PI Adaptive Fuzzy Control With Large and Fast Disturbance Rejection for a Class of Uncertain Nonlinear Systems

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***Abstract*—Design of controllers for uncertain systems is inherently paradoxical. Adaptive control approaches claim to adapt system parameters against uncertainties, but only if these uncertainties change slowly enough. Alternatively, robust control methodologies claim to ensure system stability against uncertainties, but only if these uncertainties remain within known bounds. This is while, in reality, disturbances and uncertainties remain faithfully uncertain, i.e., may be both fast and large. In this paper, a PI-adaptive fuzzy control architecture for a class of uncertain nonlinear systems is proposed that aims to provide added robustness in the presence of large and fast but bounded uncertainties and disturbances. While the proposed approach requires the uncertainties to be bounded, it does not require this bound to be known. Lyapunov analysis is used to prove asymptotic stability of the proposed approach. Application of the proposed method to a second-order inverted pendulum system demonstrates the effectiveness of the proposed approach. Specifically, system responses to fast versus slow and large versus small disturbances are considered in the presented simulation studies.**

***Index Terms*—Adaptive control, fuzzy control, Lyapunov theory, nonlinear systems.**

I. Introduction

GENERALLY, there are two kinds of uncertainties in a system to be controlled. One is caused by a lack of information about system structure and parameters, and the other is due to internal and external disturbances. Due to these uncertainties, the conventional nonlinear methods like feedback linearization usually fail, and the control of nonlinear uncertain systems remains a challenging and yet rewarding problem [1], [2].

Among the methods that address uncertainty in plant dynamics is adaptive control. The objective, in adaptive control, is to introduce an adaptation law that adjusts the parameters of the controller or the system against system uncertainties and disturbances. However, such methods generally guarantee parameter convergence only if parameter changes are slow enough [3], [4]. Moreover, the existing adaptive control approaches commonly require the general structure of the plant, such as its order, to be known.

Fuzzy logic, as an alternative to conventional control methodologies handling uncertainty, has been the focus of numerous studies in the past two decades [5]. Fuzzy logic provides an important tool for utilization of human expert knowledge in complement to mathematical knowledge. This is mainly due to the possibility of making use of fuzzy knowledge-based control to deal with systems whose dynamics are not so well understood and whose models can not be so conveniently established [6], [7]. Therefore, hybrid combinations of the fuzzy logic and adaptive control are an attractive approach for designing robust control systems with high degrees of nonlinearities and uncertainties [8]–[16]. Such hybrid approaches provide the ability to incorporate human expert knowledge about plant dynamics (indirect adaptive control), as well as its operation (direct adaptive control) into the adaptive control methodology.

Literature on adaptive fuzzy controllers has been abundant and growing, particularly during the past decade. As one of the pioneering works in this area, Wang in 1996 [8] proposed a general approach to design of stable adaptive fuzzy controllers based on a fuzzy basis function framework and Lyapunov synthesis. The concept of fuzzy basis function viewed fuzzy logic framework not only as a paradigm for formulating human knowledge, but also as a powerful nonlinear function with universal approximation capabilities. Wang applied his control approach to a second-order nonlinear model of inverted pendulum system. More recently, the problem of controlling a class of uncertain nonlinear systems to follow a reference trajectory was addressed in [9]. There, based on *a priori* information, a nominal fuzzy controller was first implemented; then, a signal to compensate for both structured and unstructured uncertainties arising from the approximation was synthesized based on the Lyapunov method. Exponential tracking to the reference trajectory up to an ultimately bounded error was achieved. Later in [10], Tang addressed a lack of robustness in conventional adaptive fuzzy methods and proposed an adaptive robust fuzzy control design for a class of nonlinear systems represented by input-output models to follow a reference trajectory in the presence of uncertainties. In 2005, Yang and Zhou [11] developed a new systematic procedure for the synthesis of stable adaptive robust fuzzy controllers for a class of continuous uncertain systems, where Takagi–Sugeno type fuzzy logic systems were used to approximate the unknown unstructured functions in the

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systems and the adaptive mechanism with minimal learning parameterization was achieved by use of Lyapunov theorem. In [12], Tong and his colleagues developed a new approach to direct and indirect adaptive output-feedback fuzzy decentralized controllers for a class of uncertain large-scale nonlinear systems. These controllers did not need the availability of the state variables, yet the stability of the closed-loop systems could be verified from Lyapunov stability analysis. Moreover, the proposed overall control schemes guaranteed that all the signals involved remained bounded and achieved H^{∞} tracking performance. In [13], a hybrid indirect and direct adaptive fuzzy output tracking control schemes were developed for a class of nonlinear multiple-input–multiple-output (MIMO) systems, which also did not need the availability of state variables. In [14], both indirect and direct adaptive control methods are developed, where the spatially localized models (in the form of Takagi–Sugeno fuzzy systems or radial basis function neural networks) are used as online approximators to learn the unknown dynamics of the system. A novel adaptive fuzzy robust tracking control (AFRTC) algorithm is proposed in [15] for a class of nonlinear systems with the uncertain system function and uncertain gain function. The Takagi–Sugeno type fuzzy logic systems are used to approximate unknown uncertain functions and the AFRTC algorithm is designed by use of the input-to-state stability approach and small gain theorem. In [16], an adaptive fuzzy sliding mode controller is proposed to suppress the sprung mass position oscillation due to road surface variation. This intelligent control strategy combines an adaptive rule with fuzzy and sliding mode control algorithms. the authors in [17] proposed a direct adaptive fuzzy PI sliding mode control for a class of uncertain nonlinear systems which could cope with chattering while guaranteeing asymptotic stability. Finally, for a survey on analysis and design of model-based fuzzy control systems, see [18].

The basic idea of this paper was earlier introduced by authors in [19]. That approach is extended here by considering disturbances that are bounded but where the magnitude of these bounds is not known. The proposed indirect-adaptive fuzzy control strategy contains a PI-type switching structure that provides a mechanism to cope with the bounded large-and-fast disturbances with unknown bound while the typical chattering of the switching law is significantly attenuated. Lyapunov synthesis of the system guarantees system asymptotic stability without a need to know the magnitude of system uncertainties. The application of the proposed strategy to a second-order nonlinear inverted pendulum confirms the above results even with fast and large disturbances.

This paper is organized as follows. Section II represents the problem formulation and assumptions such as specifying the class of nonlinear systems, fuzzy logic systems, and universal approximation theorem. In Section III, the proposed adaptive fuzzy controller is derived. In Section IV, the proposed approach is applied to an inverted pendulum system. To demonstrate the effectiveness of the proposed method, system performance is tested there under various magnitudes and frequencies of system disturbances. Finally, Section V concludes the main advantages of the proposed method.

II. Problem Formulation, Assumptions and Fuzzy Logic Systems

A. Problem Formulation and Assumptions

Consider a class of SISO nth-order nonlinear systems in the following form:

$$\begin{aligned} x^{(n)} &= f(X,t) + g(X,t)u(t) + d(X,t) \\ y &= x \end{aligned} \tag{1}$$

where f and g are unknown bounded nonlinear real continuous functions, where the bounds need not be known, $d(X,t)$ is an unknown external disturbance, $X^T = [x, \dot{x}, \cdots, x^{(n-1)}] = [x_1, x_2, \cdots, x_n] \in R^n$ is the state vector of the system which is assumed to be available for measurement, and $u \in R$ and $y \in R$ are the input and the output of the system, respectively. We have the following assumptions.

Assumption 1: External disturbance $d(X,t)$ is bounded by an unknown constant D, i.e.,

$$|d(X,t)| \leq D. \tag{2}$$

Assumption 2: System (1) is controllable, $g(X,t) \neq 0$. Without loss of generality, we assume $g(X,t) > 0$, i.e., $g(X,t)$ can be negative and the control can be similarly derived.

The control objective is to design u such that the state of the system X follows the desired state X_d in the presence of uncertainties and disturbances, that is the tracking error

$$E = X - X_d = \left[e, \dot{e}, \cdots, e^{(n-1)}\right]^T \tag{3}$$

should converge to zero.

