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IDENTIFICATION AND DAHLIN'S CONTROL FOR NONLINEAR DISCRETE TIME OUTPUT FEEDBACK SYSTEMS

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An improved identification and control for discrete time nonlinear system is presented in this paper. This design approach contains two phases i) Parameter identification phase ii) Control phase. Once this identification phase is over, the acquired parameter information can be used to implement any control algorithm. In this paper, Dahlin's control algorithm is used for control, as it is more natural than the existing deadbeat control and a new input selection procedure is also evolved to make the scheme attractive.

K e y w o r d s: active identification, discrete time system, output feedback form, Dahlin's control

1 INTRODUCTION

In recent years [1, 2] a great deal of progress has been made in the area of adaptive control of continuous-time nonlinear systems. In contrast, adaptive control of discrete-time nonlinear systems remains a largely unsolved problem. The few existing results [3–5] can only guarantee global stability under restrictive growth conditions on the nonlinearities, because they use techniques from the literature on adaptive control of continuous time linear systems [6, 7]. In reference [7, 8] that some discrete-time nonlinear systems with unknown parameters and randomly distributed noise are explained but they are not globally stabilized.

The backstepping methodology [1], which provided a crucial ingredient for the development of solutions to many continuous-time adaptive nonlinear problems, has a very simple discrete-time counterpart: one simply "looks ahead" and chooses the control law to force the states to acquire their desired values after a finite number of time steps. One can debate the merits of such a deadbeat control strategy [9], especially for nonlinear systems [10], but it seems that in order to guarantee global stability in the presence of arbitrary non-linearities, any controller will have to incorporate some form of prediction capability.

In the presence of unknown parameters, however, it is nearly impossible to calculate these "look-ahead" values of the states. Furthermore, since these calculations involve the unknown parameters as arguments of arbitrary nonlinear functions, suitable parameters estimation method is applicable, since all of them require a linear parameterization to guarantee global results. This is the biggest obstacle to providing global solutions for any of the more general discrete-time nonlinear problems.

In reference [11], a completely different approach was introduced to obtain a globally stable controller for a large class of discrete time output feed back nonlinear system with unknown parameters without imposing any growth condition on nonlinearities. This result shows that all the parameters information, necessary for control purposes, will be available after $2nr_0$ steps at most, where n is the dimension of the system and r_0 is the dimension of the regressor subspace. If the dimension of the system/regressor subspace increases it takes long time for values of the parameter to converge.

The proposed algorithm overcomes such a difficulty for the convergence of the parameters of the system. The improved scheme suggested to minimize the length of this interval, and hence it is important for transient performance considerations, as this will prevent the state from becoming too large during the identification phase. Once this active identification phase is over, the values of the acquired parameters can be used to implement any control strategy as if the parameters were completely known. Also we have developed a straightforward Dahlin's control algorithm for control purpose. The general block diagram of output feed back system is shown in Fig. 1.

2 PROBLEM FORMULATION

The systems considered in this paper is described in equation (1) which comprise of all systems that can be transformed via a global parameter-independent diffemorphism into the so-called parameter-output-feedback form [4]

$$\begin{aligned} x_1(t+1) &= x_2(t) + \Theta^\top \Psi_1\big(x_1(t)\big) \\ &\vdots \\ x_{n-1}(t+1(&= x_n(t) + \Theta^\top \Psi_{n-1}\big(x_1(t)\big) \\ x_n(t+1) &= u(t) + \Theta^\top \Psi_n\big(x_1(t)\big) \\ y(t) &= x_1(t) \end{aligned} \tag{1}$$

where $\Theta \in R^p$ is a vector of unknown constant parameters and Ψ_i, for $i = 1, \dots, n$ are $p \times 1$ nonlinear vector

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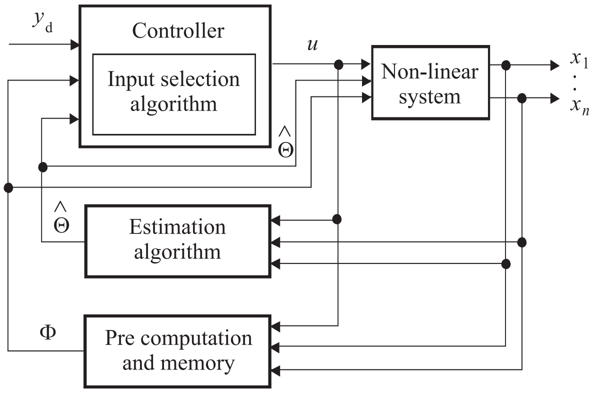

Fig. 1. Block diagram of output feedback system.

functions. The name "parameter-output-feedback-form" denotes the fact that the nonlinearities Ψ_i are multiplied by unknown parameters, which depend only on the output y, which is the only measured variable. The states $x_2, \ldots, x_n$ are not measurable. It is important to note that the functions Ψ_i are not restricted by any type of growth conditions

2.1 A second order example

To illustrate the difficulties present in this problem, let us consider the case when the system is of second order *ie*,

$$\begin{aligned} x_1(t+1) &= x_2(t) + \Theta^\top \Psi_{1,t} \\ x_2(t+1) &= u(t) + \Theta^\top \Psi_{2,t} \\ y(t) &= x_1(t) \end{aligned} \tag{2}$$

and rewrite it in the following scalar form:

$$\begin{aligned} x_1(t+2) &= x_2(t+1) + \Theta^\top \Psi_{1,t+1} = u(t) + \Theta^\top [\Psi_{2,t} + \Psi_{1,t+1}] \\ &= u(t) + \Theta^\top [\Psi_{2,t} + \Psi_1(x_2(t) + \Theta^\top \Psi_{1,t}] = u(t) + \\ &\Theta^\top [\Psi_{2,t} + \Psi_1(u(t-1) + \Theta^\top(\Psi_{2,t-1} + \Psi_{1,t}))]\,. \end{aligned} \tag{3}$$

Even if Θ is known, the control $u(t)$ would only be able to affect the output x_1 at time $t+2$. In other words given any initial conditions $x_1(0)$ and $x_2(0)$, we have no way of influencing $x_1(1)$ through $u(0)$.The best way is drive $x_1(2)$ to zero and keep it there.

