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Automatic analogue modulation recognition

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Abstract

For several reasons, modulation recognition is extremely important in communication intelligence (COMINT). In this paper, a global procedure for recognition of analogue modulation types is developed. Computer simulations for different types of band-limited analogue modulated signals corrupted by band-limited Gaussian noise have been carried out. Expressions for the instantaneous amplitude and phase as well as the Fourier transform of different analogue modulation types are derived and used to set up a recognition procedure. It is found that all types of analogue modulation have been classified with success rate $\geqslant 90\%$ at SNR = 10 dB.

Zusammenfassung

Aus verschiedenen Gründen ist die Modulationserkennung in der Kommunikationsüberwachung äußerst wichtig. In diesem Beitrag wird ein globales Verfahren zur Erkennung analoger Modulationsarten entwickelt. Rechnersimulationen mit bandbegrenzten, analog modulierten Signalen in bandbegrenztem, Gauß'schem Störrauschen sind durchgeführt worden. Ausdrücke für momentane Amplitude und Phase sowie für die Fouriertransformierte verschiedener analoger Modulationsarten werden hergeleitet und zur Konstruktion einer Erkennungsprozedur verwendet. Man findet, daß alle Arten analoger Modulation sich mit einer Trefferquote von mindestens 90% bei einem Störabstand von 10 dB klassifizieren lassen.

Résumé

Pour plusieurs raisons, la reconnaissance de modulation est extrêmement importante dans l'intelligence de communication (COMINT). Dans cet article, une procédure globale de reconnaissance des types de modulation analogique est développée. Des simulations informatiques ont été effectuées pour différents types de signaux modulés analogiques et à bande limitée, corrompus par du bruit gaussien à bande limitée. Les expressions de l'amplitude et de la phase instantanées de même que la transformée de Fourier de plusieurs types de modulation analogique sont dérivées et utilisées pour établir une procédure de reconnaissance. Les simulations montrent que tous les types de modulation analogique ont été classés avec un taux de succès $\geqslant 90\%$ pour un SNR de 10 dB.

Keywords: Analogue modulation; Modulation recognition; Signal classification

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1. Introduction

Communication signals can have different modulation types as well as different frequencies that fall in a very wide band (HF and VHF). Usually, it is required to identify and monitor these signals, either for military or for civilian purposes. In military, there are many applications such as surveillance, electronic warfare and threat analysis. In surveillance applications, the first objective is to recognize the signal characteristics of intercepted emitters against a catalog of reference characteristics or signal sorting parameters. One of the important characteristics is the signal modulation type which describes how the information is being carried in the signal. In civilian, there are many applications such as signal confirmation, interference identification, monitoring and spectrum management.

Modulation recognition is extremely important in COMINT applications for several reasons: Applying the signal to an improper demodulator may damage the signal information content. It is worth noting that any damage of the signal information content considerably confuses the following deciphering process which converts the demodulated message from its ciphered (non-intelligible) form to the deciphered (intelligible) one. Furthermore, knowing the correct modulation type helps to recognize the threat and determine the suitable jamming waveform.

Until now there are two main approaches to the modulation recognition process [1–8, 10–13]. The first is a decision theoretic approach in which probabilistic and hypothesis testing arguments are employed to formulate the recognition problem. The other approach is that of statistical pattern recognition, where the classification system is divided into two subsystems. The first is a feature extraction subsystem, whose function is to extract pre-defined features from the received data. The second subsystem is a pattern recognition subsystem, whose function is to indicate the modulation type. Current modulation recognition algorithms have been developed [1–8, 10–13] according to either approach. There are also some algorithms that combine both approaches. This paper uses the first approach.

2. Proposed modulation recognition procedure

In this paper a global procedure is developed to decide reliably whether the intercepted signal contains the information in the instantaneous amplitude, in the instantaneous phase, in the instantaneous frequency, or in a combination of them. Additionally, a criterion is developed to judge the quality of different segments of the intercepted signal according to their suitability for modulation recognition. The proposed global procedure for analogue modulation recognition comprises the following steps.

Let the duration of the intercepted signal, $x_c(t)$ be K seconds. Sampling at the rate f_s higher than four times the maximum frequency in the signal spectrum, and grouping the samples into successive sets (frames) of N_s samples result in M $(= Kf_s/N_s)$ frames. In order to classify the intercepted signal according to its information content and to discriminate good frames from bad ones, it is suggested to proceed as follows:

(a) From every available frame decide about the type of modulation. The key features proposed for this classification are derived from the instantaneous amplitude $A(t)$ and the instantaneous phase $\phi(t)$ of the intercepted signal (see Appendix A) as well as the signal spectrum [14].

The first key feature, $\gamma_{\max}$, is defined by

$$\gamma_{\max} = \max|\mathrm{DFT}(A_{\mathrm{cn}}(i))|^2, \tag{1}$$

where $A_{\mathrm{cn}}(i)$ is the value of the normalized centered instantaneous amplitude at time instants $t = i/f_s$ $(i = 1, 2, \ldots, N_s)$ and it is defined by

$$A_{\mathrm{cn}}(i) = A_{\mathrm{n}}(i) - 1, \quad \text{where } A_{\mathrm{n}}(i) = \frac{A(i)}{m_a}, \tag{2}$$

where m_a is the average value of the instantaneous amplitude over one frame, i.e.,

$$m_a = \frac{1}{N_s} \sum_{i=1}^{N_s} A(i). \tag{3}$$

Normalization of the instantaneous amplitude is necessary in order to compensate the channel gain. Thus, $\gamma_{\max}$ represents the maximum value of the spectral power density of the normalized centered instantaneous amplitude of the intercepted signal.