Assumption 3: The desired trajectory vector X_d is continuous, available, and bounded, i.e.,

$$\|X_d\| < \Psi \tag{4}$$

where Ψ is a known positive bound.

B. Fuzzy Logic Systems and Universal Approximation Theorem

The fuzzy logic system (FLS) that is detailed in [6] is used in this research and is briefly explained below for continuity of discussion. FLS performs a mapping from a compact set $U_1 \times U_2 \times \cdots \times U_n = U \subset R^n$ to a compact set $V \subset R$. Any fuzzy system consists of a fuzzifier, a fuzzy rule base, a fuzzy inference engine and a defuzzifier. The fuzzy rule base consists of a collection of fuzzy IF-THEN rules such as

$$R^{(l)}: \text{IF } \left\{x_1 \text{ } is \text{ } F_1^l \text{ and } \cdots \text{ and } x_n \text{ } is \text{ } F_n^l\right\} \text{ THEN } y \text{ } is \text{ } G^l \quad l = 1, \cdots M \tag{5}$$

where $X = [x_1, x_2, \cdots, x_n]^T \in U$ and $y \in V$ are the input and output of the fuzzy system, respectively. F_i^l and G^l are fuzzy sets in U_i and V, respectively. The fuzzy inference engine performs a mapping from fuzzy sets in U to fuzzy sets in V, based on fuzzy rule base. Furthermore, the fuzzifier maps a crisp point

$X = [x_1, x_2, \cdots, x_n]^T \in U$ to a fuzzy set in U and the defuzzifier maps fuzzy sets in V to a crisp point in V.

Using Singleton fuzzifier, product inference engine and center average defuzzifier, the output of fuzzy system can be expressed as

$$y = \frac{\sum_{l=1}^{M} \overline{y}^l \left(\prod_{i=1}^{n} \mu_{F_i^l}(x_i) \right)}{\sum_{l=1}^{M} \left(\prod_{i=1}^{n} \mu_{F_i^l}(x_i) \right)} = \theta^T \xi(X) \tag{6}$$

where M is the total number of rules, $\theta = [\overline{y}^1, \overline{y}^2, \cdots, \overline{y}^M]^T$ is the center of output fuzzy membership functions and is the adjustable parameter vector, and $\xi(X) = [\xi_1(X), \xi_2(X), \cdots, \xi_M(X)]^T$ is the fuzzy basis function defined as follows:

$$\xi_j(X) = \frac{\prod_{i=1}^{n} \mu_{F_i^j}(x_i)}{\sum_{l=1}^{M} \left(\prod_{i=1}^{n} \mu_{F_i^l}(x_i) \right)} \quad j = 1, \cdots, M. \tag{7}$$

Now, we have the following theorem.

Theorem 1 [6]: For any given real continuous function g on the compact set $U \subset R^n$ and arbitrary $\varepsilon > 0$, there exists a fuzzy system $f^*(X) = \theta^{*T} \xi^*(X)$ in the form of (6) such that

$$\sup_{X \in U} |f^*(X) - g(X)| < \varepsilon. \tag{8}$$

The above theorem represents that the fuzzy systems in the form of (6) can approximate any real continuous function to any degree of accuracy. This means the fuzzy systems in the form of (6) have universal approximation property as also reported earlier in [6].

III. Main Result

Here, our objective is to design an indirect adaptive fuzzy controller. The nonlinear system (1) can be rewritten as follows:

$$\begin{aligned} \dot{X} &= AX + B\{f(X,t) + g(X,t)u + d(X,t)\} \\ y &= CX \end{aligned} \tag{9}$$

where f, g and d are unknown, and

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & \cdots & \cdots & \cdots & 0 \end{bmatrix}_{n \times n} \tag{10}$$

and

$$B^T = [0 \cdots 0\, 1]_{1 \times n}, \quad \mathrm{C} = [1 \cdots 0\, 0]_{1 \times n}. \tag{11}$$

Let $K = [k_n, \ldots, k_1]^T$ be chosen such that the polynomial $s^n + k_1 s^{n-1} + \cdots + k_{n-1}s + k_n$ is Hurwitz. Considering (1) and (3), if f, g, and d were known then the following control could guarantee that error e converges asymptotically to zero

$$u = \frac{1}{g(X,t)} \left[-f(X,t) - d(X,t) + x_d^{(n)} - K^T E \right]. \tag{12}$$

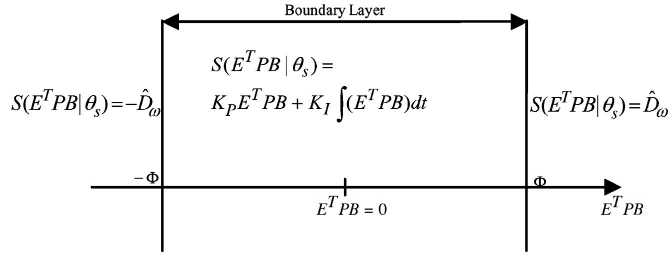

Fig. 1. Boundary layer and the adaptive PI structure.

Therefore, it is necessary to propose a robust control. The following remark can be considered.

Remark 1: From the property of K, positive matrices P and Q are chosen such that

$$(A - BK^T)^T P + P(A - BK^T) = -Q. \tag{13}$$

For increasing robustness and to cope with external disturbances, we propose the following control law

$$u = \frac{1}{\hat{g}(X|\theta_g)} \left[-\hat{f}(X|\theta_f) - S(E^T PB|\theta_s) + x_d^{(n)} - K^T E \right] \tag{14}$$

where $\hat{f}$ and $\hat{g}$ are the fuzzy systems in the form of (6) with free parameters θ_f and θ_g to approximate the unknown functions f and g, which can be described as follows:

$$\hat{f}(X|\theta_f) = \theta_f^T \xi(X) \tag{15}$$

$$\hat{g}(X|\theta_g) = \theta_g^T \xi(X). \tag{16}$$

From the universal approximation property of fuzzy systems and Theorem 1, it can be said that there exist optimal θ_f^* and θ_g^* such that

$$\theta_f^* = \arg \min_{\theta_f \in R^M} \left[\sup_{X \in R^n} \left| \hat{f}(X|\theta_f) - f(X,t) \right| \right] \tag{17}$$

$$\theta_g^* = \arg \min_{\theta_g \in R^M} \left[\sup_{X \in R^n} |\hat{g}(X|\theta_g) - g(X,t)| \right]. \tag{18}$$

Let ω be the *minimum* approximation error defined by

$$\omega = f(X,t) - \hat{f}\left(X|\theta_f^*\right) + \left(g(X,t) - \hat{g}\left(X|\theta_g^*\right)\right) u. \tag{19}$$

Based on continuity and boundedness of functions f and g and universal approximation theorem, it can be deduced $|\omega| < \Omega$, also $S(E^T PB|\theta_s)$ is a proportional-integral (PI) error feedback structure as follows:

$$S(E^T PB|\theta_s) = \begin{cases} \theta_s^T \varphi(E^T PB) = K_P E^T PB \\ \quad + K_I \int (E^T PB)dt, & |E^T PB| \le \Phi \\ \hat{D}_\omega \mathrm{sgn}(E^T PB), & |E^T PB| > \Phi \end{cases} \tag{20}$$

where P is a positive matrix that is obtained from (12), $\theta_s^T = [K_P, K_I]$ is a vector of free parameters to be adapted, and $\varphi(E^T PB) = [E^T PB, \int (E^T PB)dt]^T$. $\hat{D}_\omega = \hat{D} + \hat{\Omega}$ is an estimate of $D_\omega = D + \Omega$, D and Ω are the bounds of disturbance and minimal approximation error of fuzzy systems, respectively, and selecting Φ is a tradeoff between chattering