2.2 Deadbeat and Dahlin's control law

From the above equations we can write the control equation, the choice of deadbeat control

$$\begin{aligned} u(t) &= y_d(t+2) - \Theta^\top [\Psi_{2,t} + \Psi_{1,t+1}] = y_d(t+2) - \\ &\Theta^\top \big[\Psi_{2,t} + \Psi_1\big(u(t-1) + \Theta^\top(\Psi_{2,t-1} + \Psi_{1,t})\big)\big],\ t \ge 1 \end{aligned} \tag{4}$$

would yield $x_1(t) = y_d(t)$ for all $t \ge 2$ and would achieve the objective of global stabilization. Using the equation (4) the general deadbeat control law can be written as follows:

$$u(t) = y_d(t+n) - \Theta^\top \sum_{k=1}^{n} \Psi_{k,t+n-k} \tag{5}$$

and that will globally stabilize the system equation (1) and yield $x_1(t) = y_d(t),\ t \ge n$. Here the deadbeat control law is used only because it makes the presentation simpler. All the arguments made here are equally applicable to any other discrete-time control strategy; however, from a strictly technical point of view deadbeat control is perfectly acceptable in this case, for the following two reasons.

1. A purely discrete-time problem is only considered. Hence the well-known problems of poor inter-sample behaviour of deadbeat control of discrete-time systems do not arise.
2. Deadbeat control cannot give rise to instability because of the special structure used here, when applied to nonlinear systems.

In this paper, Dahlin's control algorithm has been developed for control, as it is practically implementable compared to the deadbeat control. It can be shown that Dahlin's control law is:

$$\begin{aligned} u(k-n+1) = \frac{1}{\tau}\Big[\big[u(k-n) + \Theta^\top \sum_{i=1}^{n} \Psi_{i,t-i}\big]\big[\tau - \Delta T\big] \\ + \Delta T k_c y_d(k+n)\Big] - \Theta^\top \sum_{i=1}^{n} \Psi_{i,k+n(i+1)} \end{aligned} \tag{6}$$

where, τ is the desired on that response of the time constant, ΔT is the sampling time and k_c is system gain. This control algorithm will yield $x_1(t) = y_d(t),\ t \ge n$. All the arguments made here are equally applicable to any other discrete-time control strategy.

3 ACTIVE IDENTIFICATION SCHEME

Let us now elaborate further on the above outlined approach by presenting in detail the two most challenging ingredients, namely the precomputation scheme and the input selection for active identification. To do this, we return to the general output-feedback form (1) and rewrite it in the following scalar form.

$$\begin{aligned} x_1(t+n) &= x_2(t+n-1) + \Theta^\top \Psi_1\big(x_1(t+n-1)\big) \\ &= x_3(t+n-2) + \Theta^\top \Psi_2\big(x_1(t+n-2)\big) + \\ &\Theta^\top \Psi_1\big(x_1(t+n-1)\big) = \cdots = \\ &u(t) + \Theta^\top \sum_{k=1}^{n} \Psi_k\big(x_1(t+n-k)\big) \end{aligned} \tag{7}$$

In order to implement the control given in equations (5) & (6), we need to calculate (at time t) $\sum_{k=1}^{n} \Psi_{k,t+n-k}$ and then $\Theta^\top \sum_{k=1}^{n} \Psi_{k,t+n-k}$. Since

$$\sum_{k=1}^{n} \Psi_{k,t+n-k} = \sum_{k=1}^{n} \Psi_k\big(x_1(t+n-k)\big) \tag{8}$$

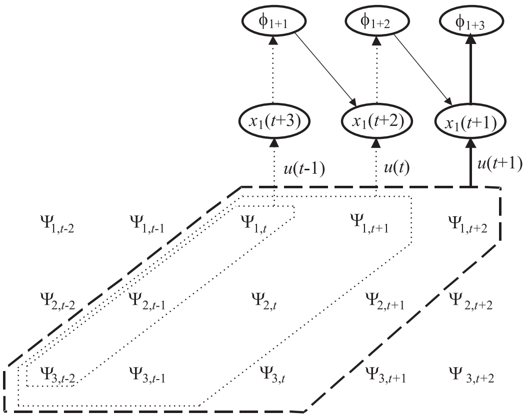

Fig. 2. Precomputation procedure.

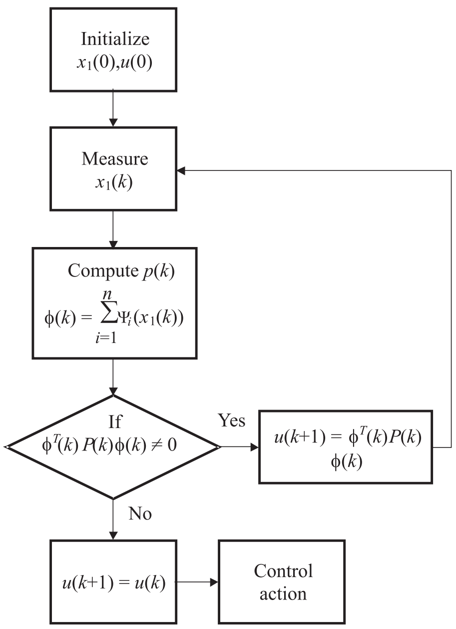

Fig. 3. Input selection algorithm.

the values of $x_1(t+1), \dots, x_1(t+n-1)$ must be computed at time t first. Using the equation (7), we express $x_1(t+1), \dots, x_1(t+n-1)$ as follows:

$$x_1(t+I) = u(t-n+i) + \Theta^\top \sum_{k=1}^{n} \Psi_l\big(x_1(t+i-k)\big), \quad (9)$$

$i = 1, \dots, n-1$. Substituting $i = 1$ into this equation, we see that the value of $x_1(t+1)$ is equal to the sum of $\Theta^\top \sum_{k=1}^{n} \Psi_{k,t+1-k}$ and $u(t-n+1)$. Since both $u(t-n+1)$ and $\sum_{k=1}^{n} \Psi_{k,t+1-k}$ are known at time t, we can precompute (at time t) $x_1(t+1)$, provided we replace Θ by $\widehat{\Theta}$. Next, to examine the calculation of $x_1(t+2)$ at time t. Substituting $i = 2$ in equation (9), we obtain

$$x_1(t+2) = u(t-n+2) + \Theta^\top \sum_{k=1}^{n} \Psi_k\big(x_1(t+2-k)\big). \quad (10)$$