The second key feature, σ_{ap}, is defined by

$$\sigma_{ap} = \left[\frac{1}{C}\left(\sum_{A_n(i) > a_t} \phi_{NL}^2(i)\right) - \left(\frac{1}{C}\sum_{A_n(i) > a_t} |\phi_{NL}(i)|\right)^2\right]^{1/2}, \tag{4}$$

where $\phi_{NL}(i)$ is the value of the non-linear component of the instantaneous phase at time instants $t = i/f_s$, a_t is a threshold for $A(t)$ below which the phase is assigned to be $\pi/2$; below this threshold the signal is very sensitive to noise and this affects the evaluation of the instantaneous phase, and C is the number of samples in $\{\phi_{NL}(i)\}$ for which $A_n(i) > a_t$. Thus, σ_{ap} is the standard deviation of the absolute value of the non-linear component of the instantaneous phase, evaluated over the non-weak segments of the signals.

The third key feature, σ_{dp}, is defined by

$$\sigma_{dp} = \left[\frac{1}{C}\left(\sum_{A_n(i) > a_t} \phi_{NL}^2(i)\right) - \left(\frac{1}{C}\sum_{A_n(i) > a_t} \phi_{NL}(i)\right)^2\right]^{1/2}. \tag{5}$$

Thus, σ_{dp} is the standard deviation of the non-linear component of the direct (not absolute) instantaneous phase evaluated over the non-weak segments of the signals.

The fourth key feature is used for measuring the spectrum symmetry around the carrier frequency, and it is based on the spectral powers for lower and upper sidebands. So, it is defined as

$$P = \frac{|P_L - P_U|}{P_L + P_U}, \tag{6}$$

where

$$P_L = \sum_{i=1}^{f_{cn}} |X_c(i)|^2, \tag{7}$$

$$P_U = \sum_{i=1}^{f_{cn}} |X_c(i + f_{cn} + 1)|^2 \tag{8}$$

and f_{cn} is the sample number corresponding to the carrier frequency.

(b) As it is possible to obtain different classifications of these M-frames, the majority logic rule is applied; i.e. select the classification with largest number of repetitions. If two or more classifications have equal maximum numbers of repetitions, they are regarded as candidates for the optimal decision. In this case it is necessary to continue as follows:

(1) Group the frames corresponding to each of the candidate decision.
(2) Determine for every frame within a group the number of samples of $A(i)$ falling below the threshold a_t. Evaluate the total numbers of these samples over the group.
(3) Adopt the decision whose corresponding group has the minimum number of samples falling below the threshold a_t.

The discrimination between the different types of analogue modulation is carried out according to the following rules:

(1) Compare γ_{max}, which is defined by Eq. (1), with a suitable threshold $t_{\gamma_{max}}$ such that

$$\gamma_{max} \underset{\text{FM}}{\overset{\text{AM, DSB, VSB, LSB, USB, Combined(AM − FM)}}{\gtrless}} t_{\gamma_{max}} \tag{9}$$

where $t_{\gamma_{max}}$ is a threshold value of γ_{max} that can be used to discriminate between the signals that have amplitude information ($\gamma_{max} \geqslant t_{\gamma_{max}}$) and that have no amplitude information ($\gamma_{max} \leqslant t_{\gamma_{max}}$).

(2) Compare σ_{ap}, which is defined by Eq. (4), with a suitable threshold $t_{\sigma_{ap}}$ such that

$$\sigma_{ap} \underset{\text{AM, DSB, VSB}}{\overset{\text{LSB, USB, Combined(AM − FM)}}{\gtrless}} t_{\sigma_{ap}}, \tag{10}$$

where $t_{\sigma_{ap}}$ is a threshold value of σ_{ap} that can be used to discriminate between the signals that have absolute phase information ($\sigma_{ap} \geqslant t_{\sigma_{ap}}$) and that have no absolute phase information ($\sigma_{ap} \leqslant t_{\sigma_{ap}}$).

(3) Compare σ_{dp}, which is defined by Eq. (5), with a suitable threshold $t_{\sigma_{dp}}$ such that

$$\sigma_{dp} \underset{\text{AM, VSB}}{\overset{\text{DSB}}{\gtrless}} t_{\sigma_{dp}}, \tag{11}$$

where $t_{\sigma_{dp}}$ is a threshold value of σ_{dp} that can be used to discriminate between the signals that have direct (not absolute) phase information ($\sigma_{dp} \geqslant t_{\sigma_{dp}}$) and that have no direct phase information ($\sigma_{dp} \leqslant t_{\sigma_{dp}}$).

