions on Systems, Man and Cybernetics, Part B 41 (2011) 1681–1692.
[30] T. Wang, S.C. Tong, Y.M. Li, Robust adaptive fuzzy control for nonlinear system with dynamic uncertainties based on backstepping, International Journal of Innovative Computing, Information and Control 5 (2009) 2675–2688.
[31] Y.F. Wang, T.Y. Chai, Y.M. Zhang, State observer-based adaptive fuzzy output-feedback control for a class of uncertain nonlinear systems, Information Sciences 180 (2010) 5029–5040.
[32] C.Y. Wen, J. Zhou, Decentralized adaptive stabilization in the presence of unknown backlash-like hysteresis, Automatica 43 (2007) 426–440.
[33] Y.S. Yang, C.J. Zhou, Adaptive fuzzy H_∞ stabilization for strict-feedback canonical nonlinear systems via backstepping and small-gain approach, IEEE Transactions on Fuzzy Systems 13 (2005) 104–114.

[34] J. Zhou, C.Y. Wen, Y. Zhang, Adaptive backstepping control of a class of uncertain nonlinear systems with unknown backlash-like hysteresis, IEEE Transactions on Automatic Control 49 (2004) 1751–1758.
[35] Q. Zhou, P. Shi, J. Lu, S. Xu, Adaptive output feedback fuzzy tracking control for a class of nonlinear systems, IEEE Transactions on Fuzzy Systems 19 (2011) 972–982.
[36] S.S. Zhou, G. Feng, C.B. Feng, Robust control for a class of uncertain nonlinear systems: adaptive fuzzy approach based on backstepping, Fuzzy Sets and Systems 151 (2005) 1–20.