Substitute $n = 2$ and Θ by the latest value $\widehat{\Theta}$

$$x_1(t+2) = u(t) + \Theta^\top \big[\Psi_1\big(x_1(t+1)\big) + \Psi_2\big(x_1(t)\big)\big]. \quad (11)$$

Since $u(t)$ is known at time t, pre-computing $x_1(t+2)$ can be achieved through the calculation of $\Psi_1\big(x_1(t+1)\big)$ and $\Psi_2\big(x_1(t)\big)$ at time t. To calculate $x_1(t+i)$ we modified the existing pre-computational procedure shown in Fig. 2. Equation (9) implies that precomputing this vector provides enough information to determine the values of $x_1(t+1)$, $x_1(t+n-1)$ at time t. To implement the equations (5) and (6), we still need the value of $\sum_{k=1}^{n} \Psi_{k,t+n-k}$. This leads finally to the conclusion that pre-computing at time t, the vector

$$\begin{bmatrix} \widehat{\Theta}^\top \sum_{k=1}^{n} \Psi_{k,t+1-k} \\ \vdots \\ \widehat{\Theta}^\top \sum_{k=1}^{n} \Psi_{k,t+n-k} \end{bmatrix} \quad (12)$$

is sufficient to implement the equations (5) and (6).

3.1 Input selection algorithm

So far we have seen how to precompute the values of the future states and the vectors associated with them. We can also prove the existence of finite time t_{t_f} by using the control input 'u' to drive the output x_1 to values that yield linearly independent directions for the vectors ϕ_i. The input selection takes place wherever necessary during the identification phase, that is, wherever we see that the system will not produce any new directions on it own. The flow chart in Fig. 3 gives detailed information about, how the $u(t)$ is being selected when the identification is carrying on.

Steps for Input Selection Algorithm:

Step 1: At time t, measure $x_1(k)$ and compute ϕ using $\Psi_{1,k}, \Psi_{2,k}, \dots, \Psi_{n,k}$.

Step 2: If $\phi_k^\top P_{k-1}\phi_k \neq 0$ is true updating is needed with update algorithm $u(k+1) = \phi_k^\top P_{k-1}\phi_k$ Go to step 1.

Step 3: Else $u(k+1) = u(k)$ and control action is taken.

3.2 Orthogonalized projection algorithm

Consider the problem of estimating an unknown parameter vector 'Θ' from a simple model of the form

$$y(t) = \phi(t-1)^\top \Theta \quad (13)$$

where, $y(t)$ denotes the scalar system output at time t, and $\phi(t-1)$ denotes a vector that is a linear or nonlinear function of past measurements

$$\begin{aligned} Y(t-1) &= \{y(t-1), y(t-2), \dots\}, \\ U(t-1) &= \{u(t-1), u(t-2), \dots\}. \end{aligned} \quad (14)$$

The orthogonalized projection algorithm for the system designed by equation (13) starts with an initial estimate Θ_0 and the $p \times p$ identity matrix P_{-1}, and then updates the estimate Θ and the covariance matrix P for $t \geq 1$ through the recursive expressions.

At each time t, the only measurement is the output $x_1(t)$.

$$\widehat{\Theta}_{t+1} = \begin{cases} \widehat{\Theta}_t + \frac{P_{t-1}\phi_t}{\phi_t^\top P_{t-1}\phi_t}\big((x_t - \phi_t^\top \widehat{\Theta}_t\big) & \text{if } \phi_t^\top P_{t-1}\phi_t \neq 0\,, \\ \widehat{\theta} & \text{if } \phi_t^\top P_{t-1}\phi_t = 0\,, \end{cases} \quad (15)$$

$$P_t = \begin{cases} P_{t-1} - \frac{P_{t-1}\phi_t\phi_t^\top P_{t-1}}{\phi_t^\top P_{t-1}\phi_t} & \text{if } \phi_t^\top P_{t-1}\phi_t \neq 0\,, \\ P_{t-1} & \text{if } \phi_t^\top P_{t-1}\phi_t = 0\,. \end{cases} \quad (16)$$

Thus, (15) and (16) are employed to compute recursively the estimate Θ_t and the covariance matrix P_t. This algorithm has the useful properties, which are given here without proof [11].

3.3 Recursive least square algorithm

The recursive least squares estimate of the unknown parameters Θ is

$$\widehat{\Theta}_{t+1} = \widehat{\Theta}_t + \frac{P_{t-1}\phi_t}{1 + \phi_t^\top P_{t-1}\phi_t}\big(x_{1,t+1} - u_{t-1} - \widehat{\Theta}_t^\top \phi_t\big) \quad (17)$$

in which P_t is computed through

$$P_t = P_{t-1} - \frac{P_{t-1}\phi_t\phi_t^\top P_{t-1}}{1 + \phi_t^\top P_{t-1}\phi_t}\,. \quad (18)$$

A standard approach to determine the control input would be to use the estimate (18) in a certainty-equivalence control law:

$$u(t) = -\widehat{\Theta}_t^\top \phi_{t-1} = -\widehat{\Theta}_t^\top t\big[\Psi_{1(x_{1,t-1})} + \Psi_{2(x_{1,t-2})}\big]. \quad (19)$$

3.4 Steps for Active Identification (Precomputation and Input Selection) and Control

For convenience the algorithm given is based on the second order system. The steps are as follows:

Step: 1 Initialize $x_1(0), x_2(0), u(0), \widehat{\Theta}(0), P(0), k$

Step: 2 Measure $x_1(k)$. Because output $y = x_1$, which is the only measured variable: the state $x_2, \ldots, x_n$ are not measurable.