(4) Compare the ratio P, which is defined by Eq. (6), with a suitable threshold t_p. It is well known that P at infinite SNR is ought to be **1** for SSB

signal, and **0** for AM, DSB and MASK signal and the VSB is an intermediate type between the AM and the SSB from the spectrum point of view. So, it is suggested that the t_p is chosen between 0 and 1. So, by using this rule, it is possible to discriminate the VSB signal from the AM one and also, it is possible to discriminate the SSB signal from the combined (AM–FM) one

$$P \underset{\text{AM}}{\overset{\text{VSB}}{\gtrless}} t_p \qquad (12)$$

and

$$P \underset{\text{Combined(AM–FM)},}{\overset{\text{LSB, USB}}{\gtrless}} t_p \qquad (13)$$

where t_p is a threshold value of P that can be used to discriminate between the perfectly symmetric signal ($P \leqslant t_p$) and the asymmetric signals ($P \geqslant t_p$).

(5) Compare P_L with P_U to discriminate the LSB signal from the USB one as

$$P_L \underset{\text{USB}}{\overset{\text{LSB}}{\gtrless}} P_U. \qquad (14)$$

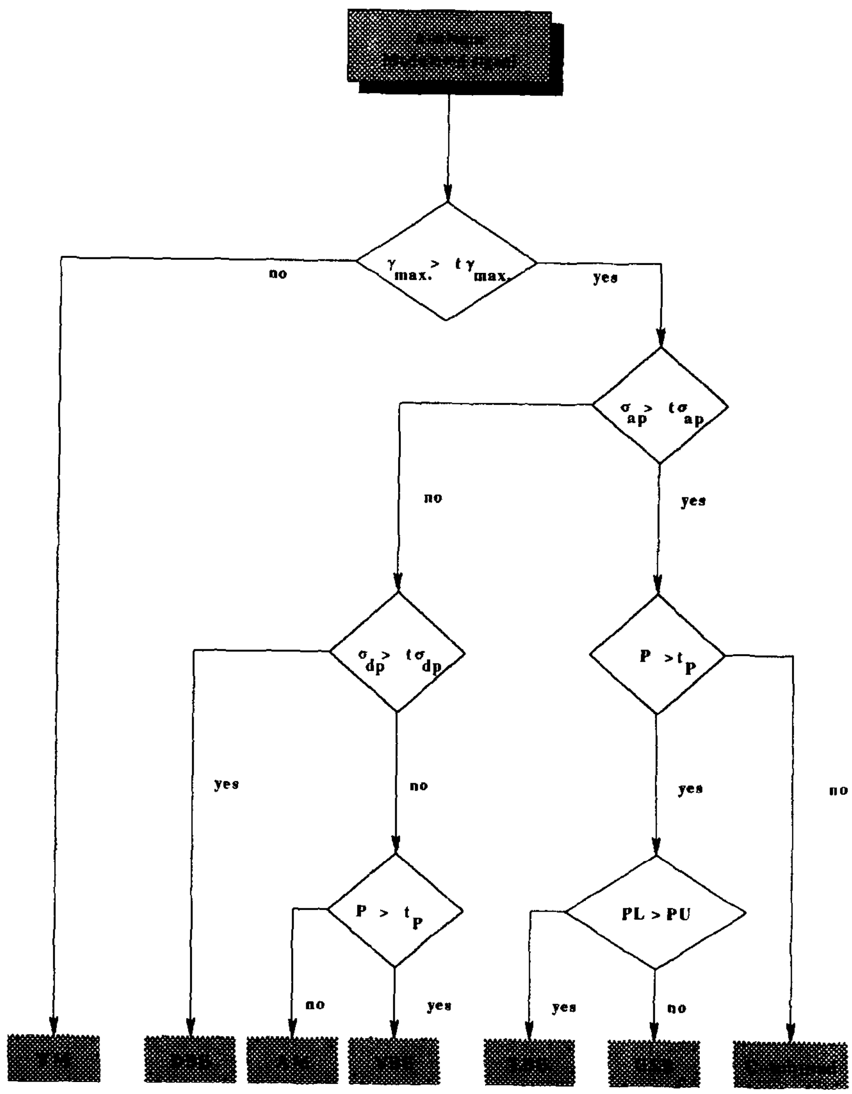


Fig. 1. Functional flowchart for automatic recognition of analogue modulations.

It is worth noting that all these thresholds are chosen such that optimal correct decision for all types of modulation can be obtained. Also, the values of these thresholds are determined from 400 realizations for each type of modulations of interest. The values of the $t_{\gamma_{\max}}$, $t_{\sigma_{ap}}$, $t_{\sigma_{dp}}$ and t_p have been chosen to be 12 000, $\pi/7$, $2\pi/7$ and 0.5, respectively. A detailed pictorial representation of the above procedure for analogue modulation recognition is shown in Fig. 1 in the form of a flowchart.

3. Computer simulations

Software generation of test signals provides a larger flexibility in selecting parameters and in adjusting specific values of the SNR, while the hardware generation of test signals lack the same degree of flexibility, it provides larger degree of realism. In the primary stage of performance evaluation of any system or algorithm, computer simulations and software generation of test signals are essential.

3.1. Analogue modulated signals simulation

The carrier frequency f_c and the sampling rate f_s were, respectively, chosen to be 150 and 1200 kHz. The procedure used for generation of a non-intelligible speech signals of length 1.7066 ms comprises the following steps:

(1) Generation of a sequence $\{n(i)\}$ of N_s ($= 2048$) independent values from a zero-mean uniformly distributed random number generator.