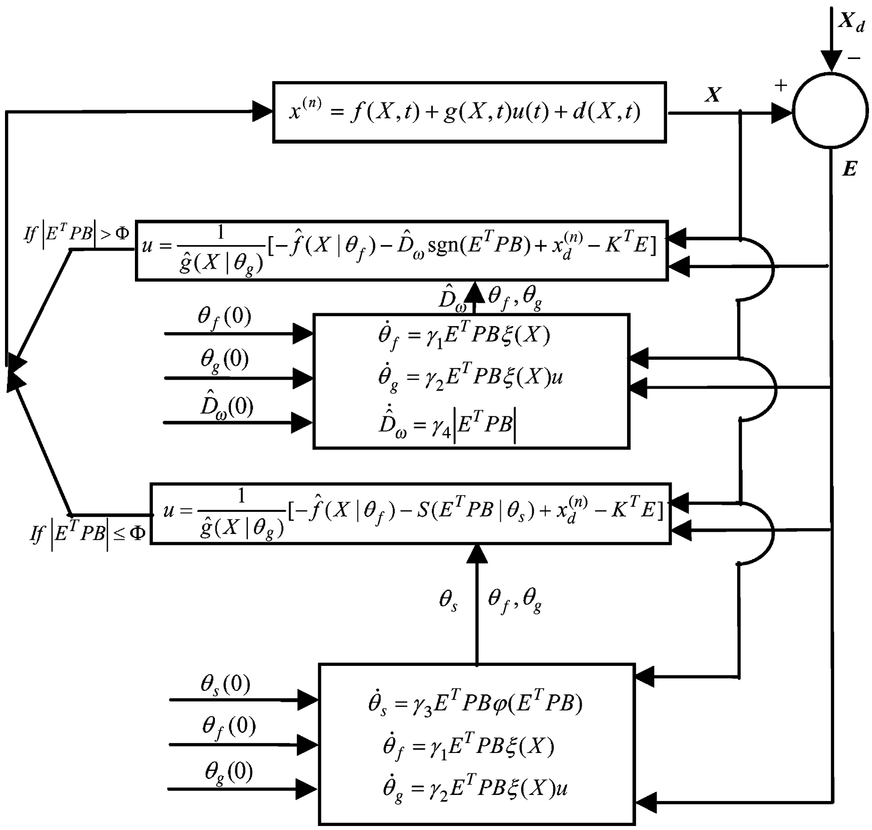

Fig. 2. Block diagram representation of the proposed method.

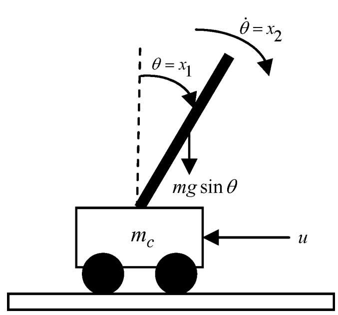

Fig. 3. Inverted pendulum system.

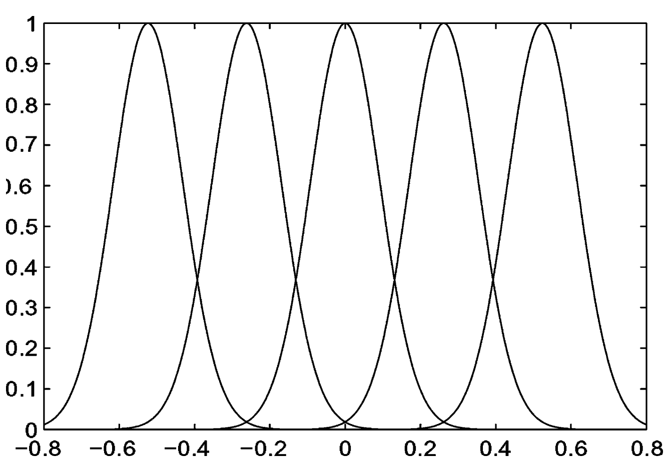

Fig. 4. Membership functions defined in universe $[-1, 1]$, from left to right $\mu_{N2}(x_i), \mu_{N1}(x_i), \mu_Z(x_i), \mu_{P1}(x_i), \mu_{P2}(x_i)$ $(i = 1, 2)$.

attenuation versus increasing the speed of convergence without affecting the stability results of this approach.

Therefore, the control procedure can be summarized as follows. First, if the state trajectory of the system is outside the boundary layer, i.e., $|E^T PB| \geq \Phi$, then it is subject to the sliding control-like $S(E^T PB|\theta_s) = \hat{D}_\omega \mathrm{sgn}(E^T PB)$ forcing the state trajectory toward the boundary layer; that is due to robustness of the discontinuous control sgn(.) as also shown in [1], [2], [17], and [20]. Second, if it enters the boundary layer region, i.e., $|E^T PB| < \Phi$, the adaptive PI structure begins operating (Fig. 1). It should be mentioned that the free parameters of fuzzy systems (15) and (16) are adapted from the beginning, but the free parameters of the PI structure are adapted when the state enters the boundary layer region.

Considering above explanations Theorem 2 is stated and proved to show that overall system is asymptotically stable.

Theorem 2: Consider the uncertain nonlinear system in the form of (1) or equivalently (9), then the control law in (14), Assumptions 1–3 and the below adaptation laws

$$\dot{\theta}_f = \gamma_1 E^T PB\xi(X) \tag{21}$$

$$\dot{\theta}_g = \gamma_2 E^T PB\xi(X)u \tag{22}$$

$$\dot{\theta}_s = \gamma_3 E^T PB\varphi(E^T PB) \tag{23}$$

$$\dot{\hat{D}}_\omega = \gamma_4 |E^T PB| \tag{24}$$

where $\gamma_i > 0, i = 1, \ldots, 4$, would yield the state tracking error $E(t)$ to asymptotically reach zero, i.e., $\lim_{t\to\infty} |E(t)| = 0$.

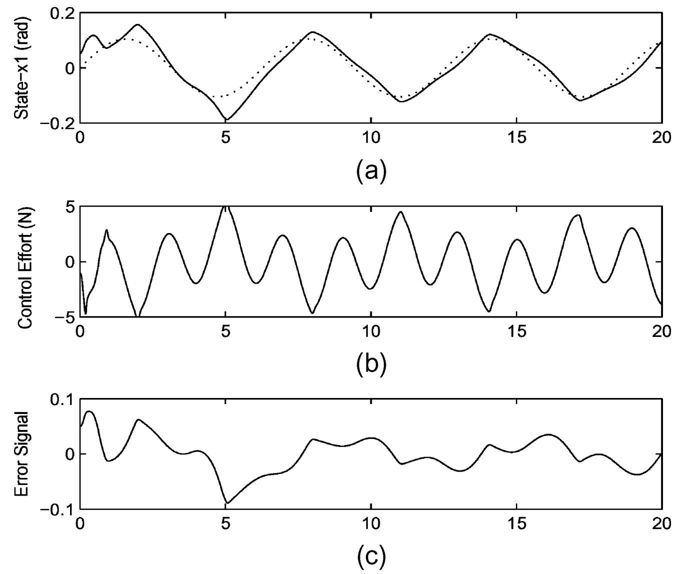

Fig. 5. $d(t) = 4\cos(\pi t)$ for the referenced adaptive fuzzy controller presented in [8]: (a) the system output (state $x_1(t)$) to solid) initial condition $[\pi/60, 0]$ versus (dashed) desired output; (b) the control effort; (c) the error signal.

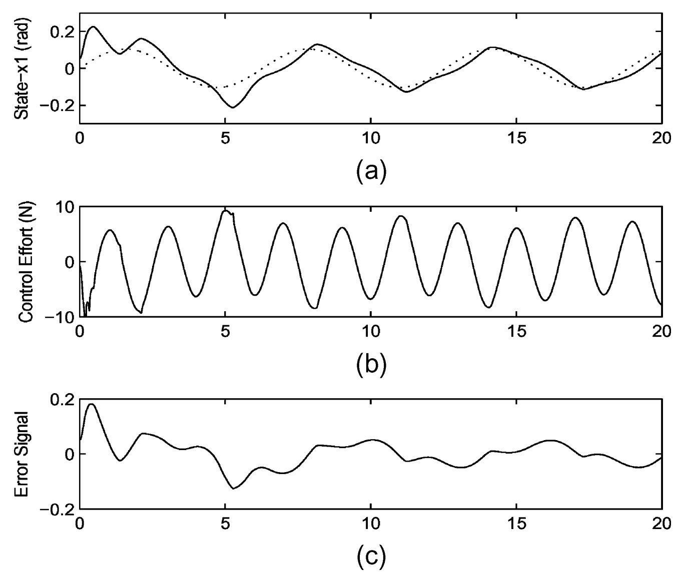

Fig. 6. $d(t) = 10\cos(\pi t)$ for the referenced adaptive fuzzy controller presented in [8]: (a) the system output (state $x_1(t)$) to (solid) initial condition $[\pi/60, 0]$ versus (dashed) desired output; (b) the control effort; (c) the error signal.