Step: 3 Calculate $\Psi_1\big(x_1(k)\big), \Psi_2\big(x_1(k)\big)$.
Compute regressor subspace
$\phi(k) = \sum_{i=1}^{n} \Psi_I\big(x_1(t+n-k)\big)$

Step: 4 Evaluate the value of $x(k), \hat{x}(k)$ using the following relationship
$x(k) = \Theta^\top \phi(k)$ and $\hat{x}(k) = \widehat{\Theta}^\top(k)\phi(k)$

Step: 5 Calculate the residue $e(k) = x(k) - \hat{x}(k)$ also calculate 'D' using regressor & covariance matrix 'P'. where, $D = \phi^\top(k)P(k)\phi(k)$

Step: 6 Check $\phi^\top P(k)\phi(k) \neq 0$ if this is true, calculate the following
$P(k+1)$, $\widehat{\Theta}(k+1)$, $u(k+1) = D$, go to step: 7
else
Updating the system parameters and controller output is not needed. $P(k+1) = P(k)$ and $\widehat{\Theta}(k+1) = \widehat{\Theta}(k)$

Step: 7 Check abs$\big(\Theta - \widehat{\Theta}\big) < 0.001$ if it is true
Calculate
$u(k+1)$ (for different control strategy)
$x_1(k) = u(k) + \Theta^\top \Psi_{1,k}$
else
$x_1(k) = x_2(k) + \Theta^\top \Psi_1\big(x1(k)\big)$
Calculate
$x_2(k+1) = u(k) + \Theta^\top \Psi_2\big(x1(k)\big)$
Update $k = k+1$, then go to step: 2

Flowchart representation of the algorithm is shown in Fig. 4.

4 SIMULATION RESULTS

4.1 Identification and Deadbeat Control

In order to evaluate the performance of the active identification procedure and Deadbeat controller design, let us consider the following example:

$$\begin{aligned} x_1(k) &= x_2(k) + \Theta^\top \Psi_1\big(x_1(k)\big) + \alpha\xi_t \\ x_2(k+1) &= u(k) + \Theta^\top \Psi_2\big(x_1(k)\big) + \alpha\zeta_t \\ y(t) &= x_1(t) \end{aligned} \quad (20)$$

where $\Theta^\top = [1 \ \ 0.8 \ \ -1 \ \ 1]$ is the unknown parameter and Ψ_1, Ψ_2 are nonlinear function of x_1.

$$\begin{aligned} \Psi_1(x_1) &= \big[0 \ \ 0 \ \ \log(1 + (x_1^2 - 2x_1)^2 \ \ x_1 \sin(x_1 - 2)\big]^\top \\ \Psi_2(x_1) &= \big[(e^{x_1} - 1)\sin(x_1 - 2)/(x12 + 3) \\ &\qquad (x_1 - 0.5x_2)\sin x_1 \ \ 0 \ \ 0\big]^\top \end{aligned} \quad (21)$$

and α is a constant. In order to illustrate the robustness of the modified algorithm with respect to additive stochastic disturbances (ξ_t, ζ_t). For a fixed value of t, ξ_t, ζ_t are uniform random variables. Further the sequence ξ_t, ζ_t are mutually independent.

To illustrate the performance of the modified algorithm we present the simulation result for two different choices of α ($\alpha = 0$; $\alpha = 0.05$). To demonstrate the algorithm we provide the explicit calculations for the intermediate steps of this procedure when system is noise free ($\alpha = 0$) and initial condition $(1.826, 2.115)$ is chosen.

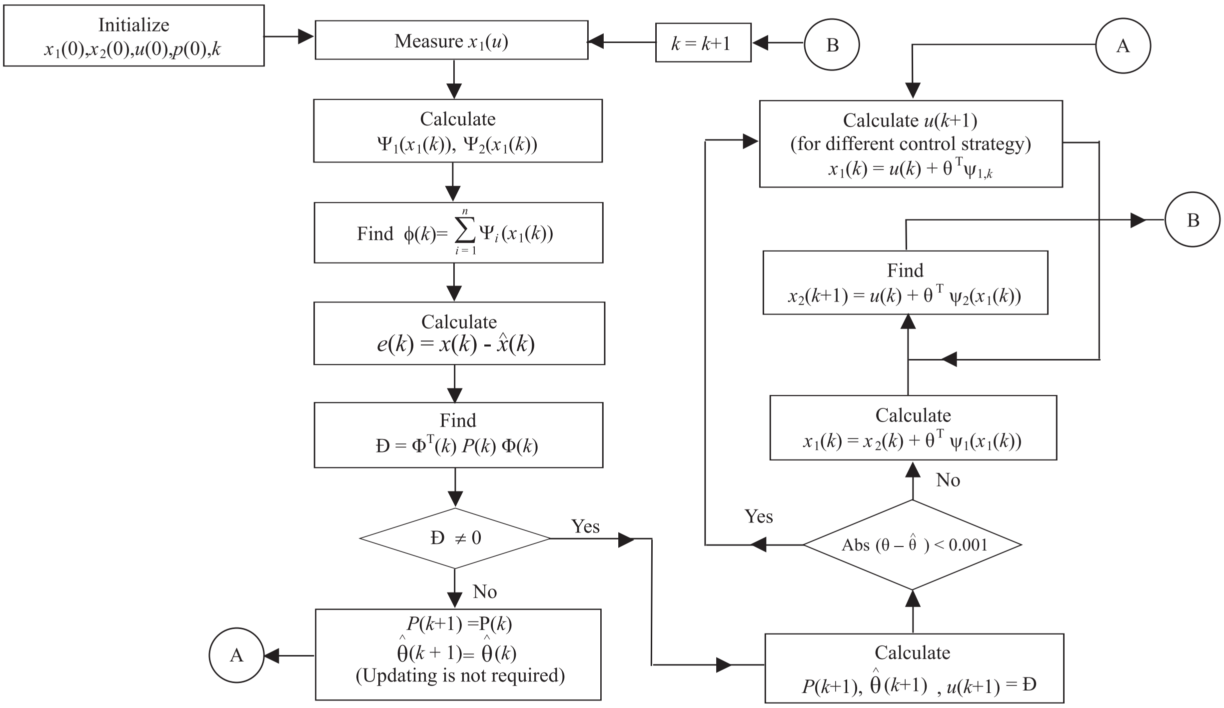

Fig. 4. Identification and Input Selection Algorithm.