(2) Calculation of another sequence $\{x_m(i)\}$ according to a difference equation representing a first-order auto-regressive model given by

$$x_m(i) = \rho x_m(i-1) + n(i), \quad x_m(0) = 0, \tag{15}$$

where the coefficient ρ was chosen such that the -3 dB spectrum bandwidth of the simulated auto-regressive signal was equal to 4 kHz. It was found that $\rho = 0.979274$ at $f_s = 1200$ kHz.

(3) The N_s-point of the sequence $\{x_m(i)\}$ is passed through a low pass filter with cut-off frequency equal to 8 kHz.

AM, DSB, FM and combined AM–FM signals have been derived from a general expression [14]

$$s(t) = (A + m x_1(t)) \cos\left[2\pi f_c t + K_f \int_{-\infty}^{t} x_2(\tau)\,d\tau\right], \tag{16}$$

where $x_1(t)$ and $x_2(t)$ are two modulating signals (simulated speech signals), m is a coefficient determined by the desired amplitude modulation depth (17), and K_f is a frequency deviation coefficient (18).

$$Q = \frac{m\{[x_1(t)]_{\max} - [x_1(t)]_{\min}\}}{2A + m\{[x_1(t)]_{\max} + [x_1(t)]_{\min}\}}, \tag{17}$$

where Q is the desired amplitude modulation depth.

$$K_f = \frac{2\pi f_x D}{[x_2(t)]_{\max}}, \tag{18}$$

where D is the desired frequency modulation index, and f_x is the maximum frequency in the spectrum of the modulating signal. Generally, the appearance of the sidebands components (spectral lines) in the FM modulated signal is related to the modulation index D. If the modulating frequency f_x is changed, for a given modulation index D, only the spacing of the spectral lines is changed, while the relative amplitudes remain unchanged. The integral in Eq. (6) was numerically approximated using the trapezoidal rule. Thus, by a suitable selection of the parameters A, m and K_f in Eq. (16), any one of the above mentioned types can be generated.

For SSB (i.e. LSB or USB) signals were generated according to the expressions in [14]. VSB signals were derived from AM signals using a VSB filter [14]. The magnitude response of the VSB filter used in the simulation is given by

$$|H_{\mathrm{VSB}}(f)| = \begin{cases} (1/2\alpha)\,[f - (f_c - \alpha)] & \text{if } f_c + \alpha \geqslant f \geqslant f_c - \alpha, \\ 1 & \text{if } f_c + f_x \geqslant f \geqslant f_c + \alpha, \\ 0 & \text{otherwise,} \end{cases} \tag{19}$$

where f_x is the maximum frequency of the simulated speech signal, and α is chosen such that

$$\frac{2\alpha}{f_c} \geqslant 0.01. \tag{20}$$

In our simulation $\alpha = 2$ kHz at $f_c = 150$ kHz.

3.2. Band limiting of simulated modulated signals

As communication transmitters have finite transmission bandwidths, the transmitted signal is band-limited. Therefore, the simulated modulated signals were band-limited to make them represent more realistic test signals for the proposed global procedure for modulation recognition. The band limitation of the simulated modulated signals was carried out in accordance with the usual implementation in practice (band limitation on the modulating signal). Finally, the theoretical expressions and the simulated values of the bandwidths for the simulated analogue modulated signals are presented in Table 1.

Usually in practice, the bandwidth of any intercepting receiver is slightly larger than signal bandwidth. So, in order to increase the degree of realism, the simulated additive Gaussian noise was band-limited to a band-width equal to 1.2 times the simulated modulated signal bandwidth.

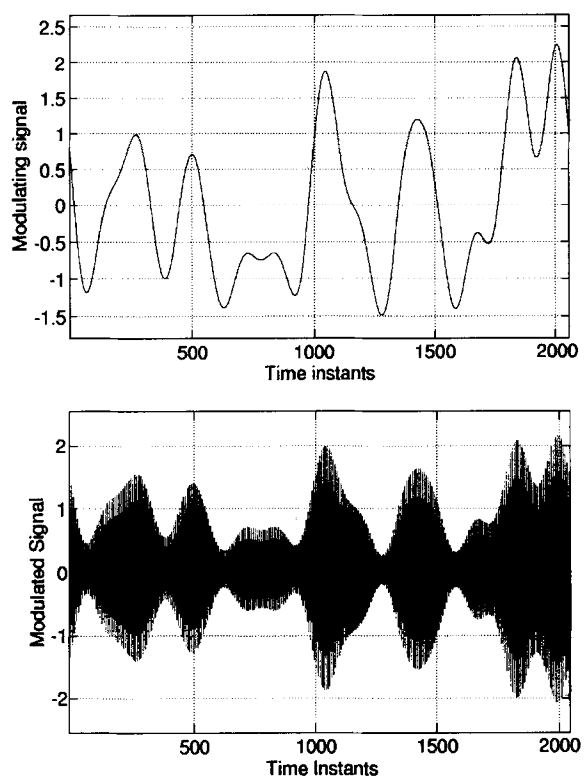
Typical examples of the modulating signal, modulated signal, and all the aforementioned key features for each type of modulation based on 1.7066 ms duration signals (equivalent to 2048 samples) are shown in Figs. 2–8.