Proof: Using the control law (14) in (9), we have

$$\begin{aligned}\dot{X} &= AX + B\left\{f(X,t) + g(X,t)u + \hat{g}(X|\theta_g)u \right.\\ &\qquad \left. -\hat{g}(X|\theta_g)u + d(X,t)\right\}\\ &= AX + Bx_d^{(n)} - BK^T E\\ &\quad + B\left\{f(X,t) - \hat{f}(X|\theta_f) + \left(g(X,t) - \hat{g}(X|\theta_g)\right)u \right.\\ &\qquad \left. -S(E^T PB|\theta_s) + d(X,t)\right\}. \end{aligned} \tag{25}$$

Considering (25) and a few manipulations yields

$$\begin{aligned}\dot{E} &= (A - BK^T)E + B\left(f(X,t) - \hat{f}(X|\theta_f)\right)\\ &+ B\left(g(X,t) - \hat{g}(X|\theta_g)\right)u + B\left(d(X,t) - S(E^T PB|\theta_s)\right). \end{aligned} \tag{26}$$

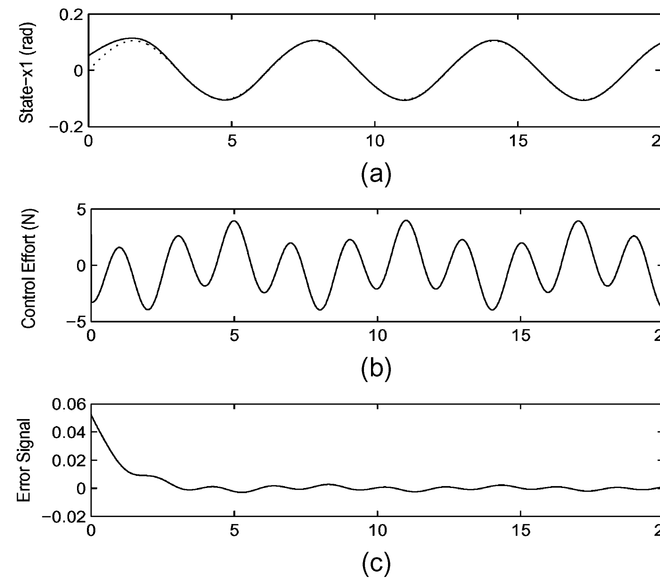

Fig. 7. $d(t) = 4\cos(\pi t)$ for proposed adaptive fuzzy controller: (a) the system output (state $x_1(t)$) to (solid) initial condition $[\pi/60, 0]$ versus (dashed) desired output; (b) the control effort; (c) the error signal.

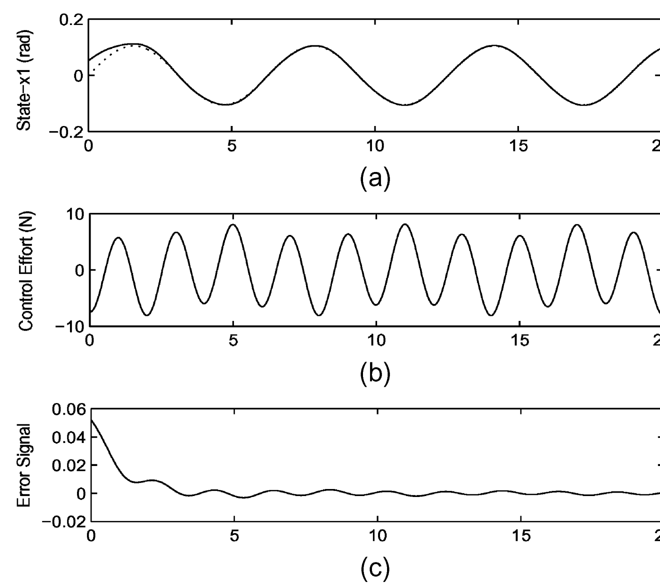

Fig. 8. $d(t) = 10\cos(\pi t)$ for the proposed adaptive fuzzy controller: (a) the system output (state $x_1(t)$) to (solid) initial condition $[\pi/60, 0]$ versus (dashed) desired output; (b) the control effort; (c) the error signal.

From (19), (26) becomes

$$\begin{aligned}\dot{E} &= (A - BK^T)E + B\left(\hat{f}\left(X|\theta_f^*\right) - \hat{f}(X|\theta_f)\right)\\ &\quad + B\left(\hat{g}\left(X|\theta_g^*\right) - \hat{g}(X|\theta_g)\right)u + B\omega\\ &\quad + B\left(d(X,t) - S(E^T PB|\theta_s)\right)\\ &= (A - BK^T)E + B\widetilde{\theta}_f^T\xi(X) + B\widetilde{\theta}_g^T\xi(X)u + B\omega\\ &\quad + B\left(S\left(E^T PB|\theta_s^*\right) - S(E^T PB|\theta_s)\right)\\ &\quad + B\left(d(X,t) - S\left(E^T PB|\theta_s^*\right)\right) \end{aligned} \tag{27}$$

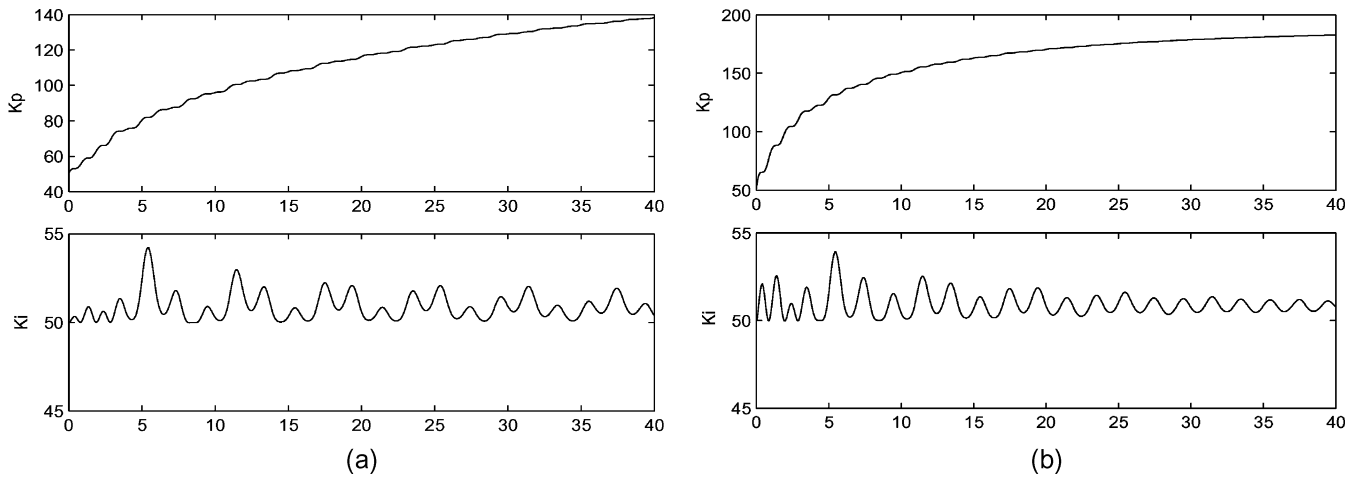

Fig. 9. PI parameters adaptation for case 1: (a) $d(t) = 4\cos(\pi t)$; (b) $d(t) = 10\cos(\pi t)$.