Case 1

Assuming the initial conditions is $(1.826, 2.115)$ for the simulation purpose. The estimator starts with $\Theta_0 = [1\ \ 1\ \ 1\ \ 2]^\top$ and $P_0 = I$, where I denotes a 4×4 identity matrix.

At $t=0$, measure $x_1(0) = 1.826$;
$\Psi_{1,0} = [0\ \ 0\ \ 0.0962\ \ -0.3161]$,
$\Psi_{2,0} = [-0.1230\ \ 0.1537\ \ 0\ \ 0]$.
Assume $u(0) = 0$.

- At time $t = 1$ measure $x_1(1) = 1.7027$; evaluate $\Psi_{1,1} = \Psi_{2,1} = [0\ \ 0\ \ 0\ \ 0]$. Using this $\Psi_{1,1} + \Psi_{2,0} = [-0.1230\ \ 0.1537\ \ 0\ \ 0]$. We see that $\widehat{\Theta}(1) = \widehat{\Theta}(0)$ and $P(1) = P(0)$. As we have not yet collected information to choose $u(1)$, we choose $u(1) = 0$, by default.
- At time $t = 2$ $u(1) = 0$; evaluate
$\Psi_{1,2} = [0\ \ 0\ \ 0.2281\ \ -0.3988]$;
$\Psi_{2,2} = [-0.1906\ \ 0.2509\ \ 0\ \ 0]$.
Using this $\Psi_{1,2} + \Psi_{2,1} = [0\ \ 0\ \ 0.2281\ \ -0.3988]$;
$\phi(2) = [-0.1230\ \ 0.1537\ \ 0.2281\ \ -0.3988]$;
$$P(2) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0.9153 & 0.2785 \\ 0 & 0 & 0.2785 & 0.0847 \end{bmatrix},$$

$\widehat{\Theta}(2) = [1\ \ 1\ \ 1.109\ \ 1.6316]$;
$u(2) = 0.1092$; $x_1(2) = -0.7269$; $x_2(2) = 0.0101$;

- At $t = 3$; $\Psi_{1,3} = [0\ \ 0\ \ 1.5951\ \ 0]$;
$\Psi_{2,3} = [0.0360\ \ 0.6586\ \ 0\ \ 0]$; Using this $\Psi_{1,3} + \Psi_{2,2} = [-0.1906\ \ 0.2509\ \ 1.5951\ \ 0.2929] = \phi(3)$;
$$P(3) = \begin{bmatrix} 0.6571 & 0.4287 & 0.1950 & 0.0593 \\ 0.4287 & 0.4640 & -0.2438 & -0.0742 \\ 0.1950 & -0.2438 & 0.8044 & 0.2447 \\ 0.0593 & -0.0742 & 0.2447 & 0.0745 \end{bmatrix}$$
$\widehat{\Theta}(3) = [1.535\ \ 0.331\ \ 0.8038\ \ 1.5391]$;
$u(3) = 0.0331$;
$x_1(3) = -1.2921$; $x_2(3) = 0.6820$;
- At $t = 4$; $\Psi_{1,4} = [0\ \ 0\ \ 2.9397\ \ -0.1938]$;
$\Psi_{2,4} = [-0.0192\ \ 2.0338\ \ 0\ \ 0]$; $\phi(4) = \Psi_{1,4} + \Psi_{2,4} = [0.0360\ \ 0.6586\ \ 2.9393\ \ -0.1938]$;
$$P(4) = \begin{bmatrix} 0.6079 & 0.4882 & -0.0039 & 0.0012 \\ 0.4482 & 0.3921 & -0.0032 & -0.0010 \\ -0.0039 & -0.0032 & 0.0000 & 0.0000 \\ -0.0012 & -0.0010 & 0.0000 & 0.0000 \end{bmatrix},$$
$\widehat{\Theta}(3) = [1.0886\ \ 0.8711\ \ -1.006\ \ 0.9998]$;
$u(4) = 1.9627$; $x_1(4) = -2.3611$; $x_2(4) = 1.6608$;
- At $t = 5$; $\Psi_{1,5} = [0\ \ 0\ \ 3.8003\ \ -2.3839]$;
$\Psi_{2,5} = [-0.0881\ \ 3.3539\ \ 0\ \ 0]$; $\phi(4) = \Psi_{1,5} + \Psi_{2,3} = [-0.0192\ \ 2.0338\ \ 3.8003\ \ -2.3839]$;
$$P(5) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \widehat{\Theta}(5) = [1\ \ \ 0.8\ \ \ -1.0\ \ \ 1];$$
$u(5) = 0.0188$; yields $x_1(5) = 0.00$
- For time $t > 5$ repeat the process presented for $t = 5$ *ie*, precomputed the next output and corresponding regressor vector and then choose the control input to drive $x_1(t+2) = 0$ for any $t > 5$;

Case 2

The values of α is changed to $\alpha = 0.05$, so that the system is perturbed by small additive random noise.

Figures 5–12 shows the simulation results corresponding to either noise free or small additive random noise of the given system which starts from initial conditions

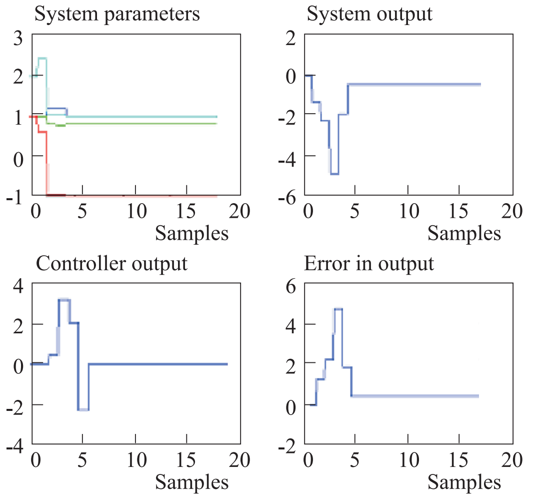

Fig. 5. Identification using Orthogonal projection algorithm ($x_1(0) = 0.5$, $x_2(0) = 0$) and $\alpha = 0$.