Table 1
Bandwidths of analogue modulated signals

Modulation type	Modulated signal bandwidth (theoretical expression)	Modulated signal bandwidth (simulated values (kHz))
AM, DSB, ISB	$2f_x$	16
SSB	f_x	8
VSB	$f_x + \alpha$	10
FM	$2(D+1)f_x$	96 if $D = 5$
		176 if $D = 10$
Combined	$2(D+2)f_x$	112 if $D = 5$
		192 if $D = 10$

4. Performance evaluations

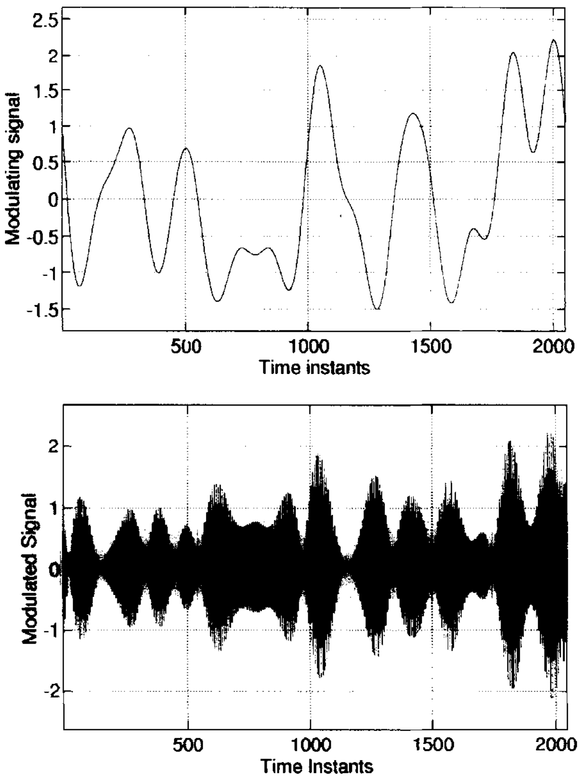
The results of the performance evaluation of the proposed procedure for analogue modulation recognition, using the complex envelope and derived from 400 realizations for each type of modulation, with $a_t = 1$, are summarized in Tables 2 and 3 for two values of SNRs (10 and 20 dB). Twelve modulation types have been simulated and sample results



Values of the key features for AM ($Q = 80\%$) signal

Key feature	Corresponding value at ∞ SNR
γ_{max}	122370
σ_{ap}	0.158
σ_{dp}	0.158
P_L, P, P_U	130630, 0.000, 130630

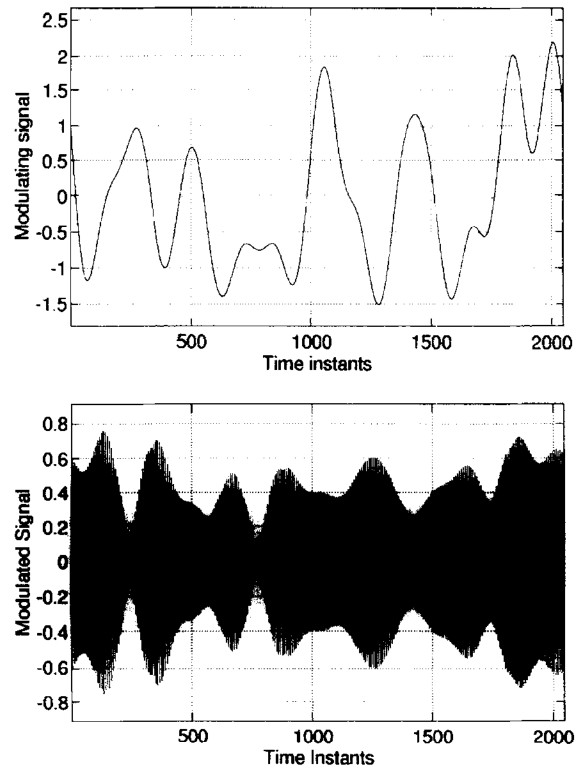
Fig. 2. Amplitude modulation.



Values of the key features for DSB signal

Key feature	Corresponding value at ∞ SNR
γ_{max}	94619
σ_{ap}	0.301
σ_{dp}	1.446
P_L, P, P_U	505500, 0.000, 505500

Fig. 3. DSB modulation.



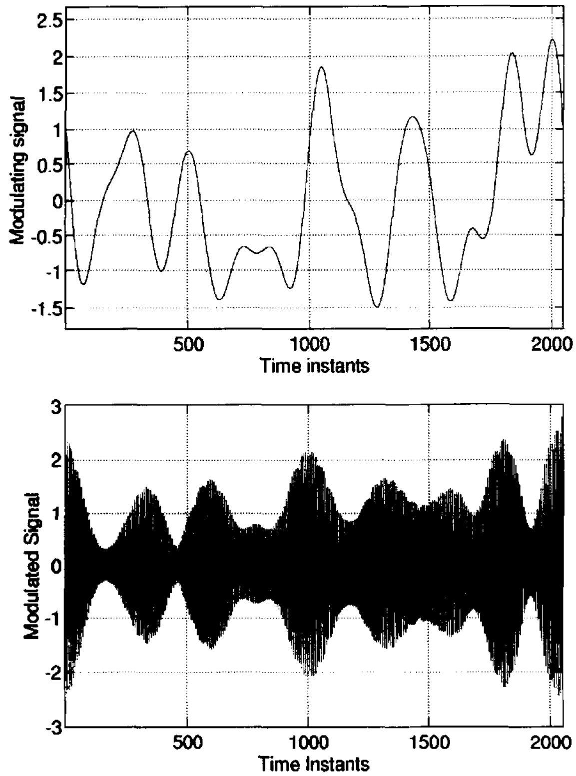
Values of the key features for VSB signal