where $\widetilde{\theta}_f = \theta_f^* - \theta_f$, $\widetilde{\theta}_g = \theta_g^* - \theta_g$. Consider the following Lyapunov function candidate:

$$V = \frac{1}{2}E^T PE + \frac{1}{2\gamma_1}\widetilde{\theta}_f^T\widetilde{\theta}_f + \frac{1}{2\gamma_2}\widetilde{\theta}_g^T\widetilde{\theta}_g + \frac{1}{2\gamma_3}\widetilde{\theta}_s^T\widetilde{\theta}_s + \frac{1}{2\gamma_4}\widetilde{D}_\omega^2 \tag{28}$$

where $\widetilde{\theta}_s = \theta_s^* - \theta_s$ and $\widetilde{D}_\omega = D_\omega - \hat{D}_\omega$. Differentiating V along the closed-loop systems trajectory (27) and using (13) leads to

$$\begin{aligned}\dot{V} = &-\frac{1}{2}E^TQE + E^TPB\omega + \widetilde{\theta}_f^T\left(E^TPB\xi(X) + \frac{1}{\gamma_1}\dot{\widetilde{\theta}}_f\right)\\ &+\widetilde{\theta}_g^T\left(E^TPB\xi(X)u + \frac{1}{\gamma_2}\dot{\widetilde{\theta}}_g\right)\\ &+\widetilde{\theta}_s^T\left(E^TPB\varphi(E^TPB) + \frac{1}{\gamma_3}\dot{\widetilde{\theta}}_s\right) + \frac{1}{\gamma_4}\widetilde{D}_\omega\dot{\widetilde{D}}_\omega\\ &+E^TPBd(X,t) - E^TPBS\left(E^TPB|\theta_s^*\right).\end{aligned}$$

From the definition of $S(E^TPB|\theta_s^*)$, we can conclude that $S(E^TPB|\theta_s^*)$ lies in the first and third quadrant, so $E^TPBS(E^TPB|\theta_s^*) = 0$ for $E^TPB = 0$, and $E^TPBS(E^TPB|\theta_s^*) \geq 0$ for all E^TPB. Therefore, $E^TPBS(E^TPB|\theta_s^*) = |E^TPB||S(E^TPB|\theta_s^*)|$, so

$$\begin{aligned}\dot{V} \leq &-\frac{1}{2}E^TQE + \widetilde{\theta}_f^T\left(E^TPB\xi(X) + \frac{1}{\gamma_1}\dot{\widetilde{\theta}}_f\right)\\ &+\widetilde{\theta}_g^T\left(E^TPB\xi(X)u + \frac{1}{\gamma_2}\dot{\widetilde{\theta}}_g\right)\\ &+\widetilde{\theta}_s^T\left(E^TPB\varphi(E^TPB) + \frac{1}{\gamma_3}\dot{\widetilde{\theta}}_s\right) + \frac{1}{\gamma_4}\widetilde{D}_\omega\dot{\widetilde{D}}_\omega\\ &+|E^TPB||\omega| + |E^TPB|\,|d(X,t)|\\ &-|E^TPB|\left|S\left(E^TPB|\theta_s^*\right)\right|\end{aligned}$$

$$\begin{aligned}\dot{V} \leq &-\frac{1}{2}E^TQE + \widetilde{\theta}_f^T\left(E^TPB\xi(X) - \frac{1}{\gamma_1}\dot{\theta}_f\right)\\ &+\widetilde{\theta}_g^T\left(E^TPB\xi(X)u - \frac{1}{\gamma_2}\dot{\theta}_g\right)\\ &+\widetilde{\theta}_s^T\left(E^TPB\varphi(E^TPB) - \frac{1}{\gamma_3}\dot{\theta}_s\right) - \frac{1}{\gamma_4}\widetilde{D}_\omega\dot{\hat{D}}_\omega\\ &+|E^TPB|D_\omega - |E^TPB|\hat{D}_\omega.\end{aligned} \tag{29}$$

Using the adaptation laws (21)–(24), we have

$$\dot{V} \leq -\frac{1}{2}E^TQE \leq 0. \tag{30}$$

From (13), Q is positive definite; therefore, $\dot{V}$ is negative semi definite, i.e., $V(E(t), \widetilde{\theta}_f(t), \widetilde{\theta}_g(t), \widetilde{\theta}_s(t), \widetilde{D}_\omega(t)) \leq V(E(0), \widetilde{\theta}_f(0), \widetilde{\theta}_g(0), \widetilde{\theta}_s(0), \widetilde{D}_\omega(0))$, implying E^TPB, θ_f, θ_g, θ_s and $\hat{D}_\omega$ are bounded. Defining $\Theta(t) = (1/2)E^TQE \leq -\dot{V}$ and integrating it with respect to time yields

$$\begin{aligned}\int_0^t \Theta(\tau)d\tau \leq &\,V\left(E(0), \widetilde{\theta}_f(0), \widetilde{\theta}_g(0), \widetilde{\theta}_s(0), \widetilde{D}_\omega(0)\right)\\ &-V\left(E(t), \widetilde{\theta}_f(t), \widetilde{\theta}_g(t), \widetilde{\theta}_s(t), \widetilde{D}_\omega(t)\right).\end{aligned} \tag{31}$$

Because $V(E(0), \widetilde{\theta}_f(0), \widetilde{\theta}_g(0), \widetilde{\theta}_s(0), \widetilde{D}_\omega(0))$ is bounded and $V(E(t), \widetilde{\theta}_f(t), \widetilde{\theta}_g(t), \widetilde{\theta}_s(t), \widetilde{D}_\omega(t))$ is nonincreasing and bounded, the following results:

$$\lim_{t\to\infty}\int_0^t \Theta(\tau)d\tau < \infty. \tag{32}$$

Also, $\dot{\Theta}(t)$ is bounded, so by using the Barbalat's lemma [1] (if the differentiable function $h(t)$ has a finite limit as $t \to \infty$, and is such that $\ddot{h}$ exists and is bounded, then $\dot{h}(t) \to 0$ as $t \to \infty$), it can be shown that $\lim_{t\to\infty}\Theta(t) = 0$. Therefore, we have $\lim_{t\to\infty}|E(t)| = 0$.

The algorithm of the proposed method is depicted in Fig. 2.

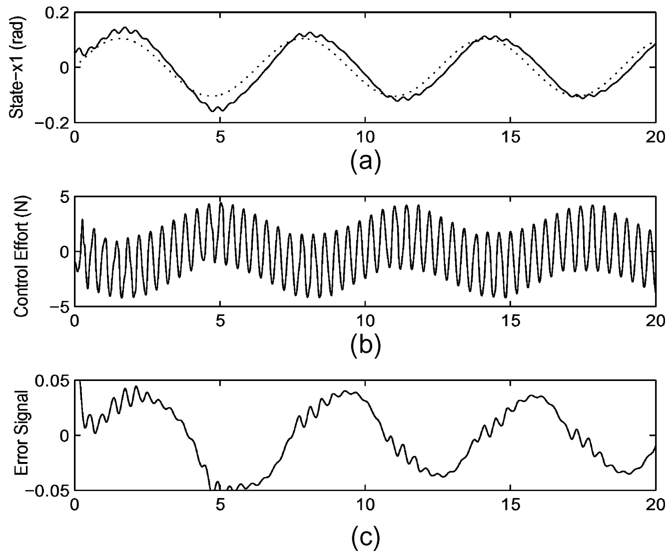

Fig. 10. $d(t) = 4\cos(5\pi t)$ for the referenced adaptive fuzzy controller in [8]: (a) the system output (state $x_1(t)$) to (solid) initial condition $[\pi/60, 0]$ versus (dashed) desired output; (b) the control effort; (c) the error signal.