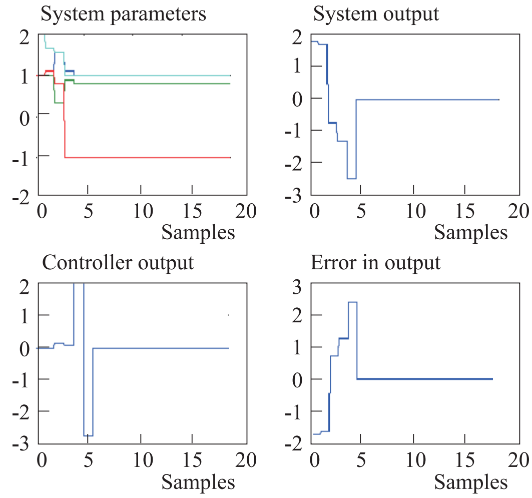

Fig. 6. Identification using Orthogonal projection algorithm ($x_1(0) = 1.826$, $x_2(0) = 2.115$) and $\alpha = 0$.

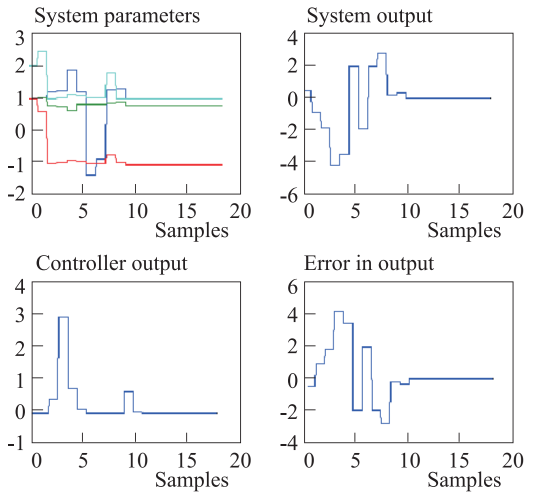

Fig. 7. Identification using Orthogonal projection algorithm (($x_1(0) = 0.5$, $x_2(0) = 0$) and $\alpha = 0.05$.

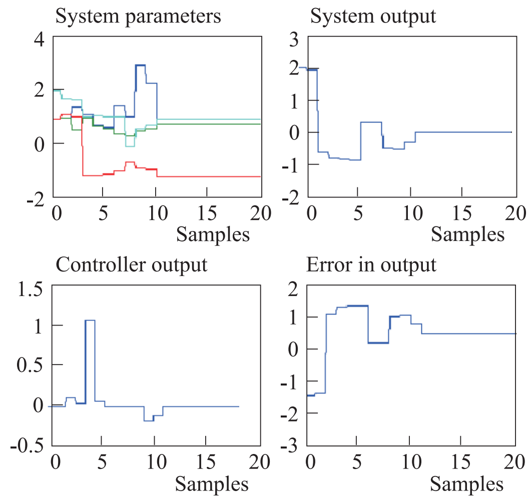

Fig. 8. Identification using Orthogonal projection algorithm ($x_1(0) = 1.826$, $x_2(0) = 2.115$) and $\alpha = 0.05$.

$(0.5, 0)$ or $(1.826, 2.115)$. For the purpose of illustrating the advantage of proposed method, these simulation results are compared with two different identification strategies. The identification procedure is used for parameter estimation then the control law is used for stabilization. The resulting system parameters (Θ), control input (u), output (y) and its corresponding error are shown in subplots 1 to 4 of Fig. 5–13.

The conclusions that can be derived from Fig. 5–13 can be summarized as follows.

1. When the system is noise free $(\alpha = 0.0)$ and starts with initial condition $(0.5, 0)$ or $(1.826, 2.115)$, then the orthogonalized projection algorithm and RLS approaches will stabilize the system. However, the transient performance of orthogonalized projection is significantly better than RLS technique. This is illustrated in Fig. 5, 6 and Fig. 9, 10. From the simulation results it is observed that all the parameter information necessary for control purposes available less than $2nr_0$ steps. In this work, identification phase is completed almost 5^{th} instant. In literature [11] it is shown that in the absence of noise RLS technique will result in instability for large initial condition. This was overcome by the proposed method. But the system parameter takes long time to convergence (sluggish in nature). From the results the input selection algorithm developed is superior to existing algorithm.
2. In presence of small additive random noise both orthogonalized projection algorithm and RLS approaches maintains its stabilization properties, there by exhibiting some degree of robustness. This is illustrated in Fig. 7, 8, and Fig 11, 12. From the results we observe that an orthogonalized projection algorithm has better transient properties and all the parameter information necessary for control purposes are available almost 10^{th} sample instant.

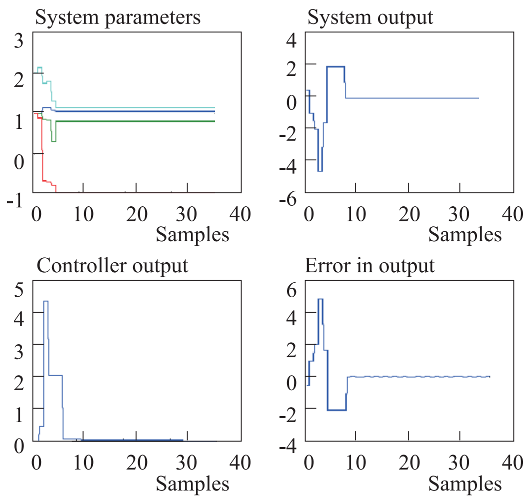

Fig. 9. Identification using RLS algorithm ($x_1(0) = 0.5$, $x_2(0) = 0$) and $\alpha = 0$.