Key feature	Corresponding value at ∞ SNR
γ_{max}	51154
σ_{ap}	0.235
σ_{dp}	0.235
P_L, P, P_U	2496.8, 0.875, 37577

Fig. 4. VSB modulation.

at two different SNR corresponding to 10 and 20 dB are presented in Tables 2 and 3, respectively. Each of the modulation types at each SNR was simulated 400 times. The values for $t_{\gamma_{max}}$, $t_{\sigma_{ap}}$, $t_{\sigma_{dp}}$ and t_p have been chosen as 12 000, $\pi/7$, $2\pi/7$ and 0.5, respectively. Consider Table 2 for example, it can be observed that 12 modulation types have been correctly classified with more than 90% success rate. Excluding VSB, LSB and USB, the success rate is $\geqslant 98\%$ at 10 dB. Indeed five of these 12 modulation types have been successfully classified every time (100% success rate). The results in Table 3 corresponds to the SNR of 20 dB. It is clear that the success rate for correct recognition has increased with increasing SNR, and now eight of the 12 modulation types have been classified with 100% success rate. Excluding LSB and USB, the success rate is $\geqslant 99\%$.

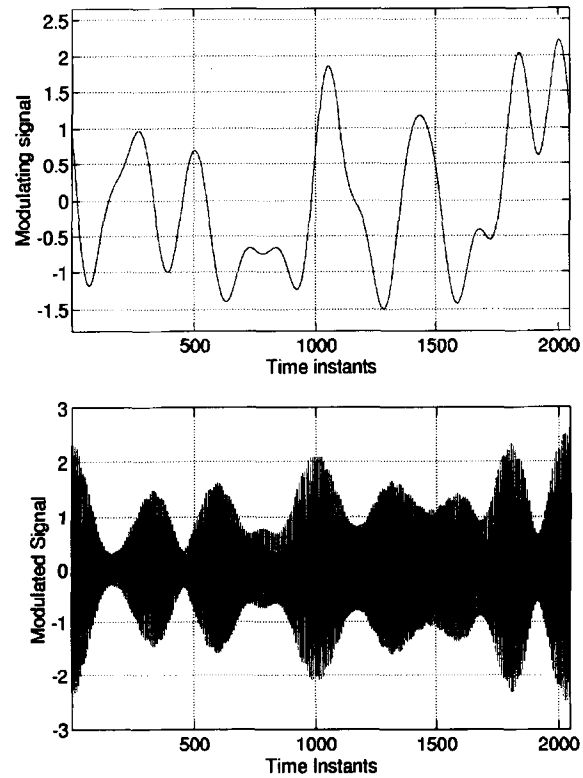
Finally, as it is important to know how long the procedure (9)–(14) takes as well as the complexity of this procedure, both the processing time and the computational power using the MATLAB software on the SUN SPARC stations **2** and **10** are measured. The values of the processing time (measured in seconds) and the computational power (measured in Megaflops) measured on the two SPARC



Values of the key features for LSB signal

Key feature	Corresponding value at ∞ SNR
γ_{max}	90824
σ_{ap}	1.448
σ_{dp}	2.884
P_L, P, P_U	2022000, 1.000, 0.000

Fig. 5. LSB modulation.



Values of the key features for USB signal

Key feature	Corresponding value at ∞ SNR
γ_{max}	90824
σ_{ap}	1.088
σ_{dp}	2.001
P_L, P, P_U	0.000, 1,000, 2022000

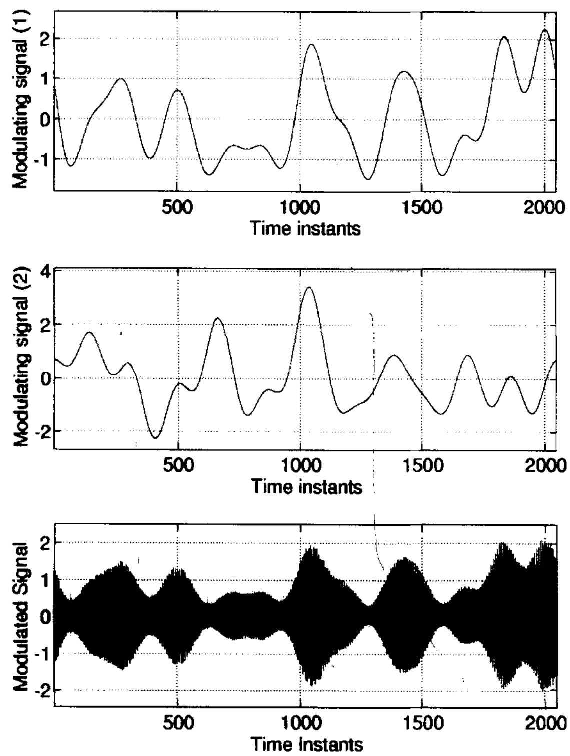
Fig. 6. USB modulation.

stations are shown in Table 4. The processing time and the number of Megaflops, required to take a decision about the modulation type, correspond to only one signal frame in each case.