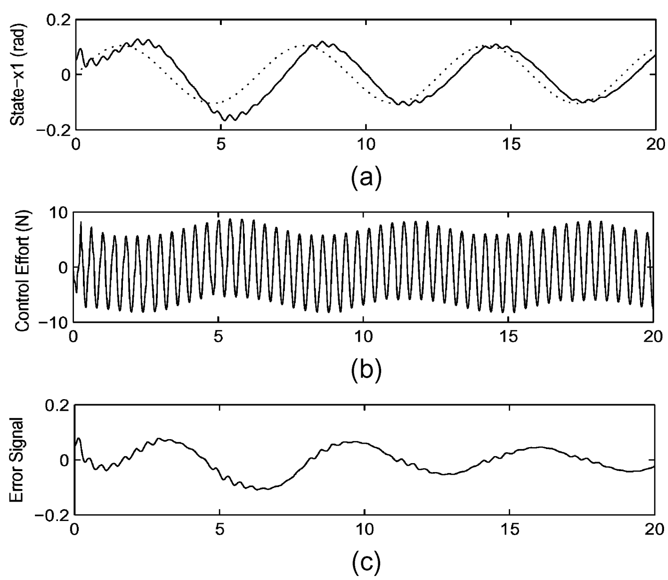

Fig. 11. $d(t) = 10\cos(5\pi t)$ for the referenced adaptive fuzzy controller in [8]: (a) the system output (state $x_1(t)$) to (solid) initial condition $[\pi/60, 0]$ versus (dashed) desired output; (b) the control effort; (c) the error signal.

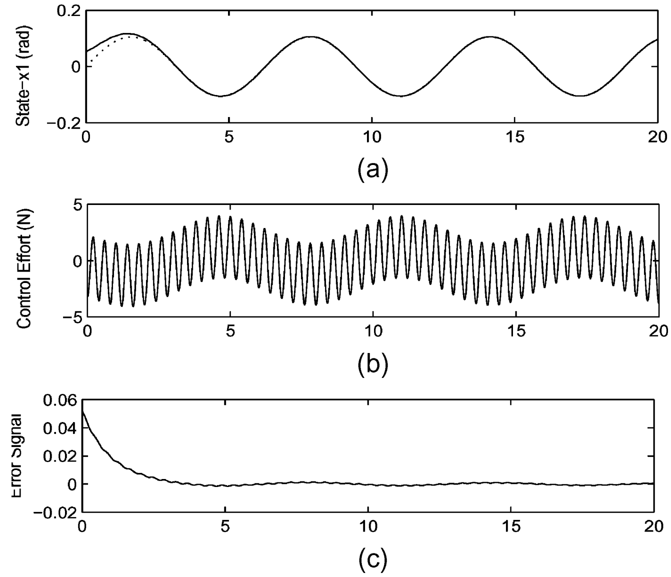

Fig. 12. $d(t) = 4\cos(5\pi t)$ for the proposed adaptive fuzzy controller: (a) the system output (state $x_1(t)$) to (solid) initial condition $[\pi/60, 0]$ versus (dashed) desired output; (b) the control effort; (c) the error signal.

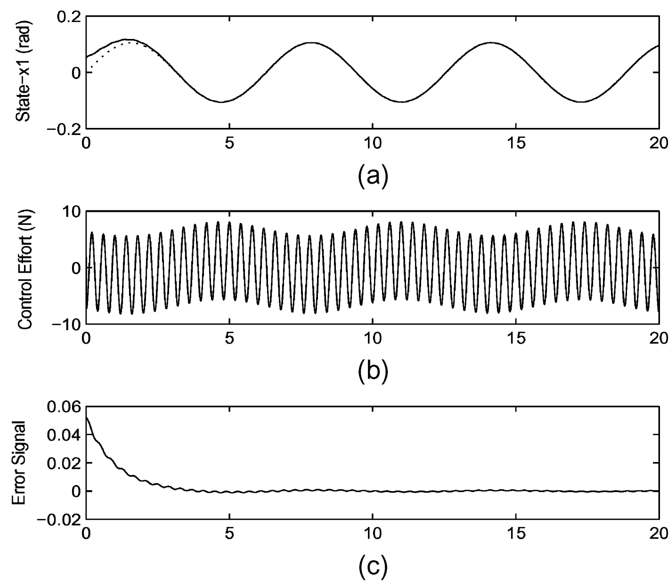

Fig. 13. $d(t) = 10\cos(5\pi t)$ for the proposed adaptive fuzzy controller: (a) the system output (state $x_1(t)$) to (solid) initial condition $[\pi/60, 0]$ versus (dashed) desired output; (b) the control effort; (c) the error signal.

IV. Simulation Example

In this section, we apply the proposed controller to an inverted pendulum system under different types and level of disturbances.

Example: Consider the inverted pendulum as depicted in Fig. 3.

Denoting $x_1 = \theta\ (\mathrm{rad})$ and $x_2 = \dot{\theta}\ (\mathrm{rad/s})$, a second-order model of the above inverted pendulum can be stated as follows [1]:

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= f(x_1, x_2) + g(x_1, x_2)u(t) + d(t)\end{aligned} \tag{33}$$

where

$$\begin{aligned}f(x_1, x_2) &= \frac{9.8\sin x_1 - \frac{m l x_2^2 \cos x_1 \sin x_1}{m_c + m}}{l\left(\frac{4}{3} - \frac{m\cos^2 x_1}{m_c + m}\right)}\\ g(x_1, x_2) &= \frac{\frac{\cos x_1}{m_c + m}}{l\left(\frac{4}{3} - \frac{m\cos^2 x_1}{m_c + m}\right)}\end{aligned} \tag{34}$$

$m_c = 1\ \mathrm{kg}$ is the mass of the cart, $m = 0.1\ \mathrm{kg}$ is the mass of the pole, $l = 0.5\ \mathrm{m}$ is half of the pole's length, and u (N) is the applied force (control). As is conventionally known, this system is open loop unstable at its equilibrium point which is (0,0), $u(t)$

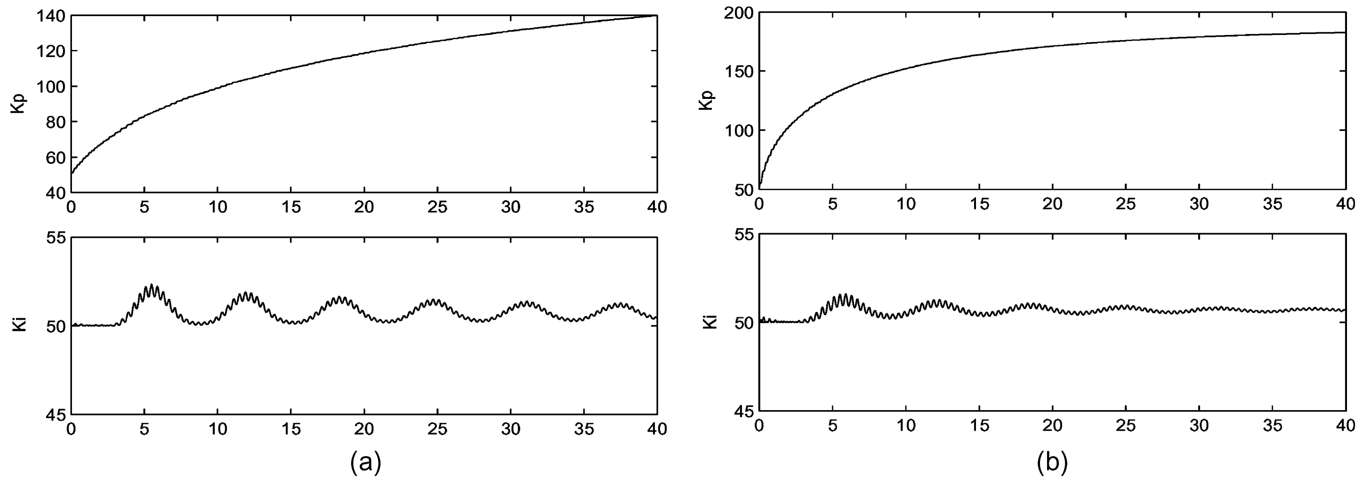

Fig. 14. PI parameter adaptation for case 2: (a) $d(t) = 4\cos(5\pi t)$; (b) $d(t) = 10\cos(5\pi t)$.