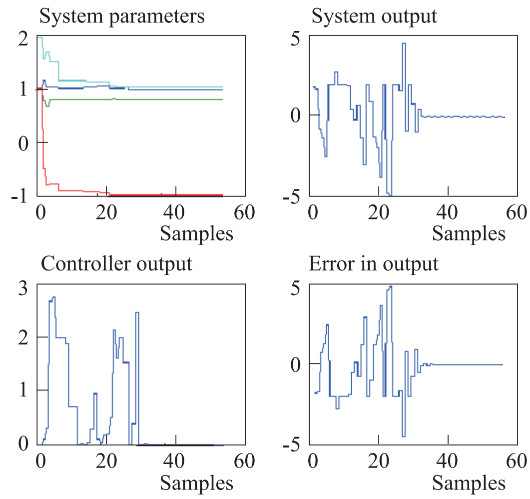

Fig. 10. Identification using RLS algorithm ($x_1(0) = 1.826$, $x_2(0) = 2.115$) and $\alpha = 0$.

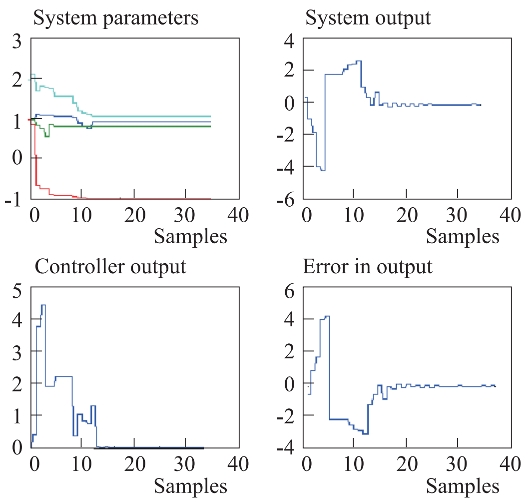

Fig. 11. Identification using RLS algorithm ($x_1(0) = 0.5$, $x_2(0) = 0$) and $\alpha = 0.05$.

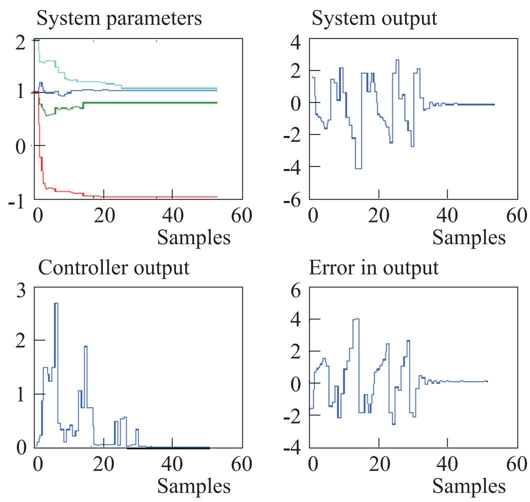

Fig. 12. Identification using RLS algorithm ($x_1(0) = 1.826$, $x_2(0) = 2.115$) and $\alpha = 0.05$.

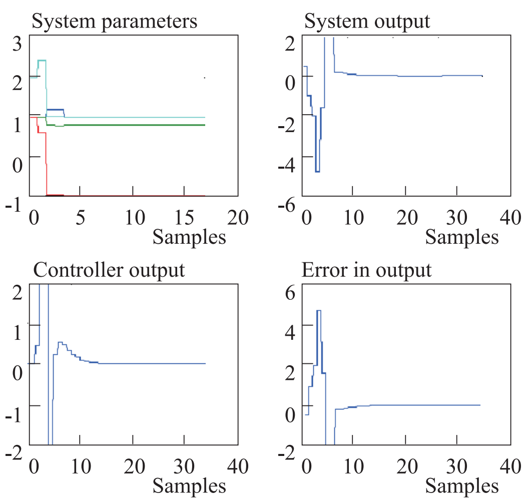

Fig. 13. Identification using Orthogonal projection algorithm ($x_1(0) = 0.5$, $x_2(0) = 0$) and $\alpha = 0$.

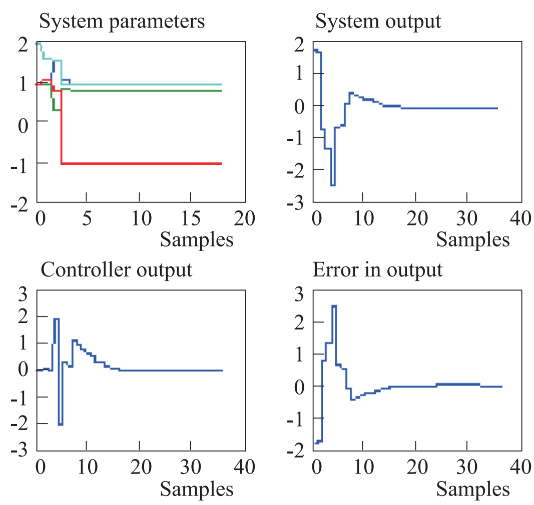

Fig. 14. Identification using Orthogonal projection algorithm ($x_1(0) = 1.826$, $x_2(0) = 2.115$) and $\alpha = 0$.

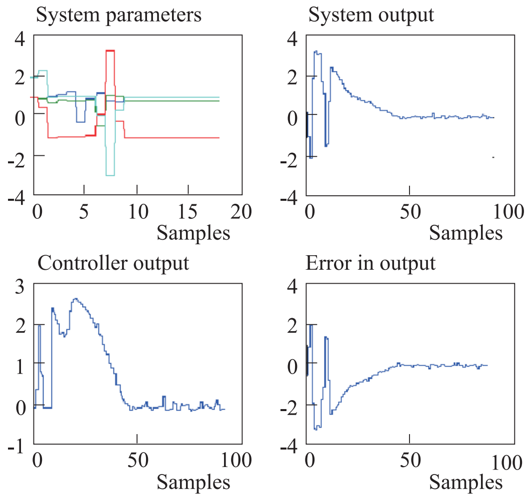

Fig. 15. Identification using Orthogonal projection algorithm ($x_1(0) = 0.5,\ x_2(0) = 0$) and $\alpha = 0.05$.