5. Concluding remarks

The aim of this paper has been to recognize automatically the types of analogue modulations in communication signals. The current approach has been to carry out this task using the decision theoretic approach. A number of parameters (key features) is proposed to fulfil the requirement of this algorithm.

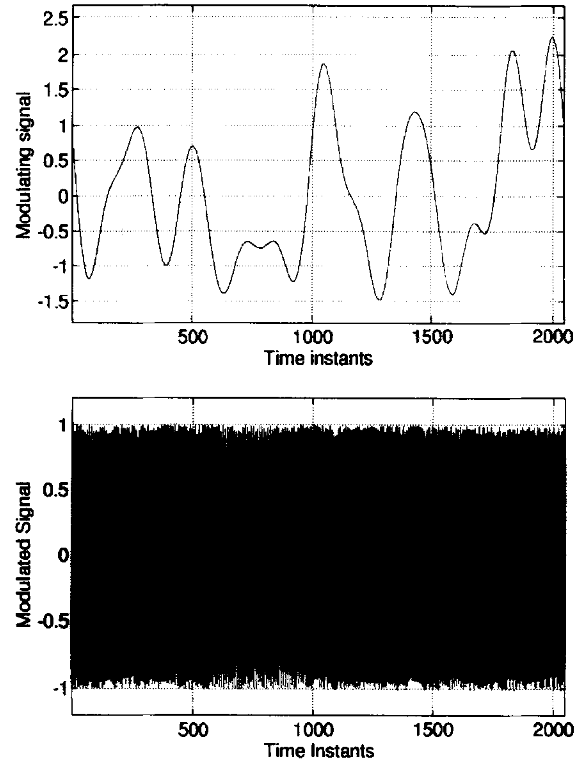
Extensive simulations of 12 analogue modulation signal types have been carried out at different SNR. Sample results have been presented at the SNR of 10 and 20 dB only. It is found that the threshold SNR for correct modulation recognition is about 10 dB, which is an improvement in the SNR threshold. Also presented are measured processing times and required computational power of the proposed algorithm for different types of modulations considered in this paper. Currently the work is under way in implementing and testing



Values of the key features for combined signal

Key feature	Corresponding value at ∞ SNR
γ_{max}	122370
σ_{ap}	3.682
σ_{dp}	5.997
P_L, P, P_U	600500, 0.078, 701680

Fig. 7. Combined modulation.



Values of the key features for FM ($D = 5$) signal

Key feature	Corresponding value at ∞ SNR
γ_{max}	0.000
σ_{ap}	4.007
σ_{dp}	6.525
P_L, P, P_U	564340, 0.083, 477780

Fig. 8. Frequency modulation.

a number of ideas on digitally modulated signals recognition.

Appendix A. Analytic expressions of the instantaneous amplitude and phase of analogue modulated single-tone signals

For AM signals, $s(t)$ takes the form

$$s(t) = [1 + Q\cos(2\pi f_x t)]\cos(2\pi f_c t), \tag{A.1}$$

where Q is the modulation depth, $0 < Q < 100\%$, f_x is the modulating frequency, and f_c is the carrier frequency. By straightforward analysis, the analytic signal, $z(t)$, is given by

$$z(t) = [1 + Q\cos(2\pi f_x t)]\cos(2\pi f_c t) + \mathrm{j}[1 + Q\cos(2\pi f_x t)]\sin(2\pi f_c t). \tag{A.2}$$

Hence, the instantaneous amplitude, $A(t)$, can be expressed as

$$A(t) = |1 + Q\cos(2\pi f_x t)|, \tag{A.3}$$

and the instantaneous phase, $\phi(t)$, is given by

$$\phi(t) = 2\pi f_c t. \tag{A.4}$$

Table 2
Confusion matrix for the analogue modulation recognition procedure (based on 400 realization) at SNR = 10 dB

	Deduced modulation type						
Simulated modulation type	AM	DSB	VSB	LSB	USB	Combined	FM
AM ($Q = 60\%$)	100%	—	—	—	—	—	—
AM ($Q = 80\%$)	100%	—	—	—	—	—	—
DSB	—	99%	—	—	—	1%	—
VSB	—	—	92.5%	—	—	7.5%	—
LSB	10%	—	—	90%	—	—	—
USB	9%	—	—	—	91%	—	—
Combined ($Q = 60\%$, $D = 5$)	—	—	—	—	—	100%	—
Combined ($Q = 60\%$, $D = 10$)	—	—	—	—	—	99.75%	0.25%
Combined ($Q = 80\%$, $D = 5$)	—	—	—	—	—	100%	—
Combined ($Q = 80\%$, $D = 10$)	—	—	—	—	—	100%	—
FM ($D = 5$)	—	—	—	—	—	1.25%	98.75%
FM ($D = 10$)	—	—	—	—	—	2%	98%

Table 3
Confusion matrix for analogue modulation recognition procedure (based on 400 realization) at SNR = 20 dB