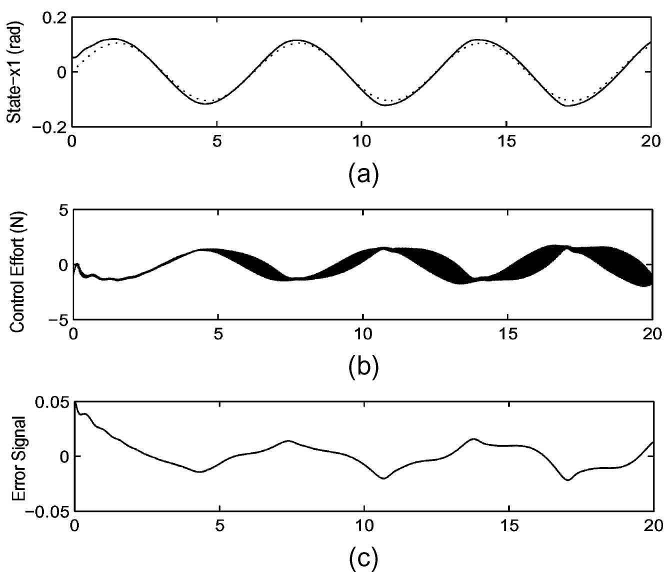

Fig. 15. $d(t) = 4\cos(50\pi t)$ for referenced adaptive fuzzy controller presented in [8]: (a) the system output (state $x_1(t)$) to (solid) initial condition $[\pi/60, 0]$ versus (dashed) desired output; (b) the control effort; (c) the error signal.

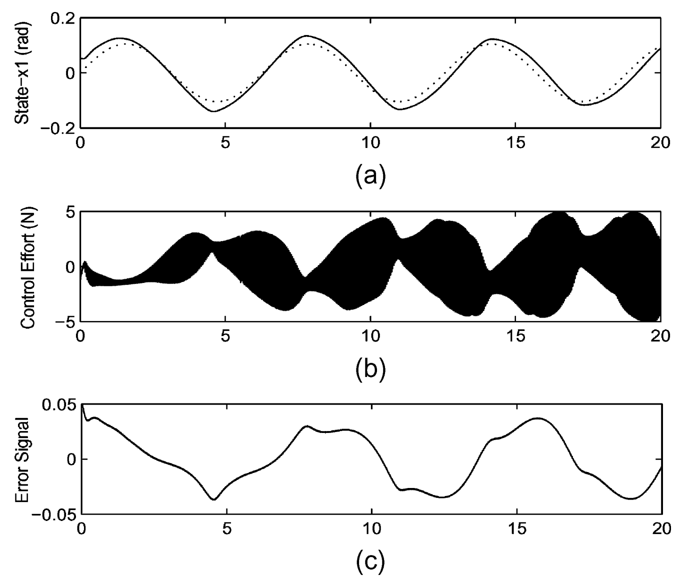

Fig. 16. $d(t) = 10\cos(50\pi t)$ for referenced adaptive fuzzy controller presented in [8]: (a) the system output (state $x_1(t)$) to (solid) initial condition $[\pi/60, 0]$ versus (dashed) desired output; (b) the control effort; (c) the error signal.

is the control signal and $d(t)$ is an unknown bounded disturbance. In order to study the robustness of the proposed control scheme against fast changes, a sinusoidal $d(t) = a\cos(2\pi bt)$ is selected for disturbance, where a and b are the gain and frequency of the disturbance, respectively. The desired trajectory is $x_d = (\pi/30)\sin(t)$. Furthermore, $K_P(0) = 50$, $K_I(0) = 50$ and $K = [1, 2]^T$ are assumed. Five fuzzy membership functions in the interval [−1,1] are assumed for x_i: $i = 1$, 2 as

$$\mu_{N2}(x_i) = \exp\left(-\left(\frac{x_i + \pi/6}{\pi/24}\right)^2\right)$$
$$\mu_{N1}(x_i) = \exp\left(-\left(\frac{x_i + \pi/12}{\pi/24}\right)^2\right)$$
$$\mu_{Z}(x_i) = \exp\left(-\left(\frac{x_i}{\pi/24}\right)^2\right)$$
$$\mu_{P2}(x_i) = \exp\left(-\left(\frac{x_i - \pi/12}{\pi/24}\right)^2\right)$$
$$\mu_{P1}(x_i) = \exp\left(-\left(\frac{x_i - \pi/6}{\pi/24}\right)^2\right) \tag{35}$$

as shown in Fig. 4. The initial conditions of fuzzy parameters are chosen randomly in the intervals [0.5, 1.5]. $\gamma_1, \gamma_2, \gamma_3$ and γ_4 are selected 50, 0.5, 8000, and 1, respectively, and $P = \begin{bmatrix} 15 & 5 \\ 5 & 5 \end{bmatrix}$ is chosen.

Three cases are considered.

Case 1) Slow disturbance with $d(t) = a\cos(\pi t)$, which has a period eight times greater than the time constant of the system.

Case 2) Fast disturbance with $d(t) = a\cos(5\pi t)$, which has a period near the time constant of the system.

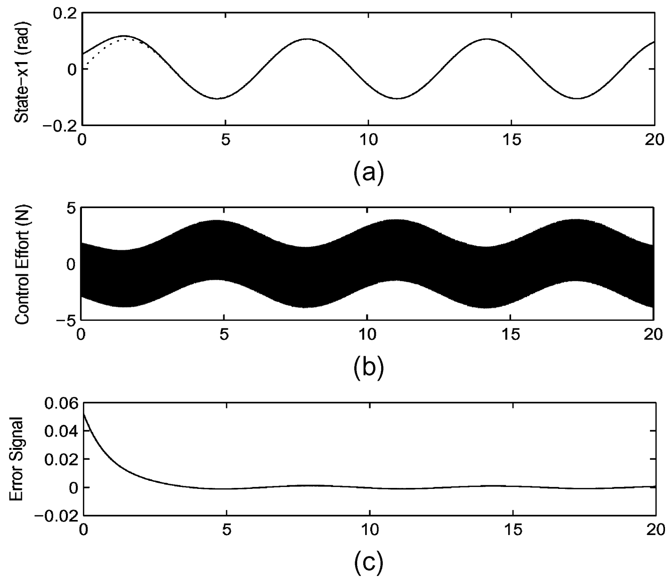

Fig. 17. $d(t) = 4\cos(50\pi t)$ for the proposed adaptive fuzzy controller: (a) the system output (state $x_1(t)$) to (solid) initial condition $[\pi/60, 0]$ versus (dashed) desired output; (b) the control effort; (c) the error signal.

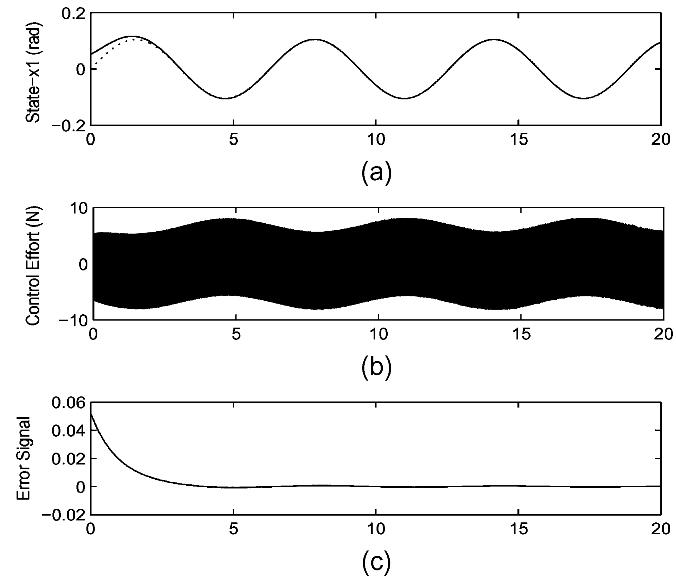

Fig. 18. $d(t) = 10\cos(50\pi t)$ for the proposed adaptive fuzzy controller: (a) the system output (state $x_1(t)$) to (solid) initial condition $[\pi/60, 0]$ versus (dashed) desired output; (b) the control effort;(c) the error signal.

Case 3) Very fast disturbance with $d(t) = a\cos(50\pi t)$, which has a period 0.2 times smaller than the time constant of the system.

In all of the above cases, robustness of the referenced adaptive fuzzy controller by Wang [8] and the proposed adaptive fuzzy controller (14) with the adaptation laws (21)–(24) is compared under smaller, $a = 4$, and larger, $a = 10$, magnitudes of disturbance. Initial conditions are set at $[\pi/60, 0]$ on all of the six studies.