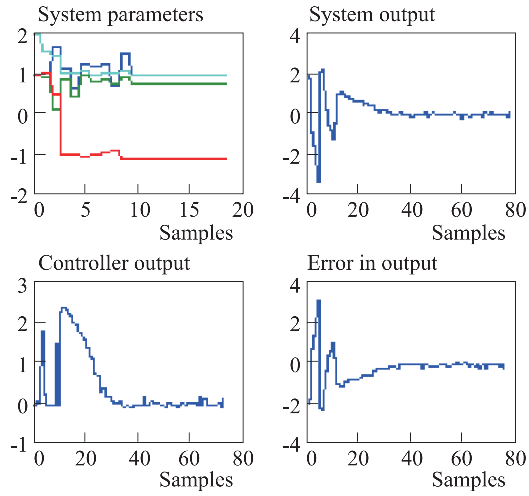

Fig. 16. Identification using Orthogonal projection algorithm ($x_1(0) = 1.826,\ x_2(0) = 2.115$) and $\alpha = 0.05$.

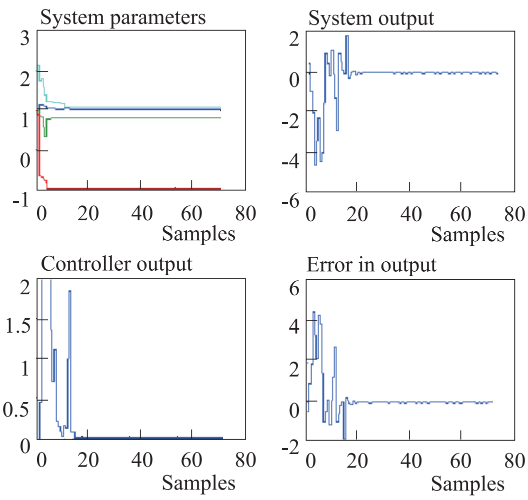

Fig. 17. Identification using RLS algorithm ($x_1(0) = 0.5,\ x_2(0) = 0$) and $\alpha = 0$.

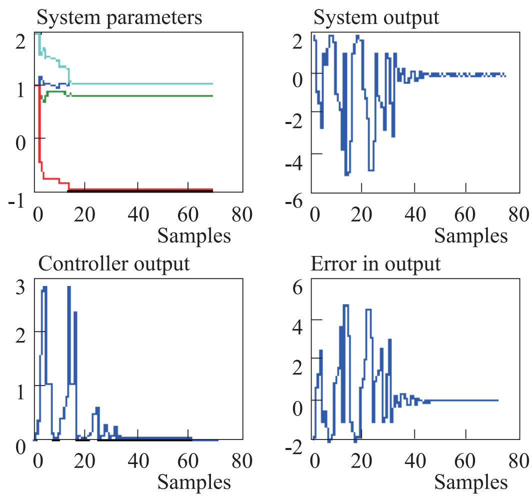

Fig. 18. Identification using RLS algorithm ($x_1(0) = 1.826,\ x_2(0) = 2.115$) and $\alpha = 0$.

4.2 Identification and Dahlin's Control

To illustrate the performance of the modified algorithm we present the simulation result for the same example with two different values of α ($\alpha = 0$; $\alpha = 0.05$). Assuming the initial conditions $(0.5, 0)$, $(1.826, 2.115)$, and $u(0) = 0$ for the simulation is carried out. Also $\tau = 1$, $\Delta T = 0.1$ and $k_c = 1$.The estimator starts with $\widehat{\Theta}_0 = [1\ \ 1\ \ 1\ \ 2]^\top$ and $P_0 = I$, where I denotes a 4×4 identity matrix.

- For time $t > 5$ repeat the process presented for $t = 5$ *ie*, precomputed the next output and corresponding regressor vector and then choose the control input to drive $x_1(t+2) = 0$ for any $t > 5$;

The values of α is changed to $\alpha = 0.05$, so that the system is perturbed by small additive random noise. Figures 13–20 shows the simulation results corresponding to either noise free or small additive random noise of the given system which starts from initial conditions $(0.5, 0)$ or $(1.826, 2.115)$. For the purpose of illustrating the advantage of proposed method, these simulation results are compared with two different identification strategies. The identification procedure is used for parameter estimation then the control law is used for stabilization. The conclusions that can be derived from Fig. 13–20 can be summarized as follows.

1. When the system is noise free ($\alpha = 0.0$) and starts with initial condition $(0.5, 0)$ or $(1.826, 2.115)$, then both the orthogonalized projection algorithm and RLS algorithm stabilize the system. This is illustrated in Fig. 13, 14 and Fig. 17, 18 (with subplots).
2. When some small noise (0.05) is added, both the algorithm gives stabilized output. However orthogonal projection algorithm settles down faster than the other one. This is observed in Fig. 15, 16 and Fig. 19, 20 (with subplots).
3. In this section Dahlin's control algorithm was used for control purpose after suitable adaptation. From the

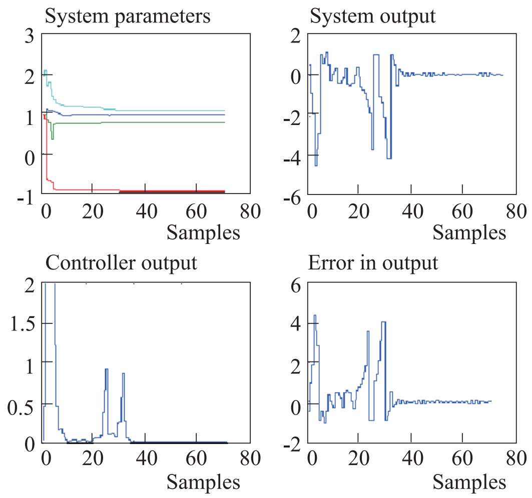

Fig. 19. Identification using RLS algorithm ($x_1(0) = 0.5$, $x_2(0) = 0$) and $\alpha = 0.05$.

simulated result it may be noted that it is practically implementable compared to the existing deadbeat control.

5 CONCLUSION

In this paper an improved identification and input selection algorithm for nonlinear discrete time output feedback system is presented. This active identification is obtained in a finite $(2r_0/2n)$ interval. Once the active identification is over the acquired parameter information can be used to implement any control algorithm as if the parameter were completely known. The input selection procedure developed in this paper guarantees that the active identification is carried out in finite duration. Also we developed a Dahlia's strategy for control purpose that is useful for practical implementation. In order to prove the proposed algorithm is robust a small additive random noise to the system is introduced and from the result we observed that the identification phase is completed faster than existing scheme.

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