	Deduced modulation type						
Simulated modulation type	AM	DSB	VSB	LSB	USB	Combined	FM
AM ($Q = 60\%$)	100%	—	—	—	—	—	—
AM ($Q = 80\%$)	100%	—	—	—	—	—	—
DSB	—	100%	—	—	—	—	—
VSB	—	—	100%	—	—	—	—
LSB	7.75%	—	—	92.25%	—	—	—
USB	6.5%	—	—	—	93.5%	—	—
Combined ($Q = 60\%$, $D = 5$)	—	—	—	—	—	100%	—
Combined ($Q = 60\%$, $D = 10$)	—	—	—	—	—	100%	—
Combined ($Q = 80\%$, $D = 5$)	—	—	—	—	—	100%	—
Combined ($Q = 80\%$, $D = 10$)	—	—	—	—	—	100%	—
FM ($D = 5$)	—	—	—	—	—	1%	99%
FM ($D = 10$)	—	—	—	—	—	1%	99%

For DSB signals, $s(t)$ takes the form

$$s(t) = \cos(2\pi f_x t)\cos(2\pi f_c t). \tag{A.5}$$

Similarly, the analytic signal is given by

$$z(t) = \cos(2\pi f_x t)\cos(2\pi f_c t) + \mathrm{j}\cos(2\pi f_x t)\sin(2\pi f_c t). \tag{A.6}$$

So, the instantaneous amplitude and phase are given by (A.7) and (A.8), respectively,

$$A(t) = |\cos(2\pi f_x t)|, \tag{A.7}$$

and

$$\phi(t) = \begin{cases} 2\pi f_c t & \text{if } \cos(2\pi f_x t) > 0, \\ 2\pi f_c t + \pi & \text{if } \cos(2\pi f_x t) < 0. \end{cases} \tag{A.8}$$

For SSB signal, $s(t)$ is expressed as

$$s(t) = x(t)\cos(2\pi f_c t) \mp y(t)\sin(2\pi f_c t), \tag{A.9}$$

where

$$x(t) = \sum_{i=1}^{N} x_i \cos(2\pi f_i t + \psi_i), \quad f_N < f_x, \tag{A.10}$$

Table 4
Measured processing times (T1 for SPARC 2 and T2 for SPARC 10) and computational power of the proposed algorithm

Modulation type	Number of Megaflops	T1	T2
AM ($Q = 60\%$)	0.49	19.5	9.7
AM ($Q = 80\%$)	0.49	19.5	9.7
DSB	0.41	19.5	9.7
VSB	0.50	19.8	9.8
LSB	0.57	19.1	9.5
USB	0.57	19.0	9.5
Combined ($Q = 60\%$, $D = 5$)	0.49	18.8	9.4
Combined ($Q = 80\%$, $D = 5$)	0.49	19.0	9.4
Combined ($Q = 60\%$, $D = 10$)	0.49	18.8	9.4
Combined ($Q = 80\%$, $D = 10$)	0.49	18.8	9.4
FM ($D = 5$)	0.32	0.8	0.4
FM ($D = 10$)	0.32	0.8	0.4

and $y(t)$ is the Hilbert transform of $x(t)$ and it is expressed as

$$y(t) = \sum_{i=1}^{N} x_i \sin(2\pi f_i t + \psi_i). \tag{A.11}$$

Thus, the SSB signal can be expanded as

$$s(t) = \sum_{i=1}^{N} x_i \cos(2\pi(f_c \pm f_i)t + \psi_i). \tag{A.12}$$

Also, the Hilbert transform $y_{SSB}(t)$ of $s(t)$ can be written as

$$y_{SSB}(t) = \sum_{i=1}^{N} x_i \sin[2\pi(f_c \pm f_i)t + \psi_i]. \tag{A.13}$$

The instantaneous amplitude and phase are given by (A.14) and (A.15), respectively,

$$A(t) = \left(\sum_{i=1}^{N} x_i^2 + 2 \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \cos[2\pi(f_i - f_j)t] \right)^{1/2} \tag{A.14}$$

and

$$\phi(t) = \tan^{-1} \frac{\sum_{i=1}^{N} x_i \sin[2\pi(f_c + f_i)t + \psi_i]}{\sum_{i=1}^{N} x_i \cos[2\pi(f_c + f_i)t + \psi_i]}. \tag{A.15}$$

It is clear that both $A(t)$ and $\phi(t)$ are time varying functions.

Finally, for FM signal, $s(t)$ takes the form

$$s(t) = \cos(2\pi f_c t + \beta \cos(2\pi f_x t)), \tag{A.16}$$

where β is the desired frequency modulation index. Note that β and D are the same. In this appendix, we use the notation β for the case of single-tone modulating signal. Also, the FM signal defined by Eq. (A.16) can be expanded as

$$s(t) = \sum_{-\infty}^{\infty} J_n(\beta) \cos(2\pi f_c t + 2\pi f_x nt), \tag{A.17}$$

where $J_n(\beta)$ is the nth order Bessel function. Its Hilbert transform, $y(t)$, is given by

$$y(t) = \sum_{-\infty}^{\infty} J_n(\beta) \sin|2\pi f_c t + 2\pi n f_x t|. \tag{A.18}$$

The instantaneous amplitude and phase are given by (A.19) and (A.20), respectively.

$$A(t) = 1, \tag{A.19}$$

and

$$\phi(t) = \tan^{-1} \frac{\sum_{-\infty}^{\infty} J_n(\beta) \sin|2\pi(f_c + nf_x)t|}{\sum_{-\infty}^{\infty} J_n(\beta) \cos[2\pi(f_c + nf_x)t]}. \tag{A.20}$$

Thus, $A(t)$ is constant and $\phi(t)$ is time varying.

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