Case 1: Slow Disturbance $(d(t) = a\cos(\pi t))$*:* Figs. 5 and 6 show the system responses, control effort and error signal with the referenced adaptive fuzzy controller [8], for $d(t) = 4\cos(\pi t)$ and $d(t) = 10\cos(\pi t)$, respectively. Comparing the two figures, it can be seen that while the system remains stable for both uncertainty levels, tracking performance deteriorates and larger control effort is spent in the presence of larger disturbances. Figs. 7 and 8 show the system responses, control efforts and error signals with the proposed adaptive fuzzy controller for $d(t) = 4\cos(\pi t)$ and $d(t) = 10\cos(\pi t)$, respectively. Comparing Figs. 5 and 7 and Figs. 6 and 8, it can also be observed that the proposed adaptive fuzzy controller achieves a significantly better tracking performance while less control effort is spent. Additionally, PI parameters adaptation in [0, 40 s] is depicted in Fig. 9. As can be seen, K_I has small changes while K_P steadily increases before converging. This is due to the property of adaptation law (23).

Case 2: Fast Disturbance $(d(t) = a\cos(5\pi t))$*:* Here, the disturbance is assumed to have a period to be approximately as fast as the unstable dynamics of the systems. Adaptive controllers are generally designed for uncertainties that are slow varying. When this condition is not met, tracking performance rapidly deteriorates. Figs. 10 and 11 show the system response, control effort and error signal with the referenced adaptive fuzzy controller in [8], for $d(t) = 4\cos(5\pi t)$ and $d(t) = 10\cos(5\pi t)$, respectively. As expected, it can be seen that the system response is further deteriorated. In comparison, Figs. 12 and 13

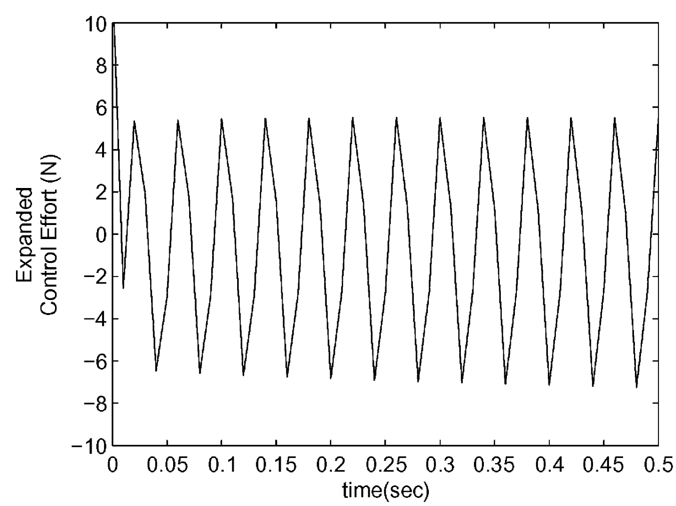

Fig. 19. Portion of the control signal in Fig. 18 with an expanded time axis view.

show the system responses, control efforts and error signal with the proposed adaptive fuzzy controller for $d(t) = 4\cos(5\pi t)$ and $d(t) = 10\cos(5\pi t)$, respectively. From the simulations, it can be seen that the proposed adaptive fuzzy controller is able to maintain system performance amid fast and large levels of disturbances. PI parameters adaptation in [0, 40 s] is depicted in Fig. 14. It can be seen there is a high-frequency sinusoid riding on a lower frequency sinusoidal that is attributed to the disturbance's frequency, which appear as a mode of oscillation.

Case 3: Very Fast Disturbance $(d(t) = a\cos(50\pi t))$*:* Figs. 15 and 16 show the system response, control effort and error signal of the referenced adaptive fuzzy controller in [8], for $d(t) = 4\cos(50\pi t)$ and $d(t) = 10\cos(50\pi t)$, respectively. In comparison, Figs. 17 and 18 show the system responses, control efforts and error signal with the proposed adaptive fuzzy controller for $d(t) = 4\cos(50\pi t)$ and $d(t) = 10\cos(50\pi t)$, respectively. Fig. 19 shows a portion of the control signal

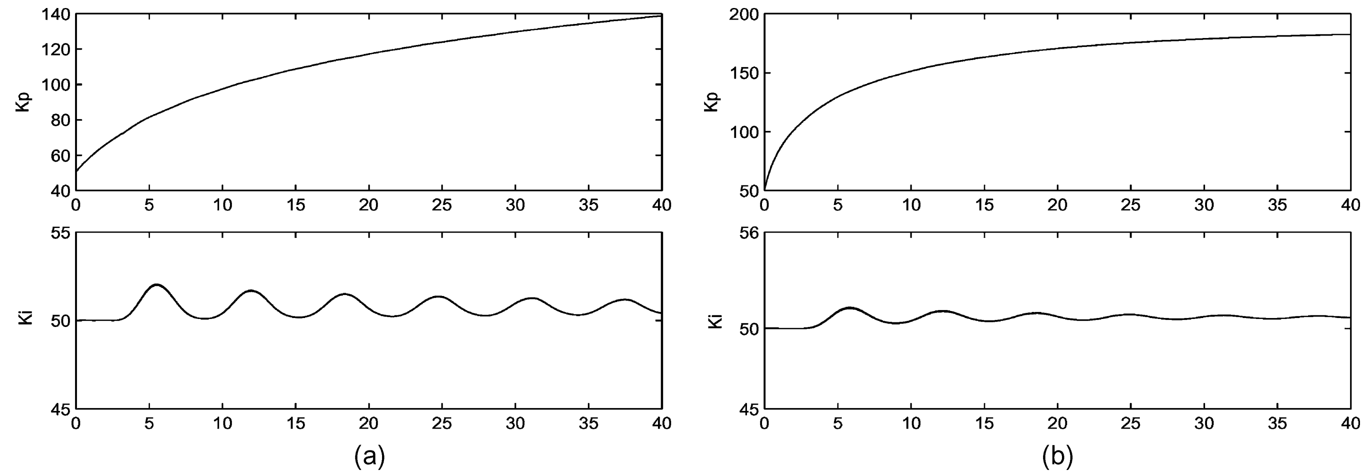

Fig. 20. PI parameters adaptation for case 3: (a) $d(t) = 4\cos(50\pi t)$; (b) $d(t) = 10\cos(50\pi t)$.

of Fig. 18 with an expanded time axis to illustrate the fast adaptation of the control effort with respect to the disturbance signal, i.e., the fast changes in the control effort is not due to chattering. Therefore, if there are such disturbances we need to use actuators that can produce such control efforts. Similar to previous cases, PI parameters adaptation in [0, 40 s] is depicted in Fig. 20.

V. CONCLUSION

Uncertainties could inherently be large, as well as fast. This is while conventional approaches typically require these uncertainties to either be slow enough (such as in adaptive control) or be contained within known bounds (such as in robust control). These, at times unreasonable, assumptions have generally been tolerated in favor of producing rigorous analysis of performance. In this paper, we extend the work of an earlier reference by proposing a PI-adaptive fuzzy controller that is robust in the presence of bounded uncertainties and external disturbances. The difference with previous results is that a bound does not need to be known here and uncertainties can be fast. The adaptive fuzzy control paradigm is used to cope with the lack of system structure and signal uncertainty. And the adaptive PI-type structure is introduced to provide robustness in the presence of large disturbances. From Lyapunov analysis, asymptotic stability is derived. In order to test the performance of the proposed algorithm, it is applied to a nonlinear model of an inverted pendulum amid large system disturbances. Results indicate a significant reduction/elimination of chattering while maintaining asymptotic convergence under various test conditions, i.e., large/small and fast/slow disturbances. In the future we hope to extend this research to uncertain nonlinear systems whose states are not available for exact measurement. Current approaches have attended to lack of direct state measurement by using only output feedback or design of state observers. However, this process can involve a great degree of uncertainty which is often ignored, while exploiting it may be instrumental for better control. There is not much research in literature about the control of such uncertain nonlinear systems. Furthermore, we also plan to broaden this methodology to control of large scale systems